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c*g-CONTINUOUS FUNCTION IN BITOPOLOGICAL SPACES

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ABSTRACT

*In this paper, we have introduced the concept of c*g –continuous functions and study some of their properties in bitopological spaces.*

Keywords: *(i, j)-c*g-continuous function.*

1. INTRODUCTION

A triple (x, τ_1, τ_2) where X is a non-empty set and τ_1 and τ_2 are topologies on X is called a bitopological space and Kelly [5] initiated the study of such spaces. In 1985 Fukutake [3] introduced the concepts of g-closed sets in bitopological spaces and after that several authors turned their attention to the generalization of various concepts of topology by considering bitopological spaces instead of topological spaces. In 2004, P.Sundaram [12] introduced the concept of g*-closed sets in bitopological spaces.

In 1970 Levine introduced the concept of generalized closed sets in topological spaces. Pushpalatha [11] introduced strongly g-closed and investigated many properties related to it. Palaniappan and Rao [10] and Gnanambal [6] investigated rg-closed, gpr-closed sets in topological spaces respectively. As well as they investigated in continuous function also.

In this chapter we present the notion of c*g-continuous maps and c*g-irresolute in bitopological spaces and we obtain some interesting results.

Throughout this chapter (X, τ_1, τ_2) (or X) and (Y, σ_1, σ_2) (or Y) denote two non empty bitopological spaces. In this section we introduce the concept of (i, j)-c*g-closed sets and we obtain some interesting results in bitopological spaces.

2. PRELIMINARIES

Definition 2.1: A function f: X→Y is called

(i) (i, j)-σ_k-weakly generalized continuous ((i,j)-σ_k-wg - continuous) if the inverse image of σ_k–closed set is (i,j)-σ_k-weakly generalized closed in X. [4]

(ii) (i, j)-σ_k-w - continuous if the inverse image of σ_k–closed set is (i, j)-σ_k-w closed in X. [5]

(iii) (i, j)-σ_k-gpr- continuous if the inverse image of σ_k–closed set is (i, j)-σ_k-gpr closed in X. [5]

(iv) (1, 2)*-gs-continuous [9] if f^{-1}(F) is (1, 2)*-gs-closed set in X for every $\sigma_{1,2}$-closed set V in Y.

3. c*g-CONTINUOUS FUNCTIONS IN BITOPOLOGICAL SPACES

Definition 3.1: A map f: (X, τ_1, τ_2) →(Y, σ_1, σ_2) from a bitopological space X into a bitopological space Y is called (i, j)- **c***g-continuous if the inverse image of every $\tau_j - \sigma_k$-closed set in Y is (i, j)- σ_k-c*g- closed in X.

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Theorem 3.2: If a map f: (X, τ_1, τ_2) →(Y, σ_1, σ_2) is $\tau_j - \sigma_k$ -continuous then it is (i, j)- σ_k -c*g- continuous but not conversely. The converse of the above theorem need not be true as seen from the following example.

Example 3.3: Let X = Y= {a,b,c} with the topologies τ_1= { φ,X,{a, b}}; τ_2= { φ,X,{a},{b},{a, b}} and σ_1= { φ,Y,{a, c}}; σ_2 ={ φ,Y,{c},{a, b}}. Let f: X→Y be the identity. Then f is (1, 2)- σ_2 -c*g continuous but not τ_2 - σ_2 -continuous, since for the τ_2 - σ_2 -closed set {a,b} in Y, f^{-1}({a,b})={a,b} is not τ_2 - σ_2 - closed in X.

Theorem 3.4: If a map f: X→Y is (i, j)- σ_k -strongly g-continuous then it is (i,j)- σ_k -c*g- continuous but not conversely. The converse of the above theorem need not be true as seen from the following example.

Example 3.5: Let X = Y ={a, b, c} with the topologies τ_1= { φ,X,{a, b}}; τ_2={ φ,X,{a},{b},{a, b}} and σ_1={ φ,Y,{b, c}}; σ_2 = { φ,Y,{c},{a ,b}}. Let f : X → Y be the identity. Then f is (1,2) - σ_2 -c*g- continuous but not (1,2)- σ_2 -strongly g-continuous, since for the σ_2 -closed set {a, b} in Y, f^{-1} ({a, b})={a, b} is not (1,2)- σ_2 - strongly g- closed in X.

Theorem 3.6: Let f: X→Y be a map. Then following statements are equivalent.

(a) f is (i,j)- σ_k -c*g- continuous

(b) The inverse image of each σ_j -open set in Y is (i, j)- σ_k -c*g-open in X.

Proof: Assume that f: X→Y is (i, j)- σ_k -c*g- continuous. Let G be $\tau_j - \sigma_j$ -open in Y, then G^c is $\tau_j - \sigma_j$ -closed in Y. Since f is (i,j)- σ_k -c*g- continuous, $f^{-1}(G^c)$ is (i,j)- σ_k -c*g- continuous in X. But $f^{-1}(G^c)$ = X- f^{-1}(G). Thus f^{-1}(G) is (i, j)- σ_k -c*g-open in X.

Conversely, Assume that the inverse image of each $\tau_j - \sigma_j$ -open set in Y is (i,j) - σ_k -c*g-open in X. Let F be any $\tau_j - \sigma_j$ -closed set in Y. Then F^c is $\tau_j - \sigma_j$ -open in Y. By assumption, $f^{-1}(F^c)$ is (i, j)- σ_k -c*g-open in X. Hence (a) and (b) are equivalent.

Theorem 3.7: If a map f: X→Y is (i, j)- σ_k -c*g- continuous, then it is (i, j)- σ_k -generalized pre regular continuous. The converse of the above theorem need not be true as seen from the following example.

Example 3.8: Let X = Y= {a,b,c} with the topologies τ_1 ={ φ,X,{a},{b},{c},{a,b},{b,c},{a,c}} and τ_2 ={ φ,X,{a, c}} and σ_1= { φ,Y,{b},{c},{b,c}} ; σ_2 = { φ,Y,{c},{a,b}}. Let f: X→Y be the identity. Then f is (1,2) - σ_2 -g pr-continuous but not (1,2)- σ_2 - c*g- continuous, since for the σ_2 -closed set {b,c} in Y, f^{-1} ({a,b})={a,b} is not (1,2)- σ_2 -c*g- closed in X.

Remark 3.9: We illustrate the relations between various generalizations of continuous functions in the following diagram.

$\tau_j - \sigma_j$ **-Continuity**

↓

(i,j)- σ_k **- strongly g- continuity**

↓

(i,j)- σ_k **-c*g- continuity**

↓

(i,j)- σ_k **-gpr – continuity.**

Figure – 3.1

In the above diagram none of the implications can be reversed.

Remark 3.10: The concept of (i, j)- σ_k -c*g -continuous is independent of the following classes of continuous sets namely (i, j)- σ_k -w-continuous, (i, j)- σ_k -αg- continuous, (i, j)- σ_k -gα- continuous, (i, j)- σ_k -sg- continuous, (i, j)- σ_k -gs- continuous, (i, j)- σ_k -wg- continuous and (i,j)- σ_k - g- continuous.

Example 3.11: Let X = Y= {a,b,c} with the topologies τ_1 ={ φ ,X,{a, b}}; τ_2 ={ φ ,X,{a},{b},{a, b}} and σ_1 ={ φ ,Y,{a, c}}; σ_2 ={ φ ,Y,{c},{a, b}}. Let f: X→Y be the identity. Then f is (1,2) - σ_2 -c*g-continuous but not(1,2)- σ_2 -g- continuous, since for the σ_2 -closed set {a,b} in Y, f^{-1} ({a,b})={a,b} is not (1,2)- σ_2 -αg- closed, (1,2)- σ_2 -gα- closed and (1,2)- σ_2 -g- closed in X. Consider the topologies τ_1 ={ φ ,X,{c},{a,b}}; τ_2 ={ φ ,X,{b}, {b,c},{a,b}} and σ_1 ={ φ ,Y,{a},{a,b}}; σ_2 = { φ ,Y,{b},{b,c}}. Let f: X→Y be the identity. Then f is (1,2)- σ_1 - g- continuous, (1,2)- σ_1 - αg- continuous and (1,2)- σ_1 - gα- continuous but not (1,2)- σ_1 -c*g- continuous, since for the σ_1 -closed set {b,c} in Y, f^{-1} ({b,c})={b,c} is not (1,2)- σ_1 -c*g- closed in X.

Example 3.12: Let X = {a, b, c} with the topologies τ_1 = { φ ,X,{a},{b},{c},{a,b},{b,c},{a,c}} ; τ_2 ={ φ ,X,{a,c}} and σ_1 = { φ ,Y,{a},{a, b}} ; σ_2 = { φ ,Y,{a,c}}. Let f: X→Y be the identity. Then f is (1,2)- σ_2 -pg- continuous but not (1,2)- σ_2 -c*g- continuous, since for the σ_2 -closed set {a,b} in Y, f^{-1} ({a,b})={a,b} is not (1,2)- σ_2 -c*g- closed in X.

Consider the topologies τ_1 = { φ ,X,{b},{c},{b, c}} ; τ_2 ={ φ ,X,{b}} and σ_1 = { φ ,Y,{c},{a, c}}; σ_2 = { φ ,Y,{a}, {b,c}}. Let f: X→Y be the identity. Then f is (1,2)- σ_1 -c*g- continuous but not (1,2)- σ_1 - pg- continuous, since for the σ_1 -closed set {b,c} in Y, f^{-1} ({b,c})={b,c} is not (1,2)- σ_1 - pg- closed in X.

Example 3.13: Let X ={a, b, c} with the topologies τ_1 ={ φ ,X,{a},{a,b},{a,c}} ; τ_2 ={ φ ,X,{c},{a,b}} and σ_1 ={ φ ,Y,{a},{a,c}}; σ_2 ={ φ ,Y,{a},{b,c}}. Let f: X→Y be the identity. Then f is (1,2)- σ_2 -c*g-continuous but not (1,2)- σ_2 -w-continuous, since for the σ_2 -closed set{a}in Y, f^{-1}({a})={a} is not (1,2)- σ_2 -w- closed in X. Consider the topologies τ_1 = { φ ,X,{b},{a, c}}; τ_2 ={ φ ,X,{a,c}} and σ_1 = { φ ,Y,{a},{c},{a,c}}; σ_2 = { φ ,Y,{a,c}}. Let f: X→Y be the identity. Then f is (1,2)- σ_1 -w -continuous but not (1,2)- σ_1 -c*g- continuous, since for the σ_1 -closed set{a,b} in Y,f^{-1}{a,b}={a,b} is not (1,2)- σ_1 -c*g-closed in X.

Example 3.14: Let X = Y= {a,b,c} with the topologies τ_1 = { φ ,X,{a}}; τ_2 ={ φ ,X,{a},{c},{a,c}} and σ_1 = { φ ,Y,{c}} ; σ_2 = { φ ,Y,{c},{a, b}}. Let f: X→Y be the identity. Then f is (1,2) - σ_2 -sg- continuous, (1,2)- σ_2 -gs- continuous and (1,2)- σ_2 -wg- continuous but not (1,2)- σ_2 -c*g- continuous, since for the σ_2 -closed set { c} in Y, f^{-1} ({c})={ c} is not (1,2)- σ_2 -c*g- closed in X.

Example 3.15: Let X =Y= {a, b, c} with the topologies τ_1 = { φ ,X,{a},{a,b},{a,c}}; τ_2 ={ φ ,X,{c},{a,b}} and σ_1 ={ φ ,Y,{b},{a,b}}; σ_2 = { φ ,Y,{a,b}}. Let f: X→Y be the identity. Then f is (1,2)- σ_1 -c*g- continuous but not (1,2)- σ_1 -gs- continuous, (1,2)- σ_1 -sg- continuous and (1,2)- σ_1 -wg- continuous, since for the σ_1 -closed set {a,c} in Y, f^{-1} ({a,c})={a,c} is not (1,2)- σ_1 -gs- closed, (1,2)- σ_1 -sg- closed and (1,2)- σ_1 -wg- closed in X.

Remark 3.16: From the above discussion and known results we have the following diagram.

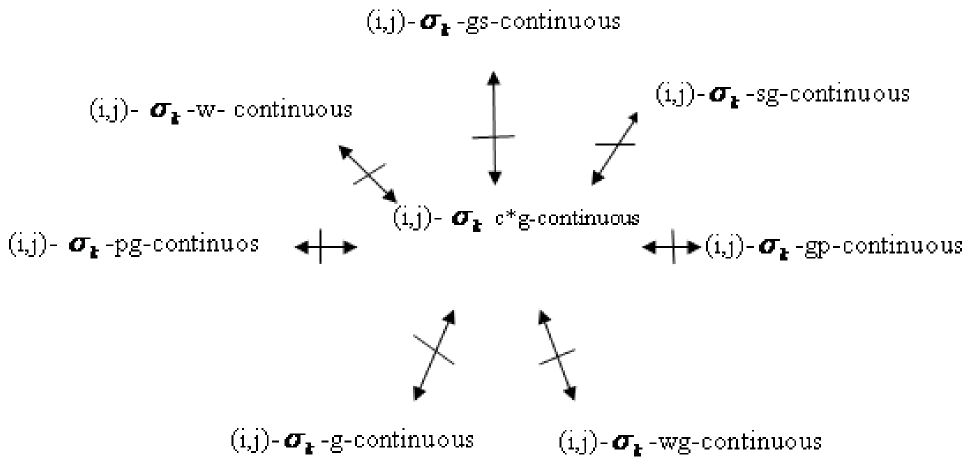

Figure- 3.2.2

Remark 3.17: In general (i,j)- σ_k -c*g-continuous is independent with (j,i)- σ_k -c*g-continuous. This is proved by the following example.

Example 3.18: Let X= Y= {a, b, c} with the topologies τ_1 = { φ ,X,{a},{b},{a,b},{b,c}}; τ_2 ={ φ ,X,{b},{c},{b,c}} and σ_1 = { φ ,Y,{a},{b,c}}; σ_2 = { φ ,Y,{c},{b,c}}. Let f: X→Y be the identity. Then f is (2, 1)- σ_1 -c*g- continuous but not (1, 2) - σ_1 - c*g- continuous. Since for the σ_1 -closed set {a, c} in Y, f^{-1} ({b, c}) = {b, c} is not (1, 2) - σ_1 - c*g- closed in X. Consider the topologies τ_1 = { φ ,X,{b}} ; τ_2 ={ φ ,X,{b,c}} and σ_1 ={ φ ,Y,{b},{a, c}}; σ_2 ={ φ ,Y,{c},{b, c}}.Let f: X→Y be the identity. Then f is (1, 2) - σ_1 -c*g -continuous but not (2, 1)- σ_1 -c*g- continuous. Since for the σ_1 -closed set {b} in Y, f^{-1}({b}) = {b} is not (2, 1) - σ_1 -c*g- closed in X.

Remark 3.19: In general, the composition of two (i,j)-c*g-continuous is not (i,j)-c*g-continuous. We have the following example.

Example 3.2.20: Let X=Y={a,b,c} with the topologies τ_1 = { φ ,X,{a, b}} ; τ_2 ={ φ ,X,{a},{b},{a, b}}; σ_1 ={ φ ,Y,{c},{a, b}}; σ_2 = { φ ,Y,{b},{b, c},{a, b}} and υ_1 ={ φ ,Z,{b}} ; υ_2 ={ φ , Z{b, c}}. Let f: (X, τ_1 , τ_2) → (Y, σ_1 , σ_2) and g : (Y, σ_1 , σ_2) → (Z, υ_1 , υ_2) be the identity. It is easily observed that f is (1, 2)- σ_1 -c*g – continuous and g is (1,2)- σ_k-c*g -continuous. But the composition function $g \circ f$: (X, τ_1 , τ_2) →(Z, υ_1 , υ_2) is not (1, 2)- υ_2 –c*g- continuous, since the τ_2 - υ_2 -closed set in Z, f^{-1} ({a})={a} is not (1, 2)- υ_2 -c*g- closed in X.

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