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BINARY SEMI OPEN SETS IN BINARY TOPOLOGICAL SPACES

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ABSTRACT

***A** binary topology from a nonempty set X to a nonempty set Y is a binary structure satisfying certain axioms that are analogous to the axioms of topology. In this paper we introduce and study binary semi open sets.*

Keywords: *Binary semi open, binary interior, binary closure, binary subspace.*

1. INTRODUCTION

The authors [2] introduced the concept of binary topology and discussed some of its basic properties. Semi open sets in topological spaces are introduced by Norman Levine [3].

In this paper, we introduce binary semi open sets in binary topological spaces and their basic properties are studied. Section 2 deals with basic concepts. Binary semi open sets in binary topological spaces are discussed in section 3. Throughout the paper, $\wp(X)$ denotes the power set of X.

2. PRELIMINARIES

Let X and Y be any two nonempty sets. A binary topology [2] from X to Y is a binary structure $\mathcal{M} \subseteq \wp(X)\times\wp(Y)$ that satisfies the axioms namely (i) $(\varnothing,\varnothing)$ and $(X,Y)\in\mathcal{M}$, (ii) $(A_1\cap A_2, B_1\cap B_2)\in\mathcal{M}$ whenever $(A_1, B_1)\in\mathcal{M}$ and $(A_2, B_2)\in\mathcal{M}$, and (iii) If $\{(A_\alpha, B_\alpha): \alpha\in\Delta\}$ is a family of members of $\mathcal{M}$, then $\left(\bigcup_{\alpha\in\Delta} A_\alpha, \bigcup_{\alpha\in\Delta} B_\alpha\right)\in\mathcal{M}$. If $\mathcal{M}$ is a binary topology from X to Y then the triplet $(X, Y, \mathcal{M})$ is called a binary topological space and the members of $\mathcal{M}$ are called the binary open subsets of the binary topological space $(X,Y,\mathcal{M})$. The elements of $X\times Y$ are called the binary points of the binary topological space $(X, Y, \mathcal{M})$. If Y=X then $\mathcal{M}$ is called a binary topology on X in which case we write $(X, \mathcal{M})$ as a binary topological space. The examples of binary topological spaces are given in [2].

Definition 2.1[2]: Let X and Y be any two nonempty sets and let (A, B) and (C, D) $\in \wp(X)\times\wp(Y)$. We say that $(A, B)\subseteq(C, D)$ if $A\subseteq C$ and $B\subseteq D$.

Definition 2.2[2]: Let $(X, Y, \mathcal{M})$ be a binary topological space and $A\subseteq X$, $B\subseteq Y$. Then (A, B) is called binary closed in $(X, Y, \mathcal{M})$ if $(X\backslash A, Y\backslash B)\in\mathcal{M}$.

Proposition 2.3 [2]: Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A,B)\subseteq(X,Y)$. Let $(A, B)^{1*} = \cap\{A_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A, B)\subseteq(A_\alpha, B_\alpha)\}$ and $(A, B)^{2*} = \cap\{B_\alpha: (A_\alpha, B_\alpha)$ is binary closed and $(A,B)\subseteq(A_\alpha,B_\alpha)\}$. Then $((A, B)^{1*}, (A, B)^{2*})$ is binary closed and $(A, B)\subseteq((A, B)^{1*}, (A, B)^{2*})$.

Definition 2.4 [2]: The ordered pair $((A, B)^{1*}, (A, B)^{2*})$ is called the binary closure of (A, B), denoted by b-*cl*(A, B) in the binary space $(X, Y, \mathcal{M})$ where $(A, B)\subseteq(X, Y)$.

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Definition 2.5: Let X and Y be any two nonempty sets and let (A, B) and (C, D) $\in \wp(X) \times \wp(Y)$. We say that (A, B) $\not\subset$ (C, D) if one of the following holds:

(i) $A \subseteq C$ and $B \not\subset D$
(ii) $A \not\subset C$ and $B \subseteq D$
(iii) $A \not\subset C$ and $B \not\subset D$.

Definition 2.6:

(i) $(A, B)^{1^\circ} = \cup\{A_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$.
(ii) $(A, B)^{2^\circ} = \cup\{B_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$.

Definition 2.7: Let (X, Y, $\mathcal{M}$) be a binary topological space and (A, B) $\subseteq$ (X,Y).

The ordered pair $((A, B)^{1^\circ}, (A, B)^{2^\circ})$ is called the binary interior of (A, B), denoted by b-*int*(A, B).

Definition 2.8: Let (X, Y, $\mathcal{M}$) be a binary topological space. Let (A, B) $\subseteq$(X,Y).

Define $\mathcal{M}_{(A, B)} = \{(A \cap U, B \cap V) : (U,V) \in \mathcal{M}\}$. Then $\mathcal{M}_{(A, B)}$ is a binary topology from A to B. The binary topological space (A, B, $\mathcal{M}_{(A, B)}$) is called a binary subspace of (X, Y, $\mathcal{M}$).

3. BINARY SEMI OPEN SETS

In this section, we begin with the definition of binary semi open set in a binary topological space.

Definition 3.1: Let (X, Y, $\mathcal{M}$) be a binary topological space. Let (A, B) $\subseteq$ (X, Y).Then (A, B) is called binary semi open if there exists a binary open set (U, V) such that (U, V) $\subseteq$ (A, B) $\subseteq$ b-*cl*(U, V).

Example 3.2: Consider X ={a, b, c, d} and Y ={1, 2, 3, 4, 5}.

Clearly $\mathcal{M}$ ={(∅,∅),({a},{1}), ({b},{2}), ({a, b},{1, 2}), (X, Y)} is a binary topology from X to Y. Also (∅, ∅), (X, Y), ({b, c, d}, {2, 3, 4, 5}), ({a, c, d},{1, 3, 4, 5}) and ({c, d}, {3, 4, 5}) are binary closed sets in (X, Y, $\mathcal{M}$). Consider (A, B) = ({a, c}, {1, 3}). Clearly a binary open set ({a},{1}) $\subseteq$ (A, B) $\subseteq$ b-*cl*({a},{1}), since b-*cl*({a},{1}) $=((\{a\}, \{1\})^{1*}, (\{a\}, \{1\})^{2*})$ =({a, c, d},{1, 3, 4, 5}).

Hence, (A, B) = ({a, c}, {1, 3}) is binary semi open. Similarly,({a, d},{1,4}) is binary semi open, since ({a},{1}) $\subseteq$ ({a, d}, {1,4}) $\subseteq$ b- *cl*({a},{1}).

Proposition 3.3: Let (X, Y, $\mathcal{M}$) be a binary topological space and A $\subseteq$ X, B $\subseteq$ Y. If (A, B) is binary open in (X, Y, $\mathcal{M}$), then A is semi open in (X, $\mathcal{M}_X$) and B is semi open in (Y, $\mathcal{M}_Y$).

Proof: By Proposition 2.14 [2], we have $\mathcal{M}_X = \{A \subseteq X : (A, B) \in \mathcal{M}$ for some $B \subseteq Y\}$ is a topology on X and $\mathcal{M}_Y = \{B \subseteq Y : (A, B) \in \mathcal{M}$ for some $A \subseteq X\}$ is a topology on Y. Since (A, B) is binary open in (X, Y, $\mathcal{M}$), we have $A \in \mathcal{M}_X$ and $B \in \mathcal{M}_Y$. That is, A is open in (X, $\mathcal{M}_X$) and B is open in (Y, $\mathcal{M}_Y$).Since every open set is semi open, we have A is semi open in (X, $\mathcal{M}_X$) and B is semi open in (Y,$\mathcal{M}_Y$).

The converse of the above Proposition need not be true which is shown in Example 3.4.

Example 3.4: Let X = {a, b, c, d} and Y = {1, 2, 3, 4, 5}.

Clearly $\mathcal{M}$ **=** **{**(∅, ∅), ({a}, {1}), ({b}, {2}), ({a, b}, {1, 2}), (X, Y)} is a binary topology from X to Y. Also, $\mathcal{M}_X$ ={∅, {a},{b},{a, b}, X} is a topology on X and $\mathcal{M}_Y$ ={∅, {1}, {2}, {1, 2}, Y} is a topology on Y. Consider A ={a} and B ={1, 2}. Then A is open in X and B is open in Y. Therefore, A is semi open in X and B is semi open in Y. But (A, B) is not binary open.

The proof of the following Proposition is obtained directly from Definition 3.1.

Proposition 3.5: If (U, V) is binary open in a binary topological space (X, Y, $\mathcal{M}$) , then (U, V) is binary semi open in (X, Y, $\mathcal{M}$).

The converse of Proposition 3.5 is not true. From Example 3.2**,** we can easily see that the binary set ({a, c}, {1, 3}) is binary semi open but not binary open.

Remark 3.6: Let (X, Y, $\mathcal{M}$) be a binary topological space and A$\subseteq$X, B$\subseteq$Y. If (A, B) is binary semi open in (X, Y, $\mathcal{M}$), then A need not be semi open in (X, $\mathcal{M}_X$) and B need not be semi open in (Y, $\mathcal{M}_Y$). For, from Proposition 3.5, (A, B) need not be binary open in (X, Y, $\mathcal{M}$).Therefore, A is not open in (X, $\mathcal{M}_X$) and B is not open in (Y, $\mathcal{M}_Y$). Hence, A need not be semi open in (X, $\mathcal{M}_X$) and B need not be semi open in (Y, $\mathcal{M}_Y$).

The following Proposition gives a characterization of binary semi open sets.

Proposition 3.7: Let (X, Y, $\mathcal{M}$) be a binary topological space. Let (A, B) $\subseteq$ (X, Y). Then (A, B) is binary semi open if and only if (A, B) $\subseteq$ b-cl(b-int(A, B)).

Proof: Let (A, B) be binary semi open. Then there exists a binary open set (U, V) such that (U, V) $\subseteq$(A, B)$\subseteq$b-*cl*(U,V). Now, (U, V) $\subseteq$ (A, B) implies b-*int*(U,V) $\subseteq$ b-*int*(A, B). Since (U, V) is binary open, we have (U, V) = b-*int*(U, V). Hence, (U, V) $\subseteq$ b-*int*(A, B). This implies b-*cl*(U,V)$\subseteq$ b-*cl*(b-*int*(A, B)). Thus, (A, B) $\subseteq$ b-*cl*(U, V) $\subseteq$ b-*cl*(b-*int*(A, B)). That is, (A, B) $\subseteq$ b-*cl*(b-*int*(A, B)).

Conversely, assume that (A, B) $\subseteq$ b-*cl*(b-*int*(A, B)).Then for (U, V)= b-*int*(A, B), we have, b-*int*(A, B)$\subseteq$(A, B)$\subseteq$ b-cl(b-*int*(A,B)). This implies that (U, V) $\subseteq$ (A, B)$\subseteq$ b-*cl*(U, V). Hence, (A, B) is binary semi open .

Proposition 3.8: Let (A, B) be a binary semi open set in a binary topological space (X, Y, $\mathcal{M}$) and suppose (A, B) $\subseteq$ (C,D) $\subseteq$ b-*cl*(A, B). Then (C, D) is binary semi open.

Proof: Since (A, B) is binary semi open, there exists a binary open set (U, V) such that (U, V) $\subseteq$ (A, B) $\subseteq$ b-*cl*(U, V). Therefore, (U, V) $\subseteq$ (C, D) and b-*cl*(A, B) $\subseteq$b-*cl*(b-*cl*(U, V)) = b-*cl*(U, V) .Since (C, D) $\subseteq$ b-*cl*(A, B), we have (C, D) $\subseteq$ b-*cl*(U,V).Thus (U, V) $\subseteq$ (C, D) $\subseteq$ b-*cl*(U, V). Hence (C, D) is binary semi open.

Proposition 3.9: Suppose (X, ρ) and (Y, σ) are two topological spaces. If A is open in X and B is open in Y. Then (A, B) binary semi open in the binary topological space (X, Y, ρ×σ).

Proof: Since A is open in X and B is open in Y, by Proposition 2.15 [2], (A, B) is binary open in ρ×σ. Hence, by Proposition 3.5, (A, B) binary semi open in (X, Y, ρ×σ).

Proposition 3.10: Let (X, Y, $\mathcal{M}$) be a binary topological space and (A, B, $\mathcal{M}_{(A, B)}$) is a binary subspace of (X, Y, $\mathcal{M}$). Let (C, D) be a binary semi open set in (X, Y, $\mathcal{M}$) and (C, D) $\subseteq$ (A, B). Then (C,D) is binary semi open in (A,B, $\mathcal{M}_{(A, B)}$).

Proof: Since (C, D) is a binary semi open set in (X,Y,$\mathcal{M}$), we have (U, V) $\subseteq$(C, D) $\subseteq$b-cl(U, V) where (U, V)$\in\mathcal{M}$.Since (U,V)$\subseteq$(A,B), (U,V)=(U$\cap$A,V$\cap$B)$\subseteq$(C$\cap$A,D$\cap$B) $\subseteq$b-$cl_{(A,B)}$(U,V). Also, since (U, V) = (U$\cap$A, V$\cap$B), we have (U, V) is open in (A, B,$\mathcal{M}_{(A, B)}$).This implies that (U, V)$\subseteq$ (C, D)$\subseteq$b-$cl_{(A,B)}$(U, V).

The converse of the above Proposition is not true. This is shown in the following Example.

Example 3.11: Consider X ={a, b, c, d} and Y ={1, 2, 3, 4, 5}.

Clearly $\mathcal{M}$={(∅, ∅), ({a},{1}), ({b},{2}), ({a, b},{1, 2}), (X, Y)} is a binary topology from X to Y. Also (∅, ∅), (X, Y), ({b, c,d},{2,3,4,5}), ({a,c,d},{1,3,4,5}) and ({c,d},{3,4,5}) are binary closed sets in (X,Y,$\mathcal{M}$). Let (A,B) =({c,d},{1,2,3}) which is a subset of (X,Y). Clearly $\mathcal{M}_{(A, B)}$ ={(∅, ∅),(∅,{1}), (∅,{2}),(∅,{1,2}),(A,B)} is a binary topology from A to B. The binary closed sets in (A,B, $\mathcal{M}_{(A, B)}$) are (∅,∅),(A,B), (A,{2,3}), (A,{1,3}) and (A,{3}). Consider (C,D)=({c},{2}) $\subseteq$(A,B)=({c, d},{1,2,3}).

Clearly, $(\varnothing,\{2\})\subseteq(\{c\},\{2\}) \subseteq$ b-$cl_{(A,B)}(\varnothing,\{2\})$ since b-$cl_{(A,B)}(\varnothing,\{2\}) = (\{c, d\},\{2,3\})$. Hence, $(\varnothing, \{2\})$ is binary semi open in $(A, B,\mathcal{M}_{(A, B)})$. But $(\varnothing, \{2\})$ is not binary semi open in $(X, Y, \mathcal{M})$.

CONCLUSION

Semi open sets in topological spaces are extended to binary topological spaces. In this paper we introduced binary semi open sets in binary topological spaces and their basic properties are studied.

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