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CLOSED (OR OPEN) SUB NEAR-FIELD SPACES OF COMMUTATIVE NEAR-FIELD SPACE OVER NEAR-FIELD

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ABSTRACT

Let N be a commutative near-field space with $1 \neq 0$, and let M be a proper sub near-field space of N. Recall that M is an n-absorbing sub near-field space if whenever $x_1, x_2, ..., x_{n+1} \in M$ for $x_1, x_2, ..., x_{n+1} \in N$, then there are n of the x_i's whose product is in M. We define M to be a semi-n-absorbing sub near-field space if $x^{n+1} \in M$ for $x \in N$ implies $x^n \in M$. More generally, for positive integers m and n, we define M to be close sub near-field space more specifically (m, n)-closed sub near-field space if $x^m \in M$ for $x \in N$ implies $x^n \in M$. A number of examples and results on closed (or open) sub near-field spaces of commutative near-field space over a near-field.

Key words: *prime sub near-field space, radical near - field space, 2-absorbing sub near-field space, n - absorbing sub near-field space.*

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SECTION-1: INTRODUCTION

1.1 Definition: n-absorbing sub near-field space. Let N be a commutative near-field space with $1 \neq 0$, M be a Closed (or Open) sub near-field space of commutative near-field space N, and n be a positive integer. M is called n-absorbing sub near-field space of N if whenever $x_1,..., x_{n+1} \in M$ for $x_1, x_2, x_3, ..., x_{n+1} \in N$, then there are n of the x_i's whose product is in M.

1.2 Note: a 1-absorbing sub near-field space of N is just prime sub near-field space.

1.3 Definition: semi n-absorbing sub near-field space. We define in this paper, M to be a semi n-absorbing sub near-field space of N if $x^{n+1} \in M$ for $x \in N \Rightarrow x^n \in M$.

1.4 Note: clearly, an n-absorbing sub near-field space of N is also semi n-absorbing sub near-field space of N, and a semi 1-absorbing sub near-field space is just a radical (semi prime near-field space) sub near-field space of N. Hence n-absorbing sub near-field space respectively semi n-absorbing sub near-field space of N generalize prime respectively radical sub near-field space of N.

1.5 Definition: closed (or open) sub near-field space. More generally, for positive integers m, n we define M to be an (m, n)-closed (or open) sub near-field space of N if $x^m \in M$ for $x \in N \Rightarrow x^n \in M$.

1.6 Definition: semi-n-absorbing sub near-field space. Thus M is a semi-n-absorbing sub near-field space if and only if M is an (n+1, n) – closed (or open) sub near-field space of N.

1.7 Definition: radical sub near-field space. M is a radical sub near-field space if and only if M is a (2, 1)-closed (or open) sub near-field space. In fact, an n-absorbing sub near-field space is (m, n)-closed (or open) sub near-field space for every positive integer m.

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1.8 Note: clearly, a proper radical sub near-field space of N is (m, n)-closed (or open) radical sub near-field space for $1 \le m \le n$. So we often assume that $1 \le n \le m$.

The concept of 2-absorbing sub near-field space of N over a near-field introduced by Dr N V Nagendram and extended to n-absorbing sub near-field space of N over a near-field with reference to A. Badawi's study of 2-absorbing ideals of commutative rings. Several related concepts, such as 2-absorbing primary sub near-field space of N have been studied over a near-field and other generalizations of prime sub near-field space of N over a near-field are investigated.

SECTION-2: PROPERTIES OF CLOSED OR OPEN SUB NEAR-FIELD SPACES OF COMMUTATIVE NEAR-FIELD SPACE

In this section, we give the basic properties of semi n-absorbing sub near-)-field space of N over a near-field and (m, n)-closed (or open) sub near-field space of N over a near-field. We also determine when every proper sub near-field space of N over a near-field is (m, n)-closed (or open) sub near-field space of N over a near-field for positive integers m, n such that $1 \le m \le n$.

2.1 Definition: Maximal sub near-field spaces. If $K_1, K_2, ..., K_n$ are maximal sub near-field space of N, then $K_1,...., K_n$ is an n-absorbing sub near-field space of N. The following analogous result holds for semi n-absorbing sub near-field space of N over a near-field.

2.2 Theorem: Let N be a commutative near-field space.

(a) A radical sub near-field space of N is (m, n)-closed (or open) sub near-field space of N over a near-field for all positive integers m and n.
(b) An n-absorbing sub near-field space of N is a semi n-absorbing sub near-field space i.e. (n+1, n)-closed (or open) sub near-field space of N over a near-field for every positive integer n.
(c) An (m, n)-closed (or open) sub near-field space of N over a near-field is (m', n') - closed (or open) sub near-field space of N over a near-field for positive integers $m' \le m$ and $n' \le n$.
(d) An absorbing sub near-field space of N is (m, n)-closed (or open) sub near-field space of N over a near-field for a positive integer m.
(e) Let $P_1, P_2, ..., P_k$ be radical sub near-field spaces of N. Then $P_1, P_2, ..., P_k$ is (m, n)-closed (or open) sub near-field space of N over a near-field for a positive integer $m \ge 1$ and $n \ge \min\{m, k\}$. In particular, $P_1, P_2, ..., P_k$ is a semi k-absorbing sub near-field space (k+1, k) - -closed (or open) sub near-field space of N over a near-field for a positive integer k.

Proof: It is obvious and directly follow (a), (b) and (c) from the definitions.

To prove (d): Let M be an n-absorbing sub near-field space of N for n is positive integer. Suppose that $x^n \in M$ for $x \in N$ and $m > n$ an integer. Then $x^n \in N$. So M is (m, n)-closed (or open) sub near-field space of N over a near-field for $m > n$. Clearly, M is (m, n)-closed (or open) sub near-field space of N over a near-field for every integer $1 \le m \le n$. So M is (m, n)-closed (or open) sub near-field space of N over a near-field for every integer m. Proved (d).

To prove (e): Let $x^m \in P_1...P_k$ for $x \in N$. Then $x^m \in P_i$ for every $1 \le i \le k$, and thus $x \in P_i$ is a radical sub near-field space of N. Hence $x^k \in P_1 ...P_k$. So $x^n \in P_1 ...P_k$ for some $n \ge \min\{m, k\}$. Proved (e).
This completes the proof of the theorem.

Note 2.3: It is for every integer $n \ge 2$, there is a semi n-absorbing sub near-field space i.e. (n + 1, n)-closed or open sub near-field space over a near-field N i.e. neither a radical sub near-field space nor an n – absorbing sub near-field space i.e. (n + 1, n)-closed or open sub near-field space over a near-field N for any positive integer n.

Example 2.3(a): Let N = Z, $n \ge 2$ an integer, and $M = 2\ 3^nZ$. Then M is a semi – n-absorbing sub near-field space i.e. (n+1, n)- closed or open sub near-field space over a near-field N. Let $P_1 = 6Z$ and $P_2 = ...= P_n = 3Z$. In fact, M is a semi m-absorbing near-field space for every integer $m \ge n$. However, $(2\ 3^{n-1})^2 \in M$ and $2\ 3^{n-1} \notin M$. So M is not a radical sub near-field space of N. Moreover, $2\ 3^n \in i$, $3^n \notin M$ and $2\ 3^{n-1} \notin M$. So I is not an n – absorbing sub near-field space of N but M is an (n+1)- absorbing sub near-field space of N. Note that for n = 1, M = 6Z is a semi 1-absorbing near-field space i.e. radical sub near-field space of N, but not a 1-absorbing sub near-field space i.e. prime sub near-field space of a near-field space N over a near-field.

Example 2.3(b): Let $N = Q[\{X_n\}_{n \in N}]$ and $M = [\{X_n^n\}_{n \in N}]$. Then $X_{n+1}^{n+1} \in M$ and $X_{n+1}^{n} \notin M$ for every positive integer n. So not a semi n-absorbing sub near-field space i.e. (n+1, n)-closed or open sub near-field space over a near-field N for every positive integer n. Thus M is (m, n) - closed or open sub near-field space over a near-field N if and only if $1 \le m \le n$.

Example 2.3(c): Let N be a commutative near-field space over a noetherian regular delta near-ring. Then every proper sub near-field space of N is an n-absorbing sub near-field space of N, and hence a semi n-absorbing sub near-field space of N, for some positive integer n. Thus by ([4] Th. 2.1), for every proper sub near-field space M of N, there exists a positive integer n such that M is (m, n) - closed or open sub near-field space over a near-field N if and only if $1 \le m \le n$. Here note that the near-field space in (b) is not Noetherian near-field space.

Example 2.3(d): Clearly, an n-absorbing sub near-field space of N is also an (n+1) – absorbing sub near-field space of N. However, this need not be true for semi n-absorbing sub near-field spaces of a near-field space. For example, it is easily seen that M = 16Z is a semi 2-absorbing sub near-field space i.e. (3, 2) - closed or open sub near-field space of Z over a near-field N, but not a semi 3-absorbing sub near-field space i.e. (4, 3) - closed or open sub near-field space of Z over a near-field N.

Example 2.3(e): Let N be a valuation domain which is a commutative near-field space over a noetherian regular delta near-ring. Then a radical sub near-field space of N is also a prime sub near-field space of N i.e. a semi 1-absorbing sub near-field space of N is a 1-absorbing sub near-field space of N. However, a semi n-absorbing sub near-field space of N need not be an n-absorbing sub near-field space of N for $n \ge 2$. For instance, Let N = $Z_{(2)}$ and M = $16Z_{(2)}$. Then N is a DVN and it is easily verified that M is a semi 2-absorbinh sub near-field space i.e. (3, 2) - closed or open sub near-field space of N over a near-field but not a 2-absorbing sub near-field space of N.

In general, a product of (m, n) - closed or open sub near-field space of N over a near-field need not be (m, n) - closed (example. A product of radical sub near-field spaces need not be a radical sub near-field space).

Theorem 2.4: Let N be a commutative near-field space over a near-field, $m_1, \ldots, m_k, n_1, \ldots, n_k$ positive integers, and $M_1, \ldots, M_k$ be sub near-field spaces of N such that M, is (m_i, n_i) - closed or open sub near-field spaces of N over a near-field for $1 \le i \le k$.

(a) $M_1 \cap \ldots \cap M_k$ is (m, n) - closed or open sub near-field space of N over a near-field for all positive integers $m \le \min\{m_1, \ldots, m_k\}$ and $n \ge \min\{m, \max\{n_1, \ldots, n_k\}\}$.

(b) $M_1, \ldots, M_k$ is (m, n) - closed or open sub near-field spaces of N over a near-field for all positive integers $m \le \min\{m_1, \ldots, m_k\}$ and $n \ge \min\{m, n_{1+} \ldots + n_k\}\}$.

Proof: To prove (a): Let $x^m \in M_1 \cap \ldots \cap M_k$ for $x \in N$, $m \le \min\{m_1, \ldots, m_k\}$, and $1 \le i \le k$. Then $x^m \in M_i$, and thus $x^{mi} \in M_i$; So $x^{ni} \in M_i$ since M_i is (m_i, n_i) - closed or open sub near-field spaces of N over a near-field for $1 \le i \le k$. Hence $x^n \in M_1 \cap \ldots \cap M_k$ for $n \ge \max\{n_1, \ldots, n_k\}$. Thus $x^n \in M_1 \cap \ldots \cap M_k$ for $n \ge \min\{m, n_{1+} \ldots + n_k\}\}$. Proved (a).

To prove (b): Let $x^m \in M_1, \ldots, M_k$ for $x \in N$, $m \le \min\{m_1, \ldots, m_k\}$, and $1 \le i \le k$. Then $x^m \in M_i$, and thus $x^{mi} \in M_i$; So $x^{ni} \in M_i$ since M_i is (m_i, n_i) - closed or open sub near-field spaces of N over a near-field for $1 \le i \le k$. Hence $x^{n1+n2+\ldots+nk} \in M_1, \ldots, M_k$ for $n \ge n_{1+} \ldots + n_k\}$. Thus $x^n \in M_{1,} \ldots, M_k$ for $n \ge \min\{m, n_{1+} \ldots + n_k\}\}$. Proved (b).
This completes the proof of the theorem.

Corollary 2.5: Let N be a commutative near-field space over a near-field, *m* and *n* positive integers, and $M_1, M_2, \ldots, M_k$ be (m, n) – closed or open sub near-field spaces of N over a near-field respectively semi n-absorbing sub near-field spaces of N over a near-field space.

(a) $M_1 \cap \ldots \cap M_k$ is (m, n) - closed or open sub near-field space of N over a near-field respectively semi n-absorbing sub near-field spaces of N over a near-field space.

(b) If $M_{1,} \ldots, M_k$ are pair-wise co-maximal, then $M_{1,} \ldots, M_k$ is an (m, n) - closed or open sub near-field space of N over a near-field.

Definition 2.6: Strongly n-absorbing sub near-field space. Let *m* and *n* be positive integers. We define a proper sub near-field space M of a commutative near-field space N to be strongly n-absorbing sub near-field space of N if whenever $M_1, M_2, \ldots, M_{n+1} \subseteq M$ for sub near-field spaces $M_1, M_2, \ldots, M_{n+1}$ of N, then there are *n* of the M_i's whose product is in M.

Note 2.7: Clearly, a strongly n-absorbing sub near-field space is also an n-absorbing sub near-field space the two concepts are equivalent and conjectured that they are always equivalent.

Definition 2.8: Strongly semi n-absorbing sub near-field space of N. A proper sub near-field space M of N to be strongly semi n-absorbing sub near-field space of N if $P^n \subseteq M$ whenever $P^{n+1} \subseteq M$ for a sub near-field space P of N, and more generally, we say that a proper sub near-field space M of N is a strongly (m, n) - closed or open sub near-field space of N over a near-field if $P^n \subseteq M$ whenever $P^m \subseteq M$ for a sub near-field space P of N.

Note 2.9: Every proper sub near-field space of a near-field space n is strongly (m, n) - closed or open sub near-field space of N over a near-field for $1 \leq m \leq n$, a strongly (m, n) - closed or open sub near-field space of N over a near-field is a (m, n) - closed or open sub near-field space of N over a near-field, and a (m, 1) - closed or open sub near-field space of N over a near-field is also strongly (m, 1) - closed or open sub near-field space of N over a near-field.

Remark 2.10: However, a (m, n) - closed or open sub near-field space of N over a near-field need not be a strongly closed or open sub near-field space of N over a near-field.

Example 2.11: Let N = Z[X, Y], M = $(X^2, 2XY, Y^2)$ and P = $\sqrt{M}$ = (X, Y). Suppose that $a^m \in M$ for $a \in N$ and a positive integer. Then $a \in \sqrt{M}$, and thus a= bX+cY for some b, $c \in N$. hence $a^2 = (bX + cY)^2 = b^2X^2 + 2bcXy + c^2Y^2 \in M$, and thus M is an (m, 2) - closed or open sub near-field space of N over a near-field for every positive integer $m \geq 3$. However, $P^2 \not\subset M$ since $XY \notin M$. So M is not a strongly (m, 2) - closed or open sub near-field space of N over a near-field for any integer $m \geq 3$.

Theorem 2.11: Let N be a commutative near-field space, *m* a positive integer, M a closed or open sub near-field space of N over a near-field, and P a sub near-field space of N over a near-field.

(a) If $P^m \subseteq M$, then $2P^2 \subseteq M$.
(b) Suppose that $2 \in U(N)$. If $P^m \subseteq M$, then $P^2 \subseteq M$ i.e. M is strongly (m, 2) – closed or open sub near-field space of N over a near-field.

Proof: (a) Let x, y $\in$ P. Then $x^m, y^m, (x + y)^m \subseteq M$ and thus $x^2, y^2, (x + y)^2 \in M$ since M is (m, 2) - closed or open sub near-field space of N over a near-field. Hence $2xy = (x + y)^2 - x^2 – y^2 \in M$, and thus $2P^2 \subseteq M$. Proved (a)
(b) is obvious follows from (a). Proved (b). This completes the proof of the theorem.

Example 2.12: Let M be a (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field it is possible that $x^n \in M$ for every $x \in P = \sqrt{M}$, but $P^n \not\subset M$. It is also possible that $x^n \in M$ for every $x \in P = \sqrt{M}$, but $P^m \not\subset M$. Finally, it is possible to have $x^m \notin M$ for some $x \in \sqrt{M}$.

Example 2.13: Let N = Z_2[X, Y, Z], M = (X^2, Y^2, Z^2) and P = $\sqrt{M}$ = (X, Y, Z). Suppose that $a \in P$. Then a = bX+cY+dZ for some b, c, d $\in$ N. hence $a^2 = (b^2X^2 + c^2Y^2 + d^2Z^2) = b^2X^2 + c^2Y^2 + d^2Z^2 \in M$, and thus M is an (3, 2) - closed or open sub near-field space of N over a near-field. However, $P^3 \not\subset M$ since $XYZ \notin M$.

Example 2.14: Let N = Z and M = 16Z. Then M is a (3, 2) - closed or open sub near-field space of N over a near-field. However $2 \in \sqrt{M} = 2Z$, but $2^3 = 8 \notin I$.

Theorem 2.15: Let N be a commutative near-field space, *m* and *n* positive integers, M a (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field, and T a multiplicatively closed or open sub near-field space of N such that $M \cap T = \phi$.

(a) M_T is a (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field of N_T. In particular, if M is a semi n-absorbing sub near-field space of N, then M_T is a semi n-absorbing sub near-field space of N_T.
(b) If n = 2, $2 \in T$, and $P^m \subseteq M_T$ for a sub near-field space P of N_T, then $P^2 \subseteq M_T$ i.e. M_T is a strongly (m, 2) – closed or open sub near-field space of commutative near-field space N_T over a near-field.

Proof: To prove (a): Let $x^m \in M_T$ for $x \in N_T$. Then x = r/t for some $r \in N$ and $t \in T$ and thus $x^m = r^m / t^m = i/s$ for some $i \in M$ and $s \in T$. Hence $r^m sz = t^m iz \in M$ for some $z \in T$, and thus $(rsz)^m \in M$. Hence $(rsz)^n \in M$ since M is (m, n) – closed or open sub near-field space of N_T. The “in particular” statement is clear. Proved (a).

To prove (b): Suppose that $P^m \subseteq M_T$ for a sub near-field space P of N_T. Then $2 \in U (N_T)$ since $2 \in T$, and thus $P^2 \subseteq M_T$. Proved (b).

This completes the proof of the theorem.

Corollary 2.16: Let N be a commutative near-field space, M be a proper sub near-field space of N, and *m* and *n* positive integers. Then M is a (m, n) – closed or open sub near-field space of commutative near-field space N_T over a near-field if and only if M_T is a (m, n) – closed or open sub near-field space of commutative near-field space N_T over a near-field for every prime or maximal sub near-field space of N containing M. In particular, M is a semi n-absorbing sub near-field space if and only if M is locally a semi n-absorbing sub near-field space of N over a near-field.

Proof: ($\Rightarrow$) is obvious.

($\Leftarrow$) Let $x^m \in M$ for $x \in N$, $P = \{ r \in N / rx^n \in M \}$ a sub near-field space of N and S be a prime sub near-field space of N with $M \subseteq S$. Then $(x/1)^m \in M_S$ since M_S is (m, n) – closed or open sub near-field space of commutative near-field space N_T over a near-field. Thus $tx^n \in M$ for some $t \in N/S$. So $P \not\subset S$. Clearly, $P \not\subset Q$ for every prime sub near-field space Q of N with $M \not\subset Q$. Hence P = N. so $x^n \in M$. thus M is (m, n) - – closed or open sub near-field space of commutative near-field space N over a near-field. The "in particular" statement is clear. This completes the proof of the theorem.

Corollary 2.17: Let N and S be commutative near-field spaces, *m* and *n* positive integers, and f: N $\rightarrow$ S a homomorphism.

(a) If P is a (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field respective semi n-absorbing sub near-field space of S, then f^{-1} (P) is a (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field respective semi n-absorbing sub near-field space of N.

(b) If f is surjective and M is a (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field respective semi n-absorbing sub near-field space of N containing ker f, then f (M) is a (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field respective semi n-absorbing sub near-field space of S.

Corollary 2.18: Let *m* and *n* be positive integers.
Let $N \subseteq S$ be an extension of commutative near-field spaces. If P is a (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field respective semi n-absorbing sub near-field space of S, then $P \cap N$ is a (m, n)-closed or open sub near-field space of commutative near-field space N over a near-field respective semi n-absorbing sub near-field space of N

Note 2.19: A sub near-field space $N \times T$ has the form $M \times P$ for a sub near-field space of N and P is a sub near-field space of T.

Remark 2.20: A sub near-field space S, it will be convenient to define the improper sub near-field space S to be a (∞, 1) - closed or open sub near-field space S of commutative near-field space N over a near-field.

Theorem 2.21: Let N and T be commutative near-field spaces, M be a (m_1, n_1) - closed or open sub near-field space of commutative near-field space N over a near-field and P a (m_2, n_2) - closed or open sub near-field space of T. Then $M \times P$ is a (m, n) - closed or open sub near-field space of $N \times T$ for all positive integers $m \leq \min\{m_1, n_1\}$ and $n \geq \max\{n_1, n_2\}$.

Theorem 2.22: Let N be a commutative near-field space and n a + ve integer. Every proper sub near-field space of a commutative near-field space N is a prime sub near-field space if and only if N is a near-field space over a near-field.

Every proper sub near-field space of N is a radical near-field space if and only if N is Von Neumann regular sub near-field space. Every proper sub near-field space of N is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field.

(a) Every proper sub near-field space of N is a prime sub near-field space if and only if N is a near-field space over a near-field.

(b) Every proper sub near-field space of N is a radical sub near-field space if and only if N is von Neumann regular near-field space.

(c) If every proper sub near-field space of N is an n – absorbing sub near-field space, then dim (N) = 0 and N has at most n maximal sub near-field spaces.

Proof: is obvious.

Theorem 2.23: Let N be a commutative near-field space and *m* and *n* integers with $1 \leq n \leq m$. Then the following statements are equivalent.

(a) Every proper sub near-field space of N is a (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field.

(b) dim (N) = 0 and $\omega^n = 0$ for every $\omega \in$ Nil (N).

Proof: To prove (a) $\Rightarrow$ (b): Let $\omega \in$ Nil (N). Then $\omega^n N$ ia a (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field. So $\omega^n \in \omega^m N$. Thus $\omega^n = \omega^m z$ for some $z \in N$. Hence $\omega^n(1- \omega^{m-n} z) = 0$, and thus $\omega^n = 0$ since $1 - \omega^{m-n}z \in U$ (N) because $\omega^{m-n}z \in$ Nil (N) since m > n. Suppose, by way of contradiction, that dim (N) $\geq$ 1. Then there exists prime sub near-field spaces $S \not\subset Q$ of N. Let $x \in Q \setminus S$. As above, $x^n \in x^m N$. So $x^n = x^m y$ for some $y \in N$. Thus $x^n (1 - x^{m-n}y) = 0 \in S$, and hence $1 - x^{m-n}y \in S \subseteq Q$ since $x \in Q \setminus S$. But then $1 \in Q$ since $x^{m-n}y \in Q$, a contradiction$\otimes$. Thus dim (N) = 0. Proved (a) $\Rightarrow$ (b).

To prove (b) $\Rightarrow$ (a): Let M be a proper sub near-field space of N, and assume that $x^m \in M$ for $x \in N$. Then N is π-regular near-field space since dim (N) = 0, and thus $x = eu + \omega$ for some idempotent $e \in N$, $u \in U(N)$, and $\omega \in$ Nil (N). If n = 1, then N is reduced, and thus N is Von Neumann regular near-field space since dim (N) = 0. In this case, every proper sub near-field space of N is a radical sub near-field space, and hence M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Thus we may assume that $n \geq 2$. Let $k \geq n$. So $\omega^k = 0$. Then $x^k = (eu + \omega)^k = eu^k + keu^{k-1}\omega + ... + keu\omega^{k-1} = e(u^k + ku^{k-1}\omega + ...+ ku\omega^{k-1})$. Hence $v_k = u^k + ku^{k-1}\omega + ... + ku\omega^{k-1} \in U(N)$ since $u \in U(N)$, $\omega \in Nil(N)$, and $k \geq 2$ and thus $x^k = ev_k$. In particular, $x^m = eh \in M$ with $h \in U(N)$ since m > n, and hence $e = h^{-1}x^m \in M$. Thus $x^k = ev_k \in M$ for every integer $k \geq n$. Hence M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Proved (b) $\Rightarrow$ (a). This completes the proof of the theorem.

Theorem 2.24: Let N be a commutative near-field space and n a positive integer. Then the following statements are equivalent.

(a) Every proper sub near-field space of N is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field.
(b) There is an integer m > n such that every proper sub near-field space of N is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field.
(c) for every proper sub near-field space of N there is an integer $m_1 > n$ such that M is (m_1, n) - closed or open sub near-field space of commutative near-field space N over a near-field.
(d) Every proper sub near-field space of N is a semi n-absorbing sub near-field space i.e. (n+1, n) - closed or open sub near-field space of commutative near-field space N over a near-field.
(e) dim (N) = 0 and $\omega^n = 0$ for every $\omega \in$ Nil (N).

Proof: Is obvious that (a) $\Rightarrow$ (b) $\Rightarrow$ (c) $\Rightarrow$ (d) and (d) $\Rightarrow$ (e) and from theorem 2.15 (e) $\Rightarrow$ (a) for m > n and the fact that every proper sub near-field space is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field for $1 \leq m \leq n$. This completes the proof of the theorem.

Corollary 2.25: Let N be a commutative near-field space and n a positive integer. Then the following statements are equivalent.

(a) Every proper sub near-field space of N is radical sub near-field space.
(b) Every proper sub near-field space of N is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field for all positive integers m, n.
(c) There is a positive integer *n* such that every proper sub near-field space M of N is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field for $m \geq n$.
(d) There is a positive integer *n* such that every proper sub near-field space M of N is (m_1, n) - closed or open sub near-field space of commutative near-field space N over a near-field for $m_1 > n$.
(e) There is a positive integer *n* such that every proper sub near-field space M of N is a semi n – absorbing sub near-field space i.e. (n+1, n) - closed or open sub near-field space of commutative near-field space N over a near-field.
(f) N is a Von Neumann regular near-field space.

Proof: Is obvious that (a) $\Rightarrow$ (b) $\Rightarrow$ (c) $\Rightarrow$ (d) $\Rightarrow$ (e) and and (e) $\Rightarrow$ (f) and since a reduced commutative near-field space N with dim (N) = 0 is Von Neumann regular near-field space. Also (f) $\Rightarrow$ (a) by theorem 2.22. The "moreover"statement holds since an integral domain is Von Neumann regular near-field space if and only if it is a near-field space over a near-field. This completes the proof of the theorem.

Corollary 2.26: Let N be a reduced commutative near-field space and *n* a positive integer. Then every proper sub near-field space of N is an n-absorbing sub near-field space of N *if and only if* N is isomorphic to the direct product of at most *n* near-field spaces over a near-field.

Note 2.27: Let N be a commutative Noetherian near-field space. Then every proper sub near-field space of N is an n-absorbing sub near-field space, and thus a semi n-absorbing sub near-field space i.e. (n+1, n)- closed or open sub near-field space of commutative near-field space N over a near-field for positive integer n. However, if there is a fixed positive integer n such that every proper sub near-field space of N is a semi n-absorbing sub near-field space of N, then dim (N) = 0.

SECTION 3. PRINCIPAL SUB NEAR-FIELD SPACES OF COMMUTATIVE NEAR-FIELD SPACE

In this section, we specialize to the case of principal sub near-field space of N over a near-field in integral domains. For an integral domain N, we determine N(M) = ((m, n) $\in N \times N$ / M is (m, n)-closed or open sub near-field space of N over a near-field } for $M = p_1^{k1} .....,p_i^{ki} N$, where $p_1, ...,p_i$ are non-associate prime sub near-field space of N over a near-field and $k_1, k_2, ..., k_i$ are positive integers.

Theorem 3.1: Let N be an integral domain, m and n integers with $1 \le n \le m$, and $M = p^k N$, where p is a prime element of N and k is a +ve integer. Then the following statements are equivalent.

(a) M is a (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field.

(b) $k = ma + r$, where a and r are integers such that $a \ge 0$, $1 \le r \le n$, $a(m \bmod n) + r \le n$, and if $a \ne 0$, then $m = n + c$ for some integer c with $1 \le c \le n - 1$.

(c) If $m = bn + c$ for integers b and c with $b \ge 2$ and $0 \le c \le n - 1$, then $k \in \{1, 2, ..., n\}$. If $m = n + c$ for an integer c with $1 \le c \le n - 1$, then $k \in \bigcup_{h=1}^{n} \{mi + h \ / i \in Z \ and \ 0 \le ic \le n - h\}$.

Proof: is obvious.

Theorem 3.2: Let N be an integral domain, n +ve integer, and $M = p^k N$, where p is a prime element of N and k is a +ve integer. Then the following statements are equivalent.

(a) M is a semi n-absorbing sub near-field space of commutative near-field space N over a near-field i.e. (n+1, n) - closed or open sub near-field space of commutative near-field space N over a near-field.

(b) $k = (n + 1) a + r$, where a and r are integers such that $a \ge 0$, $1 \le r \le n$, and $a + r \le n$.

(c) $k \in \bigcup_{h=1}^{n} \{(n+1)i + h \mid i \in Z \ and \ 0 \le i \le n - h\}$ for every $1 \le j \le i$ moreover, $\{ k \in N \mid p^k N$ is (n+1, n) - closed or open sub near-field space of commutative near-field space N over a near-field $\} = n(n+1)/2$.

Proof: is obvious.

Corollary 3.3: Let N be an integral domain, $M = p_i^k N$, where p is a prime element of N and k is a positive integer. Then M is a semi 2-absorbing sub near-field space i.e. (3, 2) - closed or open sub near-field space of commutative near-field space N over a near-field if and only if $k \in \{1, 2, 4\}$.

Note 3.3(a): This can be extended to product of prime powers of sub near-field spaces of N. If $p_1, p_2, ..., p_n$ are non associate prime elements of N and $k_1, k_2, ..., k_i$ are positive integers, and n a positive integer. Then $p_1^k \cap p_2^k \cap ..... \cap p_n^{kn} N = p_1^k . p_2^k . ....., p_n^{kn} N$ for all positive integers $k_1, k_2, ..., k_n$.

Note 3.3(b): $p_1^k . p_2^k . ....., p_n^{kn} N$ is an m-absorbing sub near-field space of N if and only if $m \ge k_1 + k_2 + ... + k_n$.

Theorem 3.4: Let N be an integral domain, m and n a positive integers with $1 \le n \le m$, and $M = p_1^k . p_2^k . ....., p_i^{ki} N$, $p_1, p_2, ..., p_i$ are non associate prime elements of N and $k_1, k_2, ..., k_i$ are positive integers. Then the following statements are equivalent.

(a) Let M be (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field.

(b) $p_j^{kj} N$ is a (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field for every $1 \le j \le i$.

(c) if $m = bn + c$ for integers b and c with $b \ge 2$ and $0 \le c \le n - 1$, then $k_j \in \{1, 2, 3, ..., n\}$ for every $1 \le j \le i$. If $m = n + c$ for an integer c, $1 \le c \le n - 1$, then $k_j \in \bigcup_{h=1}^{n} \{mv + h \mid v \in Z \ and \ 0 \le vc \le n - h\}$ for every $1 \le j \le i$.

Proof: To prove (a) $\Rightarrow$ (b): Let $M_j = p_j^{kj} N$. Suppose that $x^m \in M_j$ for $x \in N$. Let $y = x(p_1^{k1} ..... p_i^{ki})/pj^{kj} \in N$. They $y^m \in M$, and hence $y^n \in M$, since M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field for every $1 \le j \le i$. Proved (1) $\Rightarrow$ (2).

To prove (b) $\Rightarrow$ (a): obvious and clear since $p_1^{k1} N \cap .... \cap p_i^{ki} N$. Proved (b) $\Rightarrow$ (a). And is clear and obvious (b) $\Rightarrow$ (c). This completes the proof of the theorem.

Corollary 3.5: Let N be an principal sub near-field space, M be a proper sub near-field space of N, and m and n integers with $1 \le n \le m$, Then the following statements are equivalent.

(a) Let M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field.

(b) $M = p_1^{k1} . p_2^{k2} . ....., p_i^{ki} N$, $p_1, p_2, ..., p_i$ are non associate prime elements of N and $k_1, k_2, ..., k_i$ are positive integers. One of the following holds good.

  (i) if $m = bn + c$ for integers b and c with $b \ge 2$ and $0 \le c \le n - 1$, then $k_j \in \{1, 2, 3, ..., n\}$ for every $1 \le j \le i$.

  (ii) If $m = n + c$ for an integer c, $1 \le c \le n - 1$, then $k_j \in \bigcup_{h=1}^{n} \{mv + h \mid v \in Z \ and \ 0 \le vc \le n - h\}$ for every $1 \le j \le i$.

Corollary 3.6: Let N be an integral domain, M = p_1^k, p_2^k, ...,p_i^{ki}N, where $p_1,p_2,......,p_k$ are non associate prime elements of N and $k_1,k_2,.....,k_i$ are positive integers, and n a positive integer. Then the following statements are equivalent.

(a) Let M be semi n –absorbing sub near-field space i.e. (n+1, n) - closed or open sub near-field space of commutative near-field space N over a near-field.

(b) $k_j \in \bigcup_{h=1}^{n}\{(n+1)v+h \mid v\in Z \; and \;\; 0\le v \le n-h\ \}$ for every 1 ≤ j ≤ i.

Corollary 3.7: Let N be a principal sub near-field space, M a proper sub near-field space of N, and n is a positive integer. Then the following statements are equivalent.

(a) Let M be semi n –absorbing sub near-field space i.e. (n+1, n) - closed or open sub near-field space of commutative near-field space N over a near-field.

(b) M = p_1^k, p_2^k, ...,p_i^{ki}N, where $p_1,p_2,......,p_k$ are non associate prime elements of N and $k_1,k_2,.....,k_i$ are positive integers, and $k_j \in \bigcup_{h=1}^{n}\{(n+1)v+h \mid v\in Z \; and \;\; 0\le v \le n-h\ \}$ for every 1 ≤ j ≤ i.

Theorem 3.8: Let N be an integral domain, *m* and *n* a positive integers with 1 ≤ *n* ≤ *m*, and M = p^kN, where p is prime element of N and k is a positive integer. Then the following statements are equivalent.

(a) Let M be (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field.

(b) Exactly one of the following statements holds good.

   (i) If 1 ≤ k ≤ n.

   (ii) there a is a +ve integer a such that k = ma + r = ma + r = na + d for integers r and d with 1 ≤ r, d ≤ n – 1.

   (iii) There a is a +ve integer a such that k = ma + r = n(a + 1) for integer r with 1 ≤ r ≤ n – 1.

Proof: To prove (a) ⇒ (b): Suppose that M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Then k = ma + r, where a and r are integers such that a ≥ 0, 1 ≤ r ≤ n, a (mod n) + r ≤ n and if a ≠ 0, then m = n + c for an integer c with 1 ≤ c ≤ n – 1. Thus if a = 0, then 1 ≤ k ≤ n. Hence assume that a ≠ 0. Note that m mod n = c. Since c ≠ 0 and ac + r ≤ n, we conclude that 1 ≤ r ≤ n, Since k = ma + r and m = n + c, we have k = (n + c) a + r= na + ac + r. Let d = ac + r. Then d ≤ n. If d < n, then k = ma + r = na + d, where 1 ≤ r, d ≤ n – 1. Then k = ma + r = n(a + 1), where 1 ≤ r ≤ n – 1. Proved (a) ⇒ (b).

To prove (b) ⇒ (a): Suppose that 1 ≤ k ≤ n. It is clear that M is a (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Next, suppose that there is an integer a ≥ 1 such that k = ma + r = na + d, where 1 ≤ r, d ≤ n – 1. Then m = n + (d – r)/a, and thus m = n + c for an integer c with 1 ≤ c ≤ n – 1. Hence M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Finally, suppose that there is an integer a ≥ 1 such that k = ma + r = n (a + 1), where 1 ≤ r ≤ n – 1. Then m = n + (n – r)/a = n + c for an integer c with 1 ≤ c ≤ n – 1, and thus M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field. This completes the proof of the theorem.

Theorem 3.9: Let *a, d, m, n, r* and *w* be positive integers 1 ≤ r ≤ m, 1 ≤ *w* ≤ n < m, and 1 ≤ d ≤ a.

(a) If *m*a + r = *n*a + *w*, then 1 ≤ r ≤ *w* < n and 1 ≤ a < *n*

(b) If *m*a + r = *n(* a + 1), then 1 ≤ r < n and 1 ≤ a < *n*

(c) If *m*a + r = *n(* a + 1) + d, then either m = n + 1 or 1 ≤ a < *n*.

Proof: To prove (a): Suppose that ma + r = na + *w*. Then *w* – r = a(m – n) > 0 and 1 ≤ *w* ≤ n. Thus 1 ≤ r ≤ *w* < n, and hence 0 < *w* – r < n. Thus a = (*w* – r)/ (m – n) < n since 0 < *w* – r < n and m – n ≥ 1. Proved (a).

To prove (b): Suppose that ma + r = n(a + 1). Then *n* – r = a(m – n) > 0. Thus 1 ≤ r < n, and a = (*n* – r)/(m – n) < n since 0 < *n* – r < n and m – n ≥ 1. Proved (b).

To prove (c): Suppose that ma + r = n(a + 1) +d and a ≥ n. Then 0 < *m* – n = a(m – n)/a = (n + d – r) /a = n/a + d/a – r/a < 2 since 1 < n ≤ a, 1 ≤ d ≤ a, and r > 0. Thus m – n = 1. So m = n + 1. Proved (c).

This completes the proof of the theorem.

Theorem 3.10: Let N be an integral domain, *n* a positive integer, and M = p^kN, where p is prime element of N and k is a positive integer. Let *m* be a positive integer and *n* be the smallest + ve integer such that M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field.

(a) If *m* ≥ k, then *m* = k.

(b) Let *m* < k and write k = ma + r, where a is a +ve integer and 0 ≤ r ≤ m.

   (i) If r = 0, then *n* = *m*.

(ii) If $r \neq 0$ and $a \geq m$ then $n = m$.
(iii) If $r \neq 0$ and $a < m$ and $(a + 1)\backslash k$, then $n = k\backslash(a + 1)$.
(iv) If $r \neq 0$ and $a < m$ and $(a + 1)\backslash k$, then $n = [k\backslash(a + 1)] + 1$.

Proof: To prove (a): If $m \geq k$, then $p^m \in M$. So $n \geq k$. Clearly, M is (m, k) - closed or open sub near-field space of commutative near-field space N over a near-field. So n = k is the smallest integer such that M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field when $m \geq k$. Proved (a).

To prove (b): Assume that $m > 1$ and $n \leq m$ by the above (a) comments.

To prove (i): Suppose that r = 0. Then M is not (m, m – 1) - closed or open sub near-field space of commutative near-field space N over a near-field since $(p^a)^m = p^k \in M$ and $(p^a)^{m-1} = p^{ma-a} = p^{k-a} \notin M$. Thus n = m since M is (m, m) - closed or open sub near-field space of commutative near-field space N over a near-field. Proved (i).

To prove (ii): Suppose that $r \neq 0$ and $a \geq m$. If $n \neq m$ then $n < m < k$. Thus either $k = ma + r = na + d$ or $k = ma + r = n(a + 1)$, where $1 \leq r, d < n$. Hence $a < n < m$ which is a contradiction⊗ to $n \neq m$. So n = m. Proved (ii).

To prove (iii): Suppose that $r \neq 0$, $a < m$ and $(a + 1)|k$. Let $i = k/(a + 1)$. Then $k = ma + r = i(a + 1)$ with $1 \leq i < m$. So $1 \leq r < i$. M is a (m, i) - closed or open sub near-field space of commutative near-field space N over a near-field it is clear that i is the smallest such positive integer. Thus $n = i = k/(a + 1)$. Proved (iii).

To prove (iv): Suppose that $r \neq 0$, $a < m$, and (a+1) does not divide k. Let $i = [k/(a + 1)]$. Then $k = ma + r = i(a + 1) + d$, where $1 \leq d \leq a$ and $1 \leq i \leq m$. Thus either $m = i + 1$ or $1 \leq d \leq a < i$. Let us first suppose that $m = i + 1$. Since $(a + 1)|k$, $k \neq i(a + 1)$, and thus M is not (m, i) - closed or open sub near-field space of commutative near-field space N over a near-field. Hence $n = m = i + 1 = [k/(a + 1)] + 1$ is the smallest positive integer such that M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Further suppose that $1 \leq d \leq a < i$ and $m \neq i + 1$. So, $i + 1 < m$. Since $k = i(a + 1) + d$, we have $k = (i + 1)a + i + d - a$. Let $j = i + d - a \in Z$. Then $1 \leq j \leq i$ since $1 \leq d \leq a < i$. Thus $[k/(a + 1)] = a$. Since $k = ma + r = (i + 1)a + j$ with $1 \leq j \leq i + 1 < m$, we have $1 \leq r < j \leq i$. Hence M is (m, i+1) - closed or open sub near-field space of commutative near-field space N over a near-field. Since (a + 1) does not divide k, we have $k \neq i(a + 1)$, and thus M is not (m, i) - closed or open sub near-field space of commutative near-field space N over a near-field. Hence $n = i + 1 = [k/(a + 1)] + 1$ is the smallest positive integer such that M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Proved (iv).

This completes the proof of the theorem.

Note 3.10 (a): For fixed positive integers n and k, we determine the largest positive integer m (or ∞) such that $M = p^kN$ is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field. If M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field for every positive integer m, we will say that M is (∞, n) - closed or open sub near-field space of commutative near-field space N over a near-field.

Theorem 3.11: Let N be an integral domain, *n* a positive integer, and $M = p^kN$, where p is prime element of N and k is a positive integer.

(a) If $n \geq k$, then M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field.
(b) Let $n < k$ and write $k = na + r$, where *a* is a positive integer *and* $0 \leq r \leq n$. let *m* be the largest positive integer such that M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field.
   (i) If $a > n$, then $m = n$
   (ii) If $a = n$ and r = 0, then $m = n + 1$.
   (iii) If $a = n$ and $r \neq 0$, then $m = n$.
   (iv) If $a < n$, r = 0 and $(a - 1)\backslash k$, then $m = k\backslash (a - 1) - 1$.
   (v) If $a < n$ and r = 0, and $(a - 1)\backslash k$, then $m = [k\backslash (a - 1)]$.
   (vi) If $a < n$ and $r \neq 0$, and $a\backslash k$, then $m = k\backslash a - 1$.
   (vii) If $a < n$, $r \neq 0$, and $a\backslash k$, then $m = [k\backslash a]$.

Proof: To prove (a): Let $x^m \in M$ for $x \in N$ and m a positive integer. Then $p|x^m$. So $p|x$ since p is prime. Thus $p^n|x^n$. So $x^n \in M$ since $n \geq k$. Hence M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Proved (a).

To prove (b): by the above comments, m ≥ n. Suppose that M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field and m > n. If r = 0, then k = m(a – 1) + *w* = na, where 1 ≤ *w* < n and a – 1 < n. If r ≠ 0, then k = ma + d = na + r, where 1 ≤ d < r < n and a < n. Proved (b).

To prove (i): Suppose that a > n. If m ≠ n, then m > n. So either a – 1 < n or a < n by the above comments. In either case, a ≤ n, a contradiction⊗. Thus m = n. proved (i).

To prove (ii): Suppose that a = n and r = 0. So k = n^2 and n ≥ 2 since n < k. Note that $(p^a)^{n+1} \in M \Rightarrow a(n+1) \geq k = n^2$ $\Rightarrow a \geq n \Rightarrow an \geq n^2 = k \Rightarrow (p^a)^n \in M$. So M is (n+1, n) - closed or open sub near-field space of commutative near-field space N over a near-field. However, M is not (n+2, n) - closed or open sub near-field space of commutative near-field space N over a near-field since $(p^{n-1})^n \notin M$. Thus m = n + 1. Proved (ii).

To prove (iii): Suppose that a = n and r ≠ 0. If m > n, then a < n by the above comments, is a contradiction ⊗ So m = n. Proved (iii).

To prove (iv): Suppose that a< n, r = 0, and (a – 1) |k. Let *f* = k| (a – 1). So k = *f* (a – 1) and a < n < *f*. Thus k = *f* (a – 1) = (*f* - 1 + 1)(a – 1) = (*f* – 1) (a – 1) + a – 1 = na with a – 1 < n. Hence M is (*f* – 1, n) - closed or open sub near-field space of commutative near-field space N over a near-field. M is not (*f*, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Hence m = *f* – 1 = k| (a – 1) – 1 is the largest +ve integer such that M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Proved (iv).

To prove (v): Suppose that a<n, r = 0, and (a – 1) does not divides k. Let *f* = k| (a – 1). So k = *f* (a – 1)+d and 1≤d< a– 1. Since a < n < f we have 1 ≤ d < a – 1 < *f* . Since k = *f* (a – 1) + d = na with 1 ≤ d < *f*. we have d < n. Hence M is (*f*, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Note that by a contradiction of f, if k = i (a – 1) + c for some 1 ≤ c < a – 1, then i ≤ *f*. Thus m = *f* = [k|(a – 1)] is the largest +ve integer such that M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Proved (v).

To prove (vi): Suppose that a < n, r ≠ 0, and a|k. Let *f* = k/a. So k = *f* a and *f* ≥ n + 1. Then M is not (*f*, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Let us assume that *f* – 1 > n. Thus k = *f*a = (*f* – 1 + 1) a = (*f* – 1) a + a. Since a < n < *f* – 1 and k = (*f* – 1) a + a = na + r. We conclude that M is (*f* – 1, n) - closed or open sub near-field space of commutative near-field space N over a near-field. So, m = *f* – 1 = k / (a – 1) is the largest positive integer such that M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Further, we assume that *f* – 1 = n. Then clearly m = n = k/(a – 1) is again the largest positive integer such that M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Proved (vi).

To prove (vii): Suppose that a < n, r ≠ 0, and a does not divide k. Let *f* = [k|a]. So k = *f* a + d, where 1 ≤ d < a. Since a < n < *f*, we have 1 ≤ d < a < *f*. Since k = *f*a + d = na + r and 1 ≤ d < *f*, we have d < n. Thus M is (*f*, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Note that by construction of *f*, if k = ia + c for some 1 ≤ c < a, then i < *f*. Thus m = *f* = [k/a] is the largest positive integer such that M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field. Proved (vii). This completes the proof of the theorem.

Theorem 3.12: Let N be an integral domain and M = p_1^k, p_2^k, ...,p_i^{ki}N, where $p_1, p_2, \ldots\ldots, p_k$ are non associate prime elements of N and $k_1, k_2, \ldots.., k_i$ are positive integers.

(a) Let *m* be a positive integer. If n_j is the smallest positive integer such that p_j^{kj}N is (m, nj) - closed or open sub near-field space of commutative near-field space N over a near-field for 1 ≤ j ≤ i, then $n = \max\{n_1, n_2, \ldots.., n_i\}$ is the smallest positive integer such that M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field.

(b) Let *n* be a positive integer. If m_j is the largest positive integer (or ∞) such that p_j^{kj}N is (m_j, *n*) - closed or open sub near-field space of commutative near-field space N over a near-field for 1≤ j ≤ i, then $m = \min\{m_1, m_2, \ldots.,\}$ is the largest positive integer (or ∞) such that M is (m, n) - closed or open sub near-field space of commutative near-field space N over a near-field.

Proof: Is obvious.

SECTION-4: GENERAL RESULTS ON CLOSED OR OPEN SUB NEAR-FIELD SPACES OF COMMUTATIVE NEAR-FIELD SPACE.

In this section, we continue the study of (m, n)-closed or open sub near-field space of N over a near-field and give several examples to illustrate earlier results. For a proper sub near-field space M of N over a near-field we investigate the two functions f_1 and $g1$ defined by f_1 (m) = min {n/M is (m, n)-closed or open sub near-field space of N} and g_1 (n) = Sup {m / M is (m, n)-closed or open sub near-field space of N}.

We assume throughout that all closed or open sub near-field space of N are commutative with $1 \neq 0$ and that $f(1) = 1$ for all near-field homomorphism f: N $\rightarrow$ S. For such a near-field space N, dim(N) denotes the Krull dimension of N, $\sqrt{M}$ denotes the radical sub near-field space of a near-field space M of N, and nil(N), Z(N), and U(N) denote the set sub near-field space nilpotent elements, zero divisors, and units of N, respectively; and N is reduced nil (N) = {0}.

Recall that N is von Neumann regular if for every x $\in$ N, there is y $\in$ N such that $x^2y = x$, and that N is π-regular if for every x$\in$N, there is y$\in$N a positive integer n such that x^{2m} y = x^n. Moreover, N is π-regular respectively von Neumann regular if and only if dim.(N) = 0 respectively N is reduced and dim(N) = 0.

Thus N is π-regular sub near-field space if and only if N/Nil (N) is von Neumann regular sub near-field space of N over a near-field. As usual, N, Z, Z_n and Q will denote the positive integers, integers, integers modulo n, and rational numbers respectively.

Let M be a proper sub near-field space of a commutative near-field space N over a near-field. We define N(M) = {(m, n) $\in$ N $\times$ N / M is (m, n)-closed or open sub near-field space of N over a near-field}. Thus {(m, n) $\in$ N $\times$ N / 1 $\leq$ m $\leq$ n} $\subseteq$ N(M) $\subseteq$N $\times$ N and N(M) = N $\times$ N if and only if $\sqrt{M}$ = M. We start with some elementary properties of N(M). If we define N(N) = N $\times$ N, then the results in this section hold for all sub near-field spaces of N over a near-field.

Theorem 4.1: Let N be a commutative near-field space over a near-field. M and P be proper sub near-field spaces of a near-field space N over a near-field, and m, n and k positive integers.

(a) (m, n) $\in$ N(M) for all positive integers m and n with m $\leq$ n.
(b) If (m, n) $\in$ N(M), then (m, n) $\in$ N(M) for all positive integers m and n with 1 $\leq$ m′ $\leq$ m and n′ $\geq$ n.
(c) If (m, n) $\in$ N(M), then (km, kn) $\in$ N(M).
(d) If (m, n), (n, k) $\in$ N(M), then (m, k)$\in$N(M).
(e) If (m, n), (m+1, n+1) $\in$ N(M), for m$\neq$ n, then (m+1, n)$\in$N(M).
(f) If (n, 2),(n+1, 2) $\in$ N(M), for an integer n $\geq$ 3, then (n+2, 2) $\in$ N(M), and
(g) thus (m, 2) $\in$ N(M) for every positive integer m.
(h) If (m, n) $\in$ N(M), for positive integers m and n with n $\leq$ m/2, then
(i) (m+1, n) $\in$ N(M) and thus (k, n) $\in$ N(M), for every positive integer k.
(j) (m, n) $\in$ N(M), for every positive integers m if and only if (2n, n) $\in$ N(M).
(k) N(M $\times$ P) = N(M) $\cap$ N(P) $\subseteq$ N(M $\cap$ P).

Proof: To prove (a) to (d): It easily follows from the basic definitions. Hence Proved (a) to (d).

To prove (e): If m < n, then (m+1, n) $\in$ N(M) by (a). For m > n, suppose that x^{m+1} $\in$ M for x $\in$ N. then x^{n+1} $\in$ M since M is (m+1, n+1) - closed or open sub near-field space of N over a near-field. Thus x^m $\in$ M since m $\geq$ n + 1, and hence x^n $\in$ M since M is (m, n) - closed or open sub near-field space of N over a near-field. Thus M is (m+1, n) - closed or open sub near-field space of N over a near-field. Proved (e).

To prove (f): Suppose that $x^{n+2}$$\in$M for x$\in$N. Then $(x^2)^n = x^{2n}$$\in$M since 2n $\geq$ n + 2 because n $\geq$ 2. Hence $x^4 = (x^2)^2$$\in$M since (n, 2) - closed or open sub near-field space of N over a near-field. But then $x^{n+1}$$\in$M since n $\geq$ 3. Thus x^2 $\in$ M since M is (n+1, 2) - closed or open sub near-field space of N over a near-field. Hence M is (n+2, 2) - closed or open sub near-field space of N over a near-field. Similarly, (k, 2) $\in$ N(M) for every integer k $\geq$ n + 3. So by (b), M is (k, 2) - closed or open sub near-field space of N over a near-field for every positive integer k. Proved (f).

To prove (g): Let x^{m+1} $\in$ M for x $\in$ N. Then $(x^2)^m = x^{2m}$ $\in$ M, and hence $x^{2n} = (x^2)^n$ $\in$ M since M is (m, n) - closed or open sub near-field space of N over a near-field. Thus $x^m$$\in$M since 2n $\leq$ m, and hence x^n $\in$ M since M is (m, n) - closed or open sub near-field space of N over a near-field. Thus M is (m+1, n) - closed or open sub near-field space of N over a near-field. Similarly, (k, n) $\in$ N(M) for every integer k $\geq$ n, and hence (k, n) $\in$ N(M) for every positive integer k by (b). Proved (g).

To prove (h): obvious with the help of proof of (g). Proved (g).

To prove (i): Clearly M $\times$ P is (m, n) - closed or open sub near-field space of N over a near-field if and only if M and P are both (m, n) - closed or open sub near-field space of N over a near-field. Thus N(M $\times$ P) = N(M) $\cap$ N(P). Thus N(M) $\cap$ N(P) $\subseteq$ N(M $\cap$ P) follows that N(M $\times$ P) = N(M) $\cap$ N(P) $\subseteq$ N(M $\cap$ P). Hence proved (i).
This completes the proof of the theorem.

Note 4.2: The m $\neq$ n hypothesis is needed and since (n, n) $\in$ N(M) for every positive integer n.

Note 4.3: The n $\geq$ 3 hypothesis in needed and for n =1, we have (1, 2), (2, 2) $\in$ N(M) for every proper sub near-field space M of N, but in general, (3,2) $\notin$ N(M). For n = 2, we have (2,2), (3,2) $\in$ N(M) does not imply (4, 2) $\in$ N(M). For example, let N = Z and M = 16Z. Then (2,2), (3,2) $\in$ N(M), but (4,2) $\notin$ N(M).

Note 4.4: The inclusion may be strict. For example, Let N = Z, M = 8Z and P = 16Z. Then (3, 2) $\in$N(P) =N(M $\cap$ P). However, (3, 2) $\notin$ N(M). So N(M) $\cap$ N(P) $\subseteq$ N(M $\cap$ P).

Note 4.5: More generally, N(M $\times$ P) = N(M) $\cap$ N(P) for all sub near-field spaces M and P of a commutative near-field space of N and T, respectively.

Let M be a proper sub near-field space of a commutative near-field space N over a near-field and m and n be +ve integers. We define f_1 (m) = min {n/M is (m, n) - closed or open sub near-field space of N over a near-field}$\in$ {1,2,...,m} and g_1(n) = Sup {m / M is (m, n) - closed or open sub near-field space of N over a near-field} $\in$ {n, n+1,....} $\cup$ {∞}. So $f_1 : N \to N$ and $g_1 : N \to N \cup \{\infty\}$. The columns respectively rows of N(M) determine f_1 (or g_1). Then either function f_1 or g_1 is determined the other, and either function determines N(M). It is sometimes useful to view f_1 (or g_1) as an N-valued respectively N $\cup$ {∞} valued non-decreasing sequence $f_1 = (f\,1\,(m))$ (or $g_1 = g_1$ (n)). Note that f_1 = (1,1,1,...) if and only if g_1 = {∞,∞,∞...}, if and only if $\sqrt{M}$ = M. if we define N(N) = N $\times$ N, then f_N = (1,1,1,...) and g_N = (∞,∞,∞,....). Also f_1 is eventually constant if and only if g_1 is eventually constant, if and only if g_1 is eventually ∞. We next give some elementary properties of the two functions f_1 and g_1.

Theorem 4.6: Let N be a commutative near-field space, M be a proper sub near-field space of N and m and n are + ve integers. Let f_1 (m) = min { n/M is (m, n) - closed or open sub near-field space of N over a near-field} and g_1(n) = Sup {m / M is (m, n) - closed or open sub near-field space of N over a near-field }.

(a) $1 \leq f_1 (m) \leq m$
(b) $f_1 (m) \leq f_1(m + 1)$
(c) If f_1 (m) < m, then either $f_1(m + 1) = f_1(m)$ or $f_1(m+1) \geq f_1(m) + 2$.
(d) $n \leq g_1 (n) \leq \infty$.
(e) $g_1 (n) \leq g_1 (n + 1)$
(f) If g_1 (n) > n, then either $g_1 (n+1) = g_1 (n)$ or $g_1 (n+1) \geq g_1 (n) +2$.

Proof: Obvious.

Theorem 4.7: Let N be a commutative near-field space and M and P proper sub near-field spaces of N. Let f_1(m) = min {n / M is (m, n) - closed or open sub near-field space of N over a near-field } and g_1 (n) = Sup {m / M is (m, n) - closed or open sub near-field space of N over a near-field}.

(a) $f_{M \cap P} \leq f_M \vee f_P$
(b) $g_{M \cap P} \leq g_M \vee g_P$
(c) N (M $\cap$ P) = N(M) $\cap$ N(P).

Proof: Obvious.

Theorem 4.8: Let N be a sub near-field space and x, y $\in$ N co-prime elements. Then N(xyN) = N(xN $\cap$ yN) = N(xN) $\cap$ N(yN). Moreover, $f_{xyN} = f_{xN} \vee f_{yN}$ and $g_{xyN} = g_{xN} \wedge g_{yN}$.

Proof: Obvious.

Theorem 4.9: Let N be a commutative near-field space, n a positive integer, and M an n-absorbing sub near-field space of N. Then $f_1(m) \leq n$ for every positive integer m. Thus f_1 and g_1 are eventually constant. In particular, if N is Noetherian, then f_1 and g_1 are eventually constant for every proper sub near-field space M of N.

Proof: Obvious.

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