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ON FUZZY SOFT SETS

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ABSTRACT

The soft set is one of the recent topics developed for dealing with the uncertainties present in most of our real life situations. The parametrization tool of soft set theory enhance the flexibility of its applications. In this paper, we redefine fuzzy soft sets and some results are established.

Keywords: *Soft sets, Fuzzy soft sets, and Product of fuzzy soft sets.*

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1. INTRODUCTION

Molodtsov [1] pointed out that the existing theories, viz., theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of vague set, theory of interval mathematics, theory of rough sets, can be considered as mathematical tools for dealing with uncertainties but all these theories have their own limitations. The reason for this is most possibly the inadequacy of the parameterization tool of the theories. So he developed a new mathematical theory called "Soft Set" for dealing with uncertainties and soft set is free from the above limitations. The absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable in practice. Later other authors Maji et al.[2,3] have further studied the theory of softs and also introduced the concept of fuzzy soft sets. Cağman et al. [1] redefined the operations of Molodtsov's soft sets to make them more functional for improving several new results. In this paper, we have redefined the operations of Maji et al. [2] fuzzy soft set and studied some results.

2. PRELIMINARIES

Throughout this work, U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U and $A \subset E$.

Definition: 2.1[1]
A soft set (f_A, E) or F_A on the universe U is defined by the set of ordered pairs
$(f_A, E) = F_A = \{(e, f_A(e)): e \in E, f_A(e) \in P(U)\}$, where $f_A: E \to P(U)$ such that $f_A(e) = \phi$ If $e \notin A$.

Here, f_A is called an approximate function of the soft set F_A .The set $f_A(e)$ is called e-approximate value set or e-approximate set which consists of related objects of the parameter $e \in E$. Let $S(U)$ be the set of all soft sets over U.

Example: 2.1
Let $U = \{c_1, c_2, c_3\}$ be the set of three cars and E = {costly (e_1), metallic colour (e_2) cheap (e_3)} be the set of parameters,

where $A = \{e_1, e_2\} \subset E$. Then $f_A(e_1) = \{c_1, c_2, c_3\}, f_A(e_2) = \{c_1, c_3\}\}$,

then we write a crisp soft set $(f_A, E) = F_A = \{(e_1, \{c_1, c_2, c_3\}), (e_2, \{c_1, c_3)\}$over U which describes the " attractiveness of the cars" which Mr. S (say) is going to buy.

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Definition: 2.2[1]

Let $F_A \in S(U)$. If $f_A(x) = \phi$ for all $x \in E$, then F_A is called a empty soft set, denoted by F_ϕ.

Definition: 2.3[1]

Let $F_A \in S(U)$. If $f_A(x) = U$ for all $x \in A$, then F_A is called A-universal soft set, denoted by $F_{\tilde{A}}$. If A = E, then the A-universal soft set is called universal soft set denoted by $F_{\tilde{E}}$.

Definition: 2.4[1]

Let $F_A, F_B \in S(U)$. Then, F_A is a soft subset of F_B, denoted by $F_A \tilde{\subseteq} F_B$, if $f_A(x) \subseteq f_B(x)$ for all $x \in E$.

Definition: 2.5[1]

Let $F_A, F_B \in S(U)$. Then, F_A and F_B, are soft equal, denoted by $F_A = F_B$ if and only if $f_A(x) = f_B(x)$ for all $x \in E$.

Definition: 2.6[1]

Let $F_A \in S(U)$. Then the complement F_A, denoted by $F_A^{\tilde{c}}$, is a soft set defined by the approximate function $f_{A^{\tilde{c}}}(x) = f_A^C(x)$ for all $x \in E$, where $f_A^c(x)$ is the complement of the set $f_A(x)$, that is, $f_A^c(x) = U \setminus f_A(x)$ for all $x \in E$.

Definition: 2.7[1]

Let $F_A, F_B \in S(U)$. Then, union of F_A and F_B, denoted by $F_A \tilde{\cup} F_B$, is a soft set defined by the approximate function $f_{A \tilde{\cup} B}(x) = f_A(x) \cup f_B(x)$ for all $x \in E$.

Definition: 2.8[1]

Let $F_A, F_B \in S(U)$. Then, intersection of F_A and F_B, denoted by $F_A \tilde{\cap} F_B$, is a soft set defined by the approximate function $f_{A \tilde{\cap} B}(x) = f_A(x) \cap f_B(x)$ for all $x \in E$.

Definition: 2.9[1]

Let $F_A, F_B \in S(U)$. Then, difference of two soft sets F_A and F_B, denoted by $F_A \setminus F_B$, is a soft set defined by the approximate function $f_{A \setminus B}(x) = f_A(x) \setminus f_B(x)$, for all $x \in E$.

3. FUZZY SOFT SETS

In this section, we redefine what are called fuzzy soft sets and give various results on fuzzy soft set theory as extension of soft set theory redefined by Cagman, N. et al. [1].

Throughout this work, U refers to an initial universe, E is a set of parameters, $\mathcal{F}(U)$ denotes the set of all fuzzy sets of U and $A \subseteq E$.

Definition: 3.1

A fuzzy soft set (f_A, E) or F_A on the universe U is defined by the set of ordered pairs $(f_A, E) = F_A = \{(e, f_A(e)) : e \in E, f_A(e) \in \mathcal{F}(U)\}$, where $f_A: E \to \mathcal{F}(U)$ such that $f_A(e) =$ null fuzzy set of U, if $e \notin A$.

Example: 3.1

Let $U = \{c_1, c_2, c_3\}$ be the set of three cars and $E = \{\text{costly}(e_1)$, metallic colour (e_2) getup$(e_3)\}$ be the set of parameters,

where $A = \{e_1, e_2\} \subset E$ and $f_A(e_1) = \{c_1/.6, c_2/.4, c_3/.3\}$, $f_A(e_2) = \{c_1/.5, c_2/.7, c_3/.8\}$.

Then, $F_A = \{(e_1, f_A(e_1)), (e_2, f_A(e_2))\}$ is the fuzzy soft set over U describes the "attractiveness of the cars" which Mr. S (say) is going to buy.

Definition: 3.2

Let $F_A \in F(U)$. If $f_A(x) = \phi$, the null fuzzy set of U, for all $x \in E$, then F_A is called a null fuzzy soft set, denoted by F_ϕ

Definition: 3.3

Let $F_A, F_B \in F(U)$. Then, F_A is a soft subset of F_B, denoted by $F_A \tilde{\tilde{\subseteq}} F_B$ if $f_A(x)$ is a fuzzy subset of $f_B(x)$ for all $x \in E$.

Proposition: 3.1

If $F_A, F_B \in F(U)$ then

(i) $F_\phi \tilde{\tilde{\subseteq}} F_B$

(ii) $F_A \tilde{\tilde{\subseteq}} F_A$

(iii) $F_A \tilde{\tilde{\subseteq}} F_B$ and $F_B \tilde{\tilde{\subseteq}} F_C \Rightarrow F_A \subseteq F_C$

Proof: These results can be proved easily. For all $x \in E$

$(i)\, f_\phi(x) \subseteq f_A(x)$, since $\varnothing \subseteq f_A\ (x)$.

$(ii)\ f_A(x) \subseteq f_A(x)$, since $f_A(x) = f_A(x)$.

$(iii)\ f_A(x) \subseteq f_B(x)$ and $f_B(x) \subseteq f_C(x) \Rightarrow f_A(x) \subseteq f_C(x)$.

Definition: 3.4

Let $F_A, F_B \in F(U)$. Then, F_A and F_B are fuzzy soft equal, denoted by $F_A = F_B$, if and only if $f_A(x) = f_B(x)$ for all $x \in E$.

Proposition: 3.2

If $F_A, F_B, F_C \in F(U)$, then

(i) $F_A = F_B$ and $F_B = F_C \Leftrightarrow F_A = F_C$.

(ii) $F_A \tilde{\tilde{\subseteq}} f_B(x)$ and $F_B \tilde{\tilde{\subseteq}} F_A \Rightarrow F_A \subseteq F_B$

Proof: For all $x \in E$.

$(i)\ f_A(x) = f_B(x)$ and $f_B(x) = f_C(x) \Leftrightarrow f_A(x) = f_C(x)$.

$(ii)\, f_A(x) \subseteq f_B(x)$ and $f_B(x) \subseteq f_A(x) \Leftrightarrow f_A(x) = f_B(x)$.

Definition: 3.5

Let $F_A \in F(U)$. Then, complement of F_A, denoted by F_A^c is a fuzzy soft set defined by the approximate function $f_{A^c}(x) = f_A^C(x)$, for all $x \in E$.

where $f_A^C(x)$ is complement of the fuzzy set $f_A(x)$.

Definition: 3.6

Let $F_A, F_B \in F(U)$. Then, union of F_A and F_B, denoted by $F_A \tilde{\cup} F_B$, is a fuzzy soft set defined by the approximate function

$f_{A \tilde{\cup} B}(x) = \max\{f_A(x)\ ,\ f_B(x)\}$, for all $x \in E$.

Proposition: 3.3

If $F_A, F_B, F_C \in F(U)$, then

$(i)\ F_A \tilde{\bigcup} F_A = F_A\ , F_A \tilde{\bigcup} F_\phi = F_A.$

$(ii) F_A \tilde{\bigcup} F_B = F_B \tilde{\bigcup} F_A$

$(iii)\ (F_A \tilde{\bigcup} F_B) \tilde{\bigcup} F_C = F_A \tilde{\bigcup} (F_B \tilde{\bigcup} F_C).$

Definition: 3.7

Let $F_A, F_B \in F(U)$. Then, intersection of F_A *and* F_B, denoted by $F_A \tilde{\bigcap} F_B$. is a soft set defined by the aproximate function $f_{A\tilde{\bigcap}B}(x) = \min\{f_A(x), f_B(x)),$ for all $x \in E.$

Proposition: 3.4

If $F_A, F_{B,} F_C \in S(U)$. *then*

$(i)\ F_A \tilde{\bigcap} F_A = F_A.$

$(ii)\ F_A \tilde{\bigcap} F_\phi = F_\phi.$

$(iii)\ F_A \tilde{\bigcap} F_{\tilde{E}} = F_A.$

$(iv)\ F_A \tilde{\bigcap} F_A^C = F_\phi$

$(v)\ F_A \tilde{\bigcap} F_B = F_B \tilde{\bigcap} F_A.$

$(vi)(F_A \tilde{\bigcap} F_B) \tilde{\bigcap} F_C = F_A \tilde{\bigcap} (F_B \tilde{\bigcap} F_C).$

Proposition: 3.5

Let $F_A, F_B \in F(U)$. Then

(i) $\left(F_A \tilde{\tilde{\cup}} F_B \right)^C = F_A^C \tilde{\tilde{\cap}} F_B^C$

(ii) $\left(F_A \tilde{\tilde{\cap}} F_B \right)^C = F_A^C \tilde{\tilde{\cup}} F_B^C$

Proof: For all $x \in E.$

$$(i) f_{(A\tilde{\bigcup}B)^{\tilde{C}}}(x) = f^C_{A\tilde{\bigcup}B}(x)$$
$$= \max(f_A(x), f_B(x))^C$$
$$= \min((f_A(x))^C, f_B(x))^C)$$
$$= f_{\tilde{A}\tilde{\bigcap}B^{\tilde{C}}}(x).$$

(ii) Similar to (i).

Proposition: 3.6

If $F_A, F_B, F_C \in F(U)$, then

$(i) F_A \tilde{\bigcup} (\mathrm{F}_B \tilde{\bigcap} F_C) = (F_A \tilde{\bigcap} F_B) \tilde{\bigcap} (F_A \tilde{U} F_C).$

$(ii) F_A \tilde{\bigcap} (\mathrm{F}_B \tilde{\bigcup} F_C) = (F_A \tilde{\bigcap} F_B) \tilde{\bigcup} (F_A \tilde{\bigcap} F_C).$

Proof: For all $x \in E.$

$$(i) f_{A\tilde{\bigcup}(B\tilde{\bigcup}C)}(x) = \max(f_A(x), f_{B\tilde{\cap}C}(x))$$
$$= \max(f_A(x), \min(f_B(x), f_C(x)))$$
$$= \min(\max((f_A(x), f_B(x)), \max(f_A(x), f_C(x)))$$
$$= \min(f_{A\tilde{\bigcup}B}(x), f_{A\tilde{\bigcup}C}(x))$$
$$= f_{(A\tilde{\bigcup}B)\tilde{\bigcap}(A\tilde{\bigcup}C)}(x).$$

(ii) Similar to (i).

4. PRODUCT OF FUZZY SOFT SETS

Definition: 4.1
Let F_A , $F_B \in \mathcal{F}(U)$. Then, $\wedge$- product of two fuzzy soft sets F_A and F_B , denoted by $F_A \wedge F_B$, is a fuzzy soft set defined by the approximate function $f_{A\wedge B} : E\times E \to I^X$, $f_{A\wedge B}(x, y) = \min\{f_A(x), f_B(y)\}$.

Definition: 4.2
Let F_A , $F_B \in \mathcal{F}(U)$. Then, $\vee$- product of F_A and F_B , denoted by $F_A \vee F_B$, is a fuzzy soft set defined by the approximate function $f_{A\vee B} : E\times E \to I^X$, $f_{A\vee B}(x, y) = \max\{f_A(x), f_B(y)\}$ ..

Definition: 4.3
Let F_A , $F_B \in \mathcal{F}(U)$. Then, $\overline{\wedge}$- product of two fuzzy soft sets, denoted by $F_A \overline{\wedge} F_B$, is a fuzzy soft set defined by the approximate function $f_{A\overline{\wedge} B} : E\times E \to I^X$, $f_{A\overline{\wedge} B}(x, y) = \min\{f_A(x), f_B^c(y)\}$.

Definition: 4.4
Let F_A ,$F_B \in \mathcal{F}(U)$. Then, $\underline{\vee}$- product of two fuzzy soft sets, denoted by $F_A \underline{\vee} F_B$, is a fuzzy soft set defined by the approximate function $f_{A\underline{\vee} B} : E\times E \to I^X$, $f_{A\underline{\vee} B}(x, y) = \max\{f_A(x), f_B^c(y)\}$.

Example: 4.1
Let $U=\{h_1,h_2,h_3,h_4,h_5\}$ be universal set and $E=\{e_1,e_2,e_3,e_4\}$be the set of all parameters. Assume that $A= \{e_2, e_3, e_4\}$ and $B=\{e_1, e_3, e_4\}$ are two subsets of E.

Let F_A and F_B be two fuzzy soft sets.

where
$F_A= \{(e_2,\{h_1/.7,h_2/1,h_3/.8,h_4/.5,h_5/.9\}),(e_3,\{ h_1/.6,h_2/.8,h_3/.7,h_4/1,h_5/.9\}), (e_4,\{h_1/.4,h_2/.9,h_3/.4,h_4/.8,h_5/.6\})\}$
$F_B= \{(e_1,\{h_1/.6,h_2/.6,h_3/.8,h_4/1,h_5/.5\}),(e_3,\{ h_1/.7,h_2/9,h_3/.8,h_4/1,h_5/.9\}), (e_4,\{h_1/.8,h_2/.5,h_3/.7,h_4/.4,h_5/.5\})\}$,

Then $F_A \wedge F_B$ as follows

$F_A \wedge F_B$
$=\{((e_2,e_1),\{h_1/.6,h_2/.6,h_3/.8,h_4/.5,h_5/.5\}),((e_2,e_3),\{h_1/.7,h_2/.9,h_3/.8,h_4/.5,h_5/.9\}),((e_2,e_4),\{h_1/.7,h_2/.5,h_3/.7,h_4/.4,h_5/.5\}),$
$((e_3,e_1),\{h_1/.6,h_2/.6,h_3/.7,h_4/1,h_5/.5\}),((e_3,e_3),\{h_1/.6,h_2/.8,h_3/.7,h_4/1,h_5/.9\}),((e_3,e_4),\{h_1/.6,h_2/.5,h_3/.7,h_4/.4,h_5/.5\}),((e_4,e_1),$
$\{h_1/.4,h_2/.6,h_3/.4,h_4/.8,h_5/.5\}),((e_4,e_3),\{h_1/.4,h_2/.9,h_3/.4,h_4/.8,h_5/.6\}),((e_4,e_4),\{h_1/.4,h_2/.5,h_3/.4,h_4/.4,h_5/.5\})\}$.

Similarly, $F_A \vee F_B$, $F_A \overline{\wedge} F_B$ and $F_A \underline{\vee} F_B$ can be derived.

Theorem: 4.1
Let F_A ,$F_B \in \mathcal{F}(U)$. Then

$(i)\ (F_A \vee F_B)^{\tilde{c}} = F_A^{\tilde{c}} \wedge F_B^{\tilde{c}}$

$(ii)\ (F_A \wedge F_B)^{\tilde{c}} = F_A^{\tilde{c}} \vee F_B^{\tilde{c}}$

$(iii)\ (F_A \underline{\vee} F_B)^{\tilde{c}} = F_A^{\tilde{c}} \overline{\wedge} F_B^{\tilde{c}}$

$(iv)\ (F_A \overline{\wedge} F_B)^{\tilde{c}} = F_A^{\tilde{c}} \underline{\vee} F_B^{\tilde{c}}$

Proof: With the help of approximation functions, these proofs can be done. For all $x ,y \in E$

$$
\begin{aligned}
(i)\ f_{(A\wedge B)^{\tilde{c}}}(x, y) &= f^c_{(A\vee B)}(x, y) \\
&= (\max(f_A(x), f_B(y)))^c \\
&= \min(f_A^c(x), f_B^c(y)) \\
&= f^c_{(A^c\wedge B^c)}(x, y).
\end{aligned}
$$

5. CONCLUSION

Molodtsov introduced the concept of soft sets, which is one of the recent topics developed for dealing with the uncertainties present in most of our real life situations. The parametrization tool of soft set theory enhance the flexibility of its applications. In this paper, we redefine fuzzy soft sets considering the fact that the parameters(which are words or sentences) are mostly fuzzy hedges or fuzzy parameters and some results are established in continuation to the work of Çağman et al.[**1**].

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