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ON WI$_{\hat{g}}$- CONTINUOUS AND WI$_{*g}$-CONTINUOUS FUNCTIONS IN IDEAL TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce and study the notions of $wI_{\hat{g}}$-continuous and wI_{*g}-continuous, $wI_{\hat{g}}$-irresolute and wI_{*g}-irresolute in ideal topological spaces, and also we studied their properties.

Keywords: *$wI_{\hat{g}}$-closed, wI_{*g}-closed, $wI_{\hat{g}}$-continuous, wI_{*g}-continuous, $wI_{\hat{g}}$-irresolute, wI_{*g}-irresolute.*

1. INTRODUCTION AND PRELIMINARIES

Ideals in topological spaces have been considered since 1930. In 1990, Jankovic and Hamlett [2] once again investigated applications of topological ideals. The notion of I_g-closed sets was first by Dontchev.*et.al* [1] in 1999. Navaneethakrishnan and Joseph [3] further investigated and characterized I_g-closed sets and I_g-open sets by the use of local functions. The notion of I_{*g}-closed sets was introduced by Ravi.*et.al* [4] in 2013. Recently the notion of $wI_{\hat{g}}$-closed sets and wI_{*g}-closed sets was introduced and investigated by Maragathavalli.*et.al* [5]. In this paper, we introduce the notions of $wI_{\hat{g}}$-continuous and wI_{*g}-continuous functions in ideal topological spaces.

An ideal I on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following properties. (1) A ∈I and B ⊆ A implies B∈ I, (2) A ∈ I and B ∈ I implies A∪B∈I. An ideal topological space is a topological space (X, τ) with an ideal I on X and is denoted by (X, τ, I). For a subset A ⊆X, A*(I,τ)={x ∈X: A ∩ U ∉I for every U∈τ (X,x)} is called the local function of A with respect to I and τ [6]. We simply write A* in case there is no chance for confusion. A Kuratowski closure operator cl*(.) for a topology τ *(I, τ) called the *-topology, finer than τ is defined cl*(A) = A∪ A* [7]. If A ⊆ X, cl(A) and int(A) will respectively, denote the closure and interior of A in (X, τ).

Definition 1.1: A subset A of a topological space (X, τ) is called
1. g-closed [8], if cl (A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
2. $\hat{g}$-closed [9], if cl(A)⊆ U whenever A ⊆ U and U is semi open in (X, τ).
3. *g-closed [4], if A* ⊆ U whenever A ⊆ U and U is $\hat{g}$-open in (X, τ).

Definition 1.2: A subset A of a topological space (X, τ) is called
1. I_g-closed [3], if A* ⊆ U whenever A ⊆ U and U is open in X.
2. $I_{\hat{g}}$-closed [10], if A* ⊆ U whenever A ⊆ U and U is semi-open in X.
3. $wI_{\hat{g}}$-closed [5], if int(A*)⊆ U whenever A ⊆ U and U is semi-open in X.
4. wI_{*g}-closed [5], if int(A*)⊆ U whenever A ⊆ U and U is $\hat{g}$-open in X.

Definition 1.3: A function f :(X, τ, I)→ (Y, σ) is said to be
1. g-continuous [11], if for every open set V∈ σ, f^{-1}(V) is g-open in (X, τ).
2. $\hat{g}$-continuous [9], if for every open set V∈ σ, f^{-1}(V) is $\hat{g}$-open in (X, τ).

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Definition 1.4: A function f: (X, τ, I)→ (Y, σ) is said to be I_g-continuous [12], if $f^{-1}(V)$ is I_g-closed in (X, τ, I) for every closed set V in (Y, σ).

2. $wI_{\hat{g}}$-CONTINUOUS AND wI_{*g}-CONTINUOUS

Definition 2.1: A function f: (X, τ, I)→ (Y, σ) is Said to be
1. weakly$I_{\hat{g}}$-continuous (briefly $wI_{\hat{g}}$-continuous) if $f^{-1}(V)$ is weakly $I_{\hat{g}}$-closed set in (X, τ, I) for every closed set V in (Y, σ).
2. weaklyI_{*g}-continuous (briefly wI_{*g}-continuous) if $f^{-1}(V)$ is weakly I_{*g}-closed set in (X, τ, I) for every closed set V in (Y, σ).

Definition 2.2: A function f: (X, τ, I_1)→ (Y, σ, I_2) is Said to be
- **(i)** $wI_{\hat{g}}$-irresolute if $f^{-1}(V)$ is $wI_{\hat{g}}$-closed in (X, τ, I_1) for every $wI_{\hat{g}}$-closed set V in (Y, σ, I_2).
- **(ii)** wI_{*g}-irresolute if$f^{-1}(V)$ is wI_{*g}-closed in (X, τ, I_1) for every wI_{*g}-closed set V in (Y, σ, I_2).

Theorem 2.3: Every continuous function is $wI_{\hat{g}}$-continuous.

Proof: Let f be an continuous function and let V be a closed set in (Y, σ). Then $f^{-1}(V)$ is closed set in (X, τ, I). Since every closed set is $wI_{\hat{g}}$-closed. Hence $f^{-1}(V)$ is $wI_{\hat{g}}$-closed set in (X, τ, I). Therefore f is $wI_{\hat{g}}$-continuous.

Example2.4: Let X = Y = {a, b, c}, τ = {φ, {b}, {b,c}, X}, σ = {φ, {c}, Y} and I = {φ, {b}}. Let the function f:(X, τ, I)→ (Y, σ) be the idendity function. Then the function f is $wI_{\hat{g}}$-continuous but not continuous.

Theorem 2.5: Ever continuous function is wI_{*g}-continuous.

Proof: Let f be an continuous function and let V be a closed set in (Y, σ). Then $f^{-1}(V)$ is closed set in (X, τ, I). Since every closed set is wI_{*g}-closed. Hence $f^{-1}(V)$ is wI_{*g}-closed set in (X, τ, I). Therefore f is wI_{*g}-continuous.

Example2.6: Let X = Y = {a, b, c}, τ = {φ, {b}, {b, c}, X}, σ = {φ, {c}, Y} and I = {φ, {b}}. Let the function f: (X, τ, I)→ (Y, σ) be the identity function. Then the function f is wI_{*g}-continuous but not continuous.

Theorem 2.7: Ever $I_{\hat{g}}$-continuous function is $wI_{\hat{g}}$-continuous.

Proof: Let f be an $I_{\hat{g}}$-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is $I_{\hat{g}}$-closed set in (X, τ, I). Since every $I_{\hat{g}}$-closed set is $wI_{\hat{g}}$-closed. Hence $f^{-1}(V)$ is $wI_{\hat{g}}$-closed set in (X, τ, I). Therefore f is $wI_{\hat{g}}$-continuous.

Example2.8: Let X = Y = {a, b, c, d}, τ = {φ, {a, b}, {a, b, c}, X}, σ = {φ, {a, b}, {a}, Y} and I = {φ, {a}}. Let the function f: (X, τ, I)→ (Y, σ) is defined by f(a) = b, f(b) = c, f(c) = a, f(d) = d. Then the function f is $wI_{\hat{g}}$-continuous but not $I_{\hat{g}}$-continuous.

Theorem 2.9: Ever $\hat{g}$-continuous function is $wI_{\hat{g}}$-continuous.

Proof: Let f be an $\hat{g}$-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is $\hat{g}$-closed set in (X, τ, I). Since every $\hat{g}$-closed set is $wI_{\hat{g}}$-closed set. Hence $f^{-1}(V)$ is $wI_{\hat{g}}$-closed set in (X, τ, I). Therefore f is $wI_{\hat{g}}$-continuous.

Example2.10: Let X = Y = {a, b, c, d}, τ = {φ, {b}, {a, b, c}, X}, σ = {φ, {c}, {a, c}, Y} and I = {φ, {c}}. Let the function f: (X, τ, I)→ (Y, σ) be the identity function. Then the function f is $wI_{\hat{g}}$-continuous but not $\hat{g}$-continuous.

Theorem 2.11: Ever g-continuous function is $wI_{\hat{g}}$-continuous.

Proof: Let f be an g-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is g-closed set in (X, τ, I). Since every g-closed set is $wI_{\hat{g}}$-closed set. Hence $f^{-1}(V)$ is $wI_{\hat{g}}$-closed set in (X, τ, I). Therefore f is $wI_{\hat{g}}$-continuous.

Example 2.12: Let X = Y = {a, b, c, d}, τ = {φ, {b}, {c}, {b, c}, X}, σ = {φ, {c}, X} and I = {φ, {b}}. Let the function f: (X, τ, I) → (Y, σ) be the idendity function. Then the function f is $wI_{\hat{g}}$-continuous but not g-continuous.

Theorem 2.13: Ever I_{*g}-continuous function is wI_{*g}-continuous.

Proof: Let f be an wI_{*g}-continuous function and let V be a closed set in (Y, σ). Then $f^{-1}(V)$ is wI_{*g}-closed set in (X, τ, I). Since every wI_{*g}-closed set is $wI_{\hat{g}}$-closed, hence $f^{-1}(V)$ is wI_{*g}-closed set in (X, τ, I). Therefore f is wI_{*g}-continuous.

Example 2.14: Let X = Y = {a, b, c, d}, τ = {φ, {a,b},{c,d}, X}, σ = {φ, {c,d}, Y} and I = {φ, {d}}. Let the function f: (X, τ, I) → (Y, σ) be the idendity function. Then the function f is wI_{*g}-continuous but not I_{*g}-continuous.

Theorem 2.15: Ever g-continuous function is wI_{*g}-continuous.

Proof: Let f be an g-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is g-closed set in (X, τ, I). Since every g-closed set is wI_{*g}-closed set. Hence $f^{-1}(V)$ is wI_{*g}-closed set in (X, τ, I). Therefore f is wI_{*g}-continuous.

Example 2.16: Let X = Y = {a, b, c, d}, τ = {φ, {a,b},{a,b,c}, X}, σ = {φ, {d}, {c,d}, Y} and I = {φ, {a}}. Let the function f: (X, τ, I) → (Y, σ) be the idendity function. Then the function f is wI_{*g}-continuous but not g-continuous.

Theorem 2.17: Ever I_g-continuous function is $wI_{\hat{g}}$-continuous.

Proof: Let f be an I_g-continuous function and let V be a closed set in (Y, σ), then $f^{-1}(V)$ is I_g-closed set in (X, τ, I). Since every I_g-closed set is $wI_{\hat{g}}$-closed set. Hence $f^{-1}(V)$ is $wI_{\hat{g}}$-closed set in (X, τ, I). Therefore f is $wI_{\hat{g}}$-continuous.

Example 2.18: In example 2.17, let the function f: (X, τ, I) → (Y, σ) be the idendity function. Then the function f is $wI_{\hat{g}}$-continuous but not I_g-continuous.

Theorem 2.19: Ever I_g-continuous function is wI_{*g}-continuous.

Proof: Let f be an I_g-continuous function and let V be a closed set in (Y, σ). Then $f^{-1}(V)$ is I_g-closed set in (X, τ, I). Since every I_g-closed set is wI_{*g}-closed set. Hence $f^{-1}(V)$ is wI_{*g}-closed set in (X, τ, I). Therefore f is wI_{*g}-continuous.

Example 2.20: Let X = Y = {a, b, c, d}, τ = {φ, {b}, {a,b,c}, X}, σ = {φ, {a}, {a,c,d}, Y} and I = {φ, {d}}. Let the function f: (X, τ, I) → (Y, σ) be the idendity function. Then the function f is wI_{*g}-continuous but not I_g-continuous.

Theorem 2.21: Ever wI_{*g}-continuous function is $wI_{\hat{g}}$-continuous.

Proof: Let f be a wI_{*g}-continuous function and let V be a closed set in (Y, σ). Then $f^{-1}(V)$ is wI_{*g}-closed set in (X, τ, I). Since every wI_{*g}-closed set is $wI_{\hat{g}}$-closed. Hence $f^{-1}(V)$ is $wI_{\hat{g}}$-closed set in (X, τ, I). Therefore f is $wI_{\hat{g}}$-continuous.

Example 2.22: Let X = Y = {a, b, c, d}, τ = {φ, {d}, {a, b, c}, X}, σ = {φ, {a}, Y} and I = {φ, {b}}. Let the function f: (X, τ, I) → (Y, σ) be the idendity function. Then the function f is $wI_{\hat{g}}$-continuous but not wI_{*g}-continuous.

Theorem 2.23: A map f: (X, τ, I) → (Y, σ) is $wI_{s\hat{g}}$-continuous iff the inverse image of every closed set in (Y, σ) is $wI_{\hat{g}}$- closed in (X, τ, I).

Proof: Necessary: Let v be an open set in (Y, σ). Since f is $wI_{\hat{g}}$- continuous, $f^{-1}(v^C)$is $wI_{\hat{g}}$- closed in (X, τ, I). But $f^{-1}(v^C) = X - f^{-1}(v)$. Hence $f^{-1}(v)$ is $wI_{\hat{g}}$- closed in (X, τ, I).

Sufficiency: Assume that the inverse image of every closed set in (Y, σ) is $wI_{\hat{g}}$- closed in (X, τ, I). Let v be a closed set in (Y, σ). By our assumption $f^{-1}(v^C) = X - f^{-1}(v)$ is $wI_{\hat{g}}$- closed in (X, τ, I) , which implies that $f^{-1}(v)$ is $wI_{\hat{g}}$- closed in (X, τ, I). Hence f is $wI_{\hat{g}}$- continuous.

Remark 2.24:
(i) The union of any two $wI_{\hat{g}}$- continuous function is $wI_{\hat{g}}$continuous.
(ii) The intersection of any two $wI_{\hat{g}}$- continuous function is need not be $wI_{\hat{g}}$- continuous.

Theorem 2.25: Let f: $(X,\tau, I_1) \rightarrow (Y, \sigma, I_2)$ and g: $(Y, \sigma, I_2) \rightarrow (Z, \eta, I_3)$ be any two functions. Then the following hold.
(i) g ∘ f is w$I_{\hat{g}}$- continuous if f is w$I_{\hat{g}}$continuous and g is continuous.
(ii) g∘ f is w$I_{\hat{g}}$- continuous if f is w$I_{\hat{g}}$irresolute and g is w$I_{\hat{g}}$continuous.
(iii) g∘ f is w$I_{\hat{g}}$- irresolute if f is w$I_{\hat{g}}$irresolute and g is irresolute.

Proof:
(i) Let v be a closed set in Z. Since g is continuous, $g^{-1}(v)$ is closed in Y. w$I_{\hat{g}}$-continuous of f implies, $f^{-1}(g^{-1}(v))$ is w$I_{\hat{g}}$-closed in X and hence g ∘ f is w$I_{\hat{g}}$-continuous.
(ii) Let v be a closed set in Z. Since g is w$I_{\hat{g}}$-continuous, $g^{-1}(v)$ is w$I_{\hat{g}}$-closed in Y. Since f is w$I_{\hat{g}}$-irresolute, $f^{-1}(g^{-1}(V))$ is w$I_{\hat{g}}$-closed in X. Hence g ∘ f is w$I_{\hat{g}}$-continuous.
(iii) Let v be a w$I_{\hat{g}}$-closed in Z. Since g is w$I_{\hat{g}}$- irresolute, $g^{-1}(v)$ is w$I_{\hat{g}}$-closed in Y. Since f is w$I_{\hat{g}}$-irresolute, $f^{-1}(g^{-1}(v))$ is w$I_{\hat{g}}$-closed in X. Hence g ∘ f is w$I_{\hat{g}}$-irresolute.

Theorem 2.26: Let X=A∪B be a topological space with topology τ and Y be a topological space with topology σ. Let f: $(A,\tau/A) \rightarrow (Y,\sigma)$ and g: $(B, \tau/B) \rightarrow (Y,\sigma)$ be w$I_{\hat{g}}$-continuous maps such that f(x)=g(x) for every x ∈ A∩B. Suppose that A and B are w$I_{\hat{g}}$-closed sets in X. Then the combination α:$(X, \tau, I) \rightarrow (Y, \sigma)$ is w$I_{\hat{g}}$- continuous.

Proof: Let F be any closed set in Y. Clearly $\alpha^{-1}(F)=f^{-1}(F) \cup g^{-1}(F)$ = C∪D where C = $f^{-1}(F)$ and D = $g^{-1}(F)$. But C is w$I_{\hat{g}}$-closed in A and A is be w$I_{\hat{g}}$-closed in X and so C is w$I_{\hat{g}}$-closed in X. Since we have proved that if B⊆A⊆X, B is w$I_{\hat{g}}$-closed in A and A is w$I_{\hat{g}}$-closed in X, then B is w$I_{\hat{g}}$-closed in X. Also C∪D is w$I_{\hat{g}}$-closed in X. Therefore $\alpha^{-1}(F)$ is w$I_{\hat{g}}$-closed in X. Hence α is w$I_{\hat{g}}$-continuous.

Theorem 2.27: A map f: $(X, \tau, I) \rightarrow (Y, \sigma)$ is wI_{*g}-continuous iff the inverse image of every closed set in (Y, σ) is wI_{*g}- closed in (X, τ, I).

Proof: Necessary: Let v be an open set in (Y, σ). Since f is wI_{*g}- continuous, $f^{-1}(v^C)$ is wI_{*g}- closed in (X, τ, I). But $f^{-1}(v^C) = X - f^{-1}(v)$. Hence $f^{-1}(v)$ is wI_{*g}- closed in (X, τ, I).

Sufficiency: Assume that the inverse image of every closed set in (Y, σ) is wI_{*g}- closed in (X, τ, I). Let v be a closed set in (Y, σ). By our assumption $f^{-1}(v^C) = X - f^{-1}(v)$ is wI_{*g}- closed in (X, τ, I) , which implies that $f^{-1}(v)$ is wI_{*g}- closed in (X, τ, I). Hence f is wI_{*g}- continuous.

Remark 2.28:
(i) The union of any two wI_{*g}- continuous function is wI_{*g}-continuous.
(ii) The intersection of any two wI_{*g}- continuous function is need not be wI_{*g}- continuous.

Theorem 2.29: Let f: $(X,\tau, I_1) \rightarrow (Y, \sigma, I_2)$ and g: $(Y, \sigma, I_2) \rightarrow (Z, \eta, I_3)$ be any two functions. Then the following hold.
(i) g ∘ f is wI_{*g}- continuous if f is wI_{*g}continuous and g is continuous.
(ii) g∘ f is wI_{*g}- continuous if f is wI_{*g}irresolute and g is I_{*g}continuous.
(iii) g∘ f is wI_{*g}- irresolute if f is wI_{*g}irresolute and g is irresolute.

Proof:
(i) Let v be a closed set in Z. Since g is continuous, $g^{-1}(v)$ is closed in Y. wI_{*g}-continuous of f implies, $f^{-1}(g^{-1}(v))$ is wI_{*g}-closed in X and hence g ∘ f is wI_{*g}-continuous.
(ii) Let v be a closed set in Z. Since g is wI_{*g}-continuous, $g^{-1}(v)$ is wI_{*g}-closed in Y. Since f is wI_{*g}-irresolute, $f^{-1}(g^{-1}(V))$ is wI_{*g}-closed in X. Hence g ∘ f is wI_{*g}-continuous.
(iii) Let v be a wI_{*g}-closed in Z. Since g is wI_{*g}- irresolute, $g^{-1}(v)$ is wI_{*g}-closed in Y. Since f is wI_{*g}-irresolute, $f^{-1}(g^{-1}(v))$ is wI_{*g}-closed in X. Hence g ∘ f is wI_{*g}-irresolute.

Theorem 2.30: Let X=A∪B be a topological space with topology τ and Y be a topological space with topology σ. Let f: $(A,\tau/A) \rightarrow (Y,\sigma)$ and g:$(B, \tau/B) \rightarrow (Y,\sigma)$ be wI_{*g}-continuous maps such that f(x)=g(x) for every x ∈ A∩B. Suppose that A and B are wI_{*g}-closed sets in X. Then the combination α:$(X,\tau,I) \rightarrow (Y,\sigma)$ is wI_{*g}- continuous.

Proof: Let F be any closed set in Y. Clearly $\alpha^{-1}(F)=f^{-1}(F) \cup g^{-1}(F)$ = C∪D where C = $f^{-1}(F)$ and D = $g^{-1}(F)$. But C is wI_{*g}-closed in A and A is be wI_{*g}-closed in X and so C iswI_{*g}-closed in X. Since we have proved that if B⊆A⊆X, B is wI_{*g}-closed in A and A is wI_{*g}-closed in X, then B is wI_{*g}-closed in X. Also C∪D is wI_{*g}-closed in X. Therefore $\alpha^{-1}(F)$ is wI_{*g}-closed in X. Hence α is wI_{*g}-continuous.

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