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ON ECCENTRIC CONNECTIVITY INDEX OF $S[F_n], S[S_k], S[D_n]$ AND $S[L[F_n]], S[L[S_k]], S[L[D_n]]$ GRAPHS

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ABSTRACT

The eccentric connectivity index based on degree and eccentricity of the vertices of a graph is a widely used graph invariant in mathematics. In this paper we present the explicit generalized expression for the eccentric connectivity index of the subdivision graph of some special graphs such as Friendship graph, Star graph, Dipole graph and their line graphs.

Keywords: *Dipole graph, Eccentric connectivity index, Friendship graph, Star graph, Subdivision graph.*

1. INTRODUCTION

A topological index, known as eccentric connectivity index, studies the molecular structure of graphs [4]. For basic definitions and notations, refer to [1]. For any simple connected graph with n vertices and m edges, the distance between the vertices v_i and v_j is equal to the number of edges in the shortest path connecting v_i and v_j. For a given vertex v_i, its eccentricity $ecc(v_i)$ is the largest distance from v_i to any other vertices of G[2]. The radius and diameter of the graph are respectively the smallest and the largest eccentricity of all the vertices of G. The average eccentricity of a graph G is defined as $\frac{1}{n}\sum_{i=1}^{n} ecc(v_i)$. The eccentric connectivity index of any graph G is calculated using $\xi^{(c)}(G) = \sum \deg(v).ecc(v)$, where $v \in V(G)$. For a connected graph G the subdivision graph $S(G)$ is obtained by inserting an additional vertex in each edge of G [5].Equivalently, each edge of G is replaced by a path of length 2. For the definitions of Dipole, Friendship and Star graphs refer to [3].

We know that the eccentric connectivity index of the subdivision graph of the complete graph $S[K_n]$ is $\xi^{(c)} S[K_n] = (n)(n-1)(3) + \left(\frac{n(n-1)}{2}\right)(2)(4) = 3n^2 - 3n + 4n^2 - 4n = 7n(n-1)$ for $n \geq 4$.

2. ECCENTRIC CONNECTIVITY INDEX OF SOME SUBDIVISION GRAPHS

The subdivision graphs $S[F_2]$, $S[F_3]$ and $S[F_4]$ are illustrated in the following figure.

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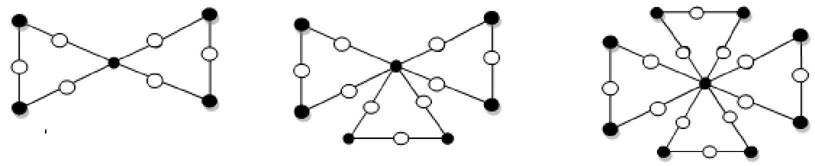


$S[F_2]$ $S[F_3]$ $S[F_4]$

Figure: 2.1

Proposition 2.1: The eccentric connectivity index of the subdivision graph of Friendship graph F_n is

$$\xi^{(c)} S[F_n] = 54n \quad \text{for all } n \geq 2$$

Proof: The cardinality of the vertex set of $S[F_n]$ is $5n+1$ in which $2n+1$ are original vertices and the remaining 3n are subdivision vertices.

In original vertices, 2n vertices are of degree 2 and eccentricity 5 and one center vertex is of degree 2n and eccentricity 3.

In subdivision vertices, 2n vertices are of degree 2 and eccentricity 4 and n vertices are of degree 2 and eccentricity 6.

For $n \geq 2$, the eccentric connectivity index of $S[F_n]$ is computed as,

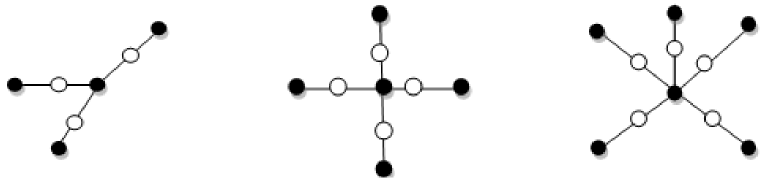
$$\xi^{(c)} S[F_n] = \{(2n)(2)(5) + (1)(2n)(3)\} + \{(2n)(2)(4) + (n)(2)(6)\} = 54n.$$

$$\xi^{(c)} S[F_n] = 54n$$

Remark 2.2: The eccentric connectivity index of the subdivision graph of Friendship graph F_1 is

$$\xi^{(c)} S[F_1] = 36$$

Subdivision graph of star graphs $S[S_3], S[S_4], S[S_5]$ are illustrated in the following figure.



$S[S_3]$ $S[S_4]$ $S[S_5]$

Figure: 2.2

Proposition 2.3: The eccentric connectivity index of the subdivision graph of Star graph S_k is

$$\xi^{(c)} S[S_k] = 12k, \text{ for all } k \geq 3$$

Proof: The cardinality of the vertex set of $S[S_k]$ is $2k+1$ in which k+1 are original vertices and the remaining k are subdivision vertices.

In original vertices, k vertices are of degree 1 and eccentricity 4 and the remaining one center vertex is of degree k and eccentricity 2.

The remaining k subdivision vertices are of degree 2 and eccentricity 3.

For $k \geq 3$, the eccentric connectivity index of $S[S_k]$ is computed as,

$$\xi^{(c)} S[S_k] = (k)(1)(4) + (1)(k)(2) + (k)(2)(3) = 12k$$

Subdivision graph of dipole graphs $S[D_2], S[D_3], S[D_4]$ are illustrated in the following figure

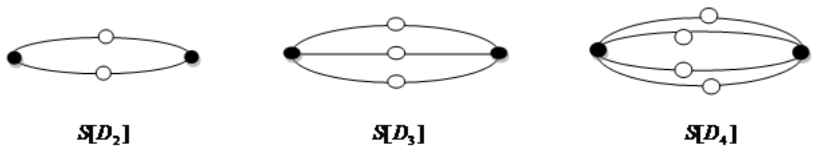


Figure: 2.3

Proposition 2.4: The eccentric connectivity index of subdivision graph of Dipole graph D_n is

$$\xi^{(c)} S[D_n] = 8n, \text{ for all } n \geq 2$$

Proof: The cardinality of the vertex set $S[D_n]$ is $n+2$ in which 2 are the original vertices and the remaining n are the subdivision vertices.

The 2 original vertices are of degree n and eccentricity 2.

The n subdivision vertices are of degree 2 and eccentricity 2.

For $n \geq 2$, the eccentric connectivity index of $S[D_n]$ is computed as,

$$\xi^{(c)} S[D_n] = (2)(n)(2) + (n)(2)(2)$$
$$= 8n$$

3. ECCENTRIC CONNECTIVITY INDEX OF SUBDIVISION GRAPHS OF SOME LINE GRAPHS

The graphs of $S[L[F_2]]$, $S[L[F_3]]$ and $S[L[F_4]]$ are illustrated in the following figure

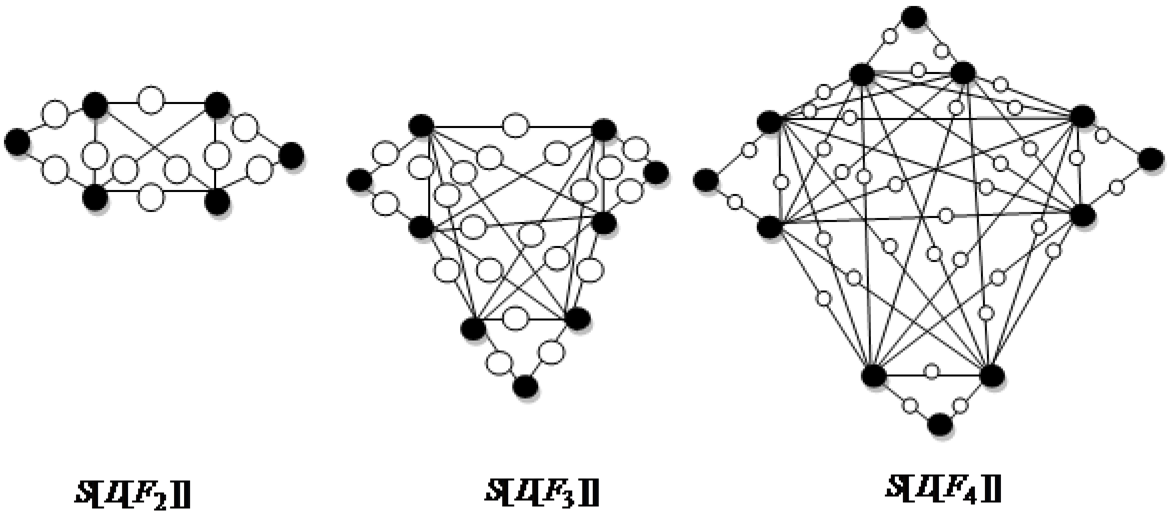


Figure: 3.1

Proposition 3.1: The eccentric connectivity index of the subdivision graph of line graph of friendship graph is

$$\xi^{(c)} S[L[F_n]] = 2n\ (18n+11) \quad \text{for all } n \geq 3$$

Proof: The cardinality of the vertex set of $S[L[F_n]]$ is $2n(n+2)$.

$S[L[F_n]]$ contains 3n original vertices and n(2n+1) subdivision vertices of $L[F_n]$.

Among the original vertices, n vertices are of degree 2 &eccentricity 6 and 2n vertices are of degree 2n & eccentricity 4.
n(2n+1) subdivision vertices are of degree 2 and eccentricity 5.

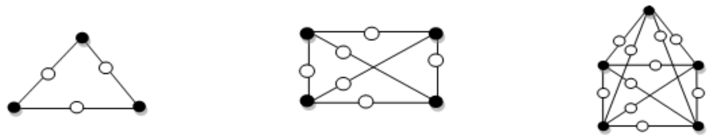
For $n \geq 3$, the eccentric connectivity index of $S[L[F_n]]$ is computed as,

$$\xi^{(c)} S[L[F_n]] = \{(n)(2)(6) + (2n)(2n)(4)\} + \{n(2n+1)(2)(5)\}$$

$$\xi^{(c)} S[L[F_n]] = 2n(18n+11) \quad .$$

Remark 3.2: The eccentric connectivity index of the subdivision graph of line graph of Friendship graphs F_1 and F_2 are $\xi^{(c)} S[L[F_1]] = 36$ and $\xi^{(c)} S[L[F_2]] = 180.$

Subdivision graph of line graph of star graphs $S[L[S_3]], S[L[S_4]], S[L[S_5]]$ are illustrated in the following figure



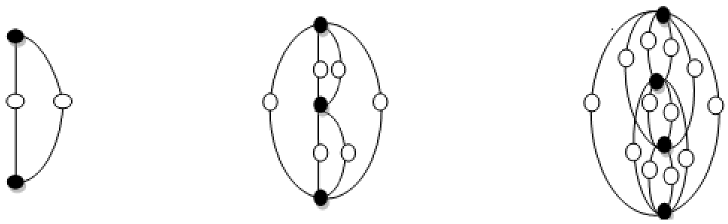
$S[L[S_3]]$ $S[L[S_4]]$ $S[L[S_5]$

Figure: 3.2

Remark 3.3: Subdivision graph of $L[S_k]$ turns out to be the subdivision graph of a complete graph. Therefore, the eccentric connectivity index of $S[L[S_k]]$ is the same as the eccentric connectivity index of $S[K_k]$. That is

$$\xi^{(c)} S[L[S_k]] = \xi^{(c)} S[K_k] = \begin{Bmatrix} 7\mathrm{k}(\mathrm{k}-1) & \text{where } \mathrm{k} \geq 4 \\ 36 & \text{where } \mathrm{k} = 3 \end{Bmatrix}$$

Subdivision graph of line graph of dipole graphs $S[L[D_2]], S[L[D_3]], S[L[D_4]]$ are illustrated in the following figure



$S[L[D_2]]$ $S[L[D_3]]$ $S[L[D_4]]$

Figure: 3.3

Proposition 3.4: The eccentric connectivity index of subdivision graph of line graph of dipole graph is

$$\xi^{(c)} S[L[D_n]] = 2n(n^2 + 2n - 3), \text{ for all } n \geq 3$$

Proof: The cardinality of the vertex set of $S[L[D_n]]$ is n^2 in which n are original vertices and the remaining n^2-n are subdivision vertices of $[L[D_n]]$.

All the n original vertices are of degree 2n-2 and eccentricity 3.

The remaining n^2-n subdivision vertices are of degree 2 and eccentricity 3.

For $n \geq 3$, the eccentric connectivity index of $S[L[D_n]]$ is computed as,

$$\xi^{(c)} S[L[D_n]] = \{(n)(2n-2)(3)\} + \{(n^2 - n)(2)(n)\}$$

$$= 2n(n^2 + 2n - 3)$$

Remark 3.5: The eccentric connectivity index of subdivision graph of line graph of Dipole graph D_2 is $\xi^{(c)} S[L[D_2]] = 16.$

CONCLUSION

The eccentric connectivity index of $S[F_n], S[S_k], S[D_n]$ and $S[L[F_n]], S[L[S_k]], S[L[D_n]]$ graphs are computed in this article. Further, the index could be computed for their subdivision-related graphs.

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