International Journal of Mathematical Archive-7(8), 2016, 144-147
Available online through www.ijma.info ISSN 2229 – 5046

UNCOUNTABLY MANY POSITIVE SOLUTIONS OF FIRST ORDER NONLINEAR NEUTRAL DELAY DIFFERENCE EQUATIONS

V. ANANTHAN*
Assistant Professor, Department of Mathematics,
Aarupadai Veedu Institute of Technology, Vinayaka Missions University,
Kanchipuram-603104, Tamilnadu, India.

S. KANDASAMY
Prof. & Head, Dept.of Mathematics, Vinayaka Missions
Kirupanantha Variar Engineering College, Salem-636308, Tamilnadu, India.

(Received On: 08-08-16; Revised & Accepted On: 30-08-16)

ABSTRACT

We consider the difference equation

$$\Delta (x_n - P_n x_{n-\tau}) + q_{(n)} f(x_{n-\sigma}) = 0 \qquad (1)$$

Where $n \geq n_0$, $\tau > 0$ & $\sigma \geq 0$ *are integers.*

Also $a \in C([t_0, \infty), (0, \infty))$, $p_n, q_n \in C(R, (0, \infty))$ *and* $f \in C(R, R)$, *where f is non decreasing function for* $f(x) > 0, x > 0$.

INTRODUCTION

We are concerned with the first order neutral Delay nonlinear Difference equation

$$\Delta(x_n - P_n x_{n-\tau}) + q_{(n)} f(x_{n-\sigma}) = 0 \qquad (1)$$

(H1) $r_n \in c'[(n_0, \infty), (0, \infty)), \sum_{s=n}^{\infty} \frac{1}{rs} = \infty$
(H2) $p_n \in c((n_0, \infty)(0, \infty))\ p \equiv 0$
(H3) $\emptyset(\lambda) \in c'((-\infty, \infty), (0, \infty))\ \ \emptyset(\lambda) \neq 0 \mid x \neq 0)$
(H4) $f(x) \in c'((-\infty, \infty), (-\infty, \infty))\ \ \lambda f(x) > 0 \mid x \neq 0)$
(H5) $G(x) = \frac{\Delta f(x)}{\emptyset(\lambda)} > 0\ (x \neq 0)$: G(x) is non decreasing $(0, \infty)$ and non increasing is $(-\infty, 0)$
(H6) $g(n) \in c[(n_0, \infty) \to (0, \infty))\ g(n) \geq n$

A non trivial solution $\{x_n\}$ is said to be oscillatory if it has arbitrarily large Zeros otherwise $\{x_n\}$ is said to be non oscillatory The proof is an adaptation of that given (1) where the special case g(n) = n was consider

Lemma 1.1: (Krasnoselskii's fixed point theorem)

Let X be a Banach space, Let Ω be a bounded close convex subset of x and let s_1 s_2 be maps ofΩ into x such that $s_1x + s_2y \in \Omega$ for every x, y ϵ Ω.

If s_1 is contractive and s_2 is completely continuous. Then the equation $s_1x+s_2x=x$ solution in Ω

Theorem: Suppose that there exist bounded from below and from above by the function $u_n, v_n \in c'([n_0, \infty), (0, \infty))$ constant c > 0, $k_2 > k_1 \geq 0$ & $n_1 \geq n_0 + m$ such that

$$u_n \leq v_n, \quad n \geq n_0 \qquad (2)$$

$$v_n - v_{n1} - u_n + u_{n1} \geq 0, \quad n_0 \leq n \leq n_1 \qquad (3)$$

$$\frac{1}{u(n-\tau)} \left(u_n - k_1 + \sum_{s=n}^{\infty} p_s f(v_s - \sigma)\right) \leq a_n < \frac{1}{v(n-\tau)} \left(v_n - k_2 + \sum p_s f(u_s - \sigma)\right) \leq c \leq 1\ n \geq n_1 \qquad (4)$$

Then eq. (1) has uncountable many positive solution which are bounded by the Functions u, v.

*Corresponding Author: V. Ananthan**
Assistant Professor, Department of Mathematics,
Aarupadai Veedu Institute of Technology, Vinayaka Missions University,
Kanchipuram-603104, Tamilnadu, India.

Proof: Let c $((k_0, \infty)$, R) be the set of all continuous bounded functions with The norm $||x|| = \sup_{n\geq n0} |x_n|$. Then $c([n_0,\infty),R)$ is Banach space.

We define a close bounded an convex subset of c $((n_0, \infty)$, R) as

$$\Omega = \{x = x_n \epsilon\, c((n_0, \infty), R): u_n \leq x_n \leq v_n, n \geq n_0\}$$

For kϵ $[k_1, k_2]$ we define two maps s_1 & s_2: $\Omega \to c((n_0, \infty))$ as follows

$$s_1 x_n = \begin{cases} k + a_n x_{n-\tau} & n \geq n_1 \\ s_1 x_{n_1} & n_0 \leq t \leq n_1 \end{cases} \quad (5)$$

$$s_2 x_n = \begin{cases} -\sum_{s=n}^{\infty} p_s f(x_{s-\sigma}) & n \geq n_1 \\ s_2 x_{n_1} + v_n - v_{n_1} & n_0 \leq t \leq n_1 \end{cases}$$

We will show that for any x, y $\epsilon\Omega$ we have $s_1x+s_2y\epsilon\Omega$ for every x, y ϵ Ω and $t \geq t_1$ with regard to (4) we obtain

$$\begin{aligned} s_1x_n + s_2y_n &= k + a_n x_{n-\tau} - \sum_{s=n}^{\infty} p_s f(y_{s-\sigma}) \\ &\leq k + a_n v_{n-\tau} - \sum_{s=n}^{\infty} p_s f(y_{s-\sigma}) \\ &\leq k + v_n - k_2 \leq v_n \end{aligned}$$

For n $\epsilon[n_0,n_1]$ we have

$$\begin{aligned} s_1x_n + s_2y_n &= s_1x_{n_1} + s_2y_{n_1} + v_n - v_{n_1} \\ &\leq v_{n_1} + v_n - v_{n_1} = v_n \end{aligned}$$

Further more for $n \geq n_1$ we get

$$\begin{aligned} s_1x_n + s_2y_n &\geq k + a_n u_{n-\tau} - \sum_{s=n}^{\infty} p_s f(v_{s-\sigma}) \\ &\geq k + u(t) - k \geq u_n \end{aligned}$$

Let n ϵ (n_0, n_1) with regards to (3) we get

$$v_n - v_{n_1} + u_{n_1} \geq u_n, \quad n_0 \leq t \leq n_1$$

Then n ϵ $[n_0, n_1]$ and any x, y ϵ Ω we obtain

$$\begin{aligned} s_1x_n + s_2y_n &= s_1x_{n_1} + s_2y_{n_1} 1 + u_t - u_{t_1} \\ &= u_{n_1} + u_n - u_{n_1} \geq u_n \end{aligned}$$

Then we have prove that s_1x+s_2y ϵ Ω for any x, y ϵ Ω

We will show that s_1 is a contraction mapping on Ω for x, y ϵ Ω & $n \geq n_1$ we have

$$|s_1x_n - s_1y_n| = |a_n||x_{n-\tau} + y_{n-\tau}| \leq c||x-y||$$

This implies

$$||s_1x - s_2y|| \leq c\,||x-y||$$

Also for n ϵ (n_0, n_1) the above inequalities is valid.

We conclude that S_1 is a contraction mapping on Ω

We now show that s_2 is completely continuous. First we show that s_2 is continuous. Let $x^i = \{x_n^{(i)}\}$ ϵ Ω be such that $x_n^{(i)} \to x_n$ as $n \to \infty$ Because x is close x =(x_n) ϵ Ω for $n \geq n_1$ we have

$$\begin{aligned} |(s_2 x_n^i - s_2x_n) &\leq |\sum_{s=n}^{\infty} p_s[fx^i_{s-\sigma} - f(x_{s-\sigma})] \\ &\leq \sum_{s=n1}^{\infty} p_s\, |fx^i_{s-\sigma} - f(x_{s-\sigma})| \end{aligned}$$

Since $|f(x^i_{s-\sigma}) - f(x_{s-\sigma})| \to 0$ as $i \to \infty$ be applying the lebesgue dominant Convergence their we obtain $\lim_{i\to\infty} |s_2 x^i - s_2 x|| = 0$ This means the s_2 is continuous.

we now show that s_2 is relatively compact in Ω, it is sufficient to share

By Arzela ascolic theorem that the family of functions $\{s_2x: x \epsilon \Omega\}$ is uniformly

$$\sum_{s=n}^{\infty} p_s f(x_{s-\sigma}) < \epsilon/2$$

The x ϵ Ω, $N_2 > N_1 \geq n$
where

$$\begin{aligned} |(s_2 x)(N_2) - (s_2 x)(N_1)| &\leq \sum_{s=N_2}^{\infty} p_s f(x_{s-\sigma}) + \sum_{s=N_1}^{\infty} p_s f(x_{s-\sigma}) \\ &\leq \mathcal{E}/2 + \mathcal{E}/2 = \mathcal{E} \end{aligned}$$

$|(s_2 x)(N_2) - (s_2 x)(N_1)| \leq \sum_{s=n1}^{N2\infty} p_s f(x_{s-\sigma})$
$\leq \text{Max}\{p_s f(x_{s-\tau})\} (N_2 - N_1), \; n_1 \leq \& \leq n$

Then there exist $s_1 = \epsilon/M$ when $M = \max p_x f(x_{s-\sigma})$ there exist $n_1 \leq \& \leq n$
$|(s_2x)(N_2) - (s_2x)(n_1)| < \epsilon$ if $0 < N_2 - N_1 < s_1$

Next we show that equation (1) has uncountable many bounded positive solution Ω.

Let $\bar{k} \in [k_1, k_2]$ be such that $\bar{k} \neq k$.

We assume that $x, y \in \Omega$
$s_1x + s_2x = x, \bar{s_1}y + \bar{s_2}y = y$
$x_n = k + a_n x_{(n-\sigma)} - \sum_{s=n}^{\infty} p_s f(x_{s-\sigma}), \quad n \geq n_1$
$y_n = \bar{k} + an\, y_{(n-\sigma)} - \sum_{s=n}^{\infty} p\, s\, f(y_{s-\sigma}), \quad n \geq n_1$

Is follow that there exist a $n_2 > n_1$ satisfy
$\sum_{s=n2}^{\infty} p_s[f(x_{s-\sigma}) + f(y_{s-\sigma})] \leq |k - \bar{k}|$

In order to prove that the set of bounded positive solution of equation (1) is Constant it is sufficient to very that $x \neq y$ for $n \geq n_{2.}$

We get $|x_n - y_n|(1+x) \| x-y \| \geq |k - \bar{k}| - \sum_{s=n}^{\infty} p_s (f(x_{s-\sigma}) + f(y_{s-\sigma}))$

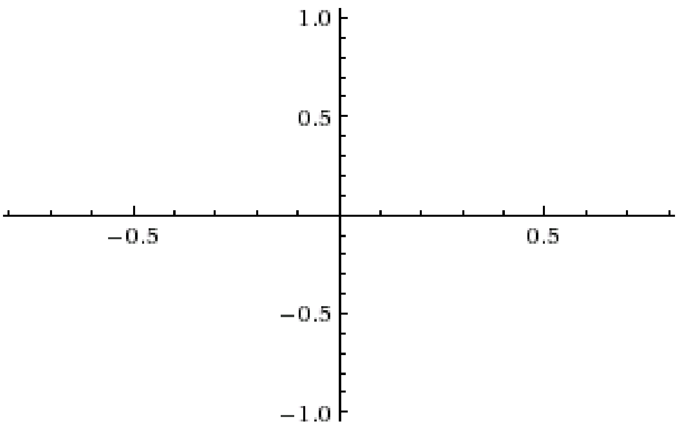
Corollary: Suppose that their exist bounded from below and from above by Function u and $v \in C[(n_0, \infty)(0, \infty))$ that $c > 0$ $k_2 > k_1 \geq 0$, $n_1 \geq n_0 + m$ such that (2) (4) holds
$\Delta u_n - \Delta u_n \leq 0 \; n_1 \leq n \leq n_1$
$H(t) = v_{n+1} - u_n - u_{n+1} + u_n$
$H^1(t) = \Delta v_n - \Delta u_n \leq 0$
$H_n(t) = 0$

Example: $\Delta(x_n - x_{n-1}) + \frac{1}{n} x_{n-1} = 0$

Input:

$$x(n+1) - 2x(n) + x(n-1) + \frac{1}{n} x(n-1) = 0$$

Plot:



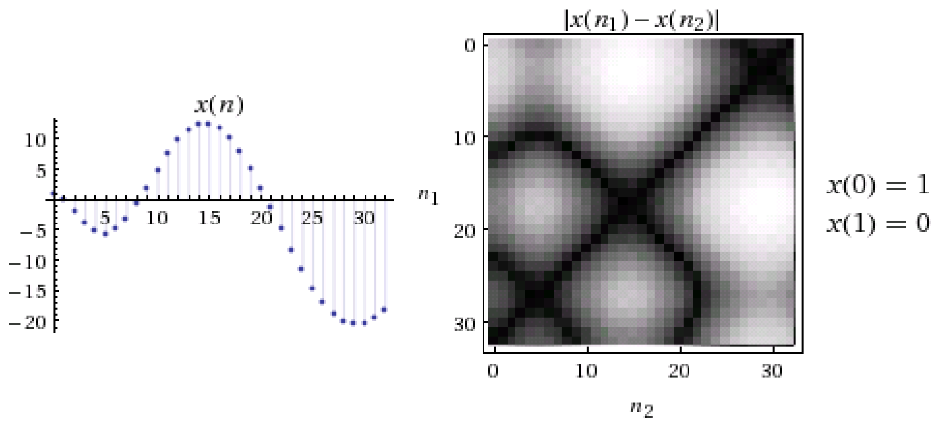
Alternate forms:

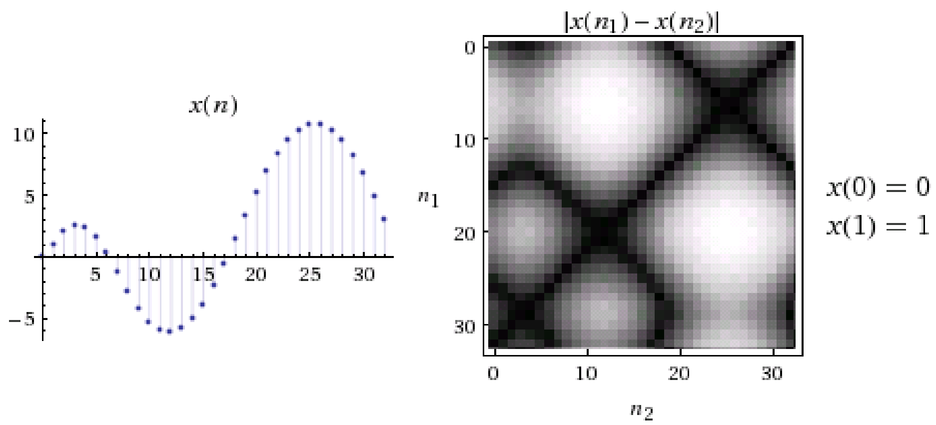
$$\left(\frac{1}{n}+1\right) x(n-1) + x(n+1) = 2x(n)$$

$$nx(n+1) = (-n-1)\; x(n-1) + 2nx(n)$$

$$\frac{n\; x(n-1) + x(n-1) - 2n\; x(n) + n\; x(n+1)}{n} = 0$$

Value plot and recurrence plot:





Values:

n	0	1	2	3	4
$x(n)$	0	1	2	2.5	2.33333

REFERENCES

1. R.P. Agarwal, Difference Equations and Inequalities: theory, Method and Application. Marcel Dekker, New York, (1992).
2. G.Ladas and C.Qian. Comparison results and linearized oscillation for higher order difference equations Intl.j.Math. and Math.sci s15(1).129-142 (1992).
3. B.S.Lalli, Oscillation theorems for neutral difference equations, computers math application 28 (1-3) 191-202. (1994).
4. R.P.Agarwal and P.J.Y.Wong, Advanced Topics in Difference Equations, Kluwer Academic. Dordrecht. (1997).
5. R.P.Agarwal,E.Thandapani and P.J.Y.Wong, oscillations of higher –order neutral difference equations. Appl. Math. Lett 10(1), 71-78 (1997).
6. R.P.Agarwal and J.Popenda, on the oscillation of recurrence equations, Nonlinear Analysis 36, 231-268, (1999).
7. R.P.Agarwal. "Differencce Equations and Inequalities-Theory, Methods and Applications", second edition(2000).
8. E.Thandapani and B.Selvaraj.' Existence and Asymptotic Behavior of Non Oscillation of Certain Nonlinear Difference Equations'. Far East Journal of Mathematical Sciences (FJMS), 14(1), 9-25(2004).
9. R.P.Agarwal,Martin Bohner, Said R.Grace, Donal O'Regan.'Discrete Oscillation theory', CMIA Book series, Volume 1, ISBN; 977-5945-19-4, (2005).

Source of support: Nil, Conflict of interest: None Declared