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On αg*p-Continuous and αg*p-irresolute Maps in Topological Spaces

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ABSTRACT

*The aim of this paper is to introduce a new type of functions called the αg*p-continuous. Here αg*p-continuous, αg*p-irresolute, αg*p-closed functions are defined. Characterizations for these functions are given. Further their fundamental properties are investigated.*

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Keywords: *αg*p-continuous, αg*p-irresolute, αg*p-open, αg*p-closed functions.*

1. INTRODUCTION

In 1982, A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb[14] introduced the concept of pre-continuity in topological spaces. Later, K.Balachandran, P.Sundram and H.Maki[4] introduced and studied the concept of generalized continuous functions. I.Arokirani, K.Balachandran and Julian Dontchev [2] defined gp-irresolute and gp-continuous functions and investigated their properties. M.K.R.S.Veerakumar[24] introduced g*p-closed sets, g*p-continuous maps, g*p-irresolute maps and their properties. Recently, the authors [20] have introduced αg*p-closed sets and their properties. In this paper we study a new class of functions, namely, αg*p-continuous functions and αg*p-irresolute functions. Also, we study some of the characterization and basic properties of αg*p-continuous functions.

In the present paper, the spaces X, Y and Z always mean topological spaces (X, τ) , (Y, σ) and (Z, η) respectively. For a subset A of X, the closure of A and interior of A will be denoted by cl(A) and int(A) respectively. The union of all preopen sets of X contained in A is called pre-interior of A and it is denoted by pint(A). The intersection of all preclosed sets of X containing A is called pre-closure of A and it is denoted by pcl (A).

2. PRELIMINARIES

The definition of preclosed ,semi-closed, α-closed, semi-preclosed, regular closed, g-closed, sg-closed, gs-closed, gα-closed, αg-closed, gp-closed, gsp-closed, gpr-closed, rg-closed, wg-closed, rwg-closed, g*-closed, mildly g-closed, g*p-closed set, gp*-closed, αg*-closed are mentioned by the authors in [20].

We recall the following definitions which are useful in the sequel.

Definition 2.1 [20]: A subset A of a topological space (X, τ) is called alpha generalized star preclosed set (briefly, αg*p-closed) if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is αg-open in X.

Definition 2.2: For a subset A of (X, τ) ,the intersection of all αg*p-closed sets containing A is called the αg*p-closure of A and is denoted by αg*p-cl(A).
That is, αg*p-cl(A) = $\cap$\{F : F is αg*p-closed in X , A $\subseteq$ F\}.

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Definition 2.3: A function f: (X, τ) → (Y, σ) is said to be

(i) precontinuous [14] if $f^{-1}(V)$ is preclosed in X for every closed subset V of Y.
(ii) semi-continuous [11] if $f^{-1}(V)$ is semi-closed in X for every closed subset V of Y.
(iii) α-continuous [15] if $f^{-1}(V)$ is α-closed in X for every closed subset V of Y.
(iv) regular continuous [3] if $f^{-1}(V)$ is regular closed in X for every closed subset V of Y.
(v) semi-precontinuous [1] if $f^{-1}(V)$ is semi-preclosed in X for every closed subset V of Y.
(vi) g-continuous [4] if $f^{-1}(V)$ is g-closed in X for every closed subset V of Y.
(vii) g*-continuous [23] if $f^{-1}(V)$ is g*-closed in X for every closed subset V of Y
(viii) αg-continuous [13] if $f^{-1}(V)$ is αg-closed in X for every closed subset V of Y.
(ix) gα-continuous [12] if $f^{-1}(V)$ is gα-closed in X for every closed subset V of Y.
(x) gs-continuous [6] if $f^{-1}(V)$ is gs-closed in X for every closed subset V of Y.
(xi) sg-continuous [21] if $f^{-1}(V)$ is sg-closed in X for every closed subset V of Y.
(xii) gp-continuous [2] if $f^{-1}(V)$ is gp-closed in X for every closed subset V of Y.
(xiii) gsp-continuous [7] if $f^{-1}(V)$ is gsp-closed in X for every closed subset V of Y.
(xiv) gpr-continuous [9] if $f^{-1}(V)$ is gpr-closed in X for every closed subset V of Y.
(xv) gp*-continuous if $f^{-1}(V)$ is gp*-closed in X for every closed subset V of Y.
(xvi) g*p-continuous [24] if $f^{-1}(V)$ is g*p-closed in X for every closed subset V of Y.
(xvii) αg*-continuous if $f^{-1}(V)$ is αg*-closed in X for every closed subset V of Y.
(xviii) wg-continuous [16] if $f^{-1}(V)$ is wg-closed in X for every closed subset V of Y.
(xix) rwg-continuous [16] if $f^{-1}(V)$ is rwg-closed in X for every closed subset V of Y.
(xx) mg-continuous [17] if $f^{-1}(V)$ is mg-closed in X for every closed subset V of Y.

Definition 2.4: A function f: (X, τ)→ (Y, σ) is said to be

(i) pre-irresolute [19] if $f^{-1}(V)$ is preclosed in X for every preclosed subset V of Y.
(ii) α-irresolute [22] if $f^{-1}(V)$ is α-closed in X for every α-closed subset V of Y.
(iii) αg -irresolute [5] if $f^{-1}(V)$ is αg-closed in X for every αg-closed subset V of Y.
(iv) preclosed [8] if f (V) is preclosed in Y for every closed subset V of X.

3. ALPHA GENERALIZED STAR PRECONTINUOUS MAPS

Definition 3.1: A function f: (X, τ) → (Y, σ) is called αg*p-continuous if $f^{-1}(V)$ is αg*p-closed set in (X, τ) for every closed set V in (Y, σ).

Example 3.2: Let X = Y= {a, b, c, d}, τ ={ϕ ,{a},{b, c},{a, b, c}, X} and σ ={ϕ,{a},{b},{a, b},{b, c},{a, b, c}, Y}.

Define a function f: X→Y by f(a) = b, f(b) = c, f(c) = a, f(d) = d. Then f is αg*p-continuous.

Theorem 3.3: Every continuous map is αg*p-continuous.

Proof: Let f: X→Y be a continuous map. Let V be a closed set in Y. Since f is continuous, $f^{-1}(V)$ is closed in X. Also every closed set is an αg*p-closed set, $f^{-1}(V)$ is αg*p-closed in X. Therefore f is αg*p-continuous map.

The converse of the above theorem need not be true as seen in the following example.

Example 3.4: Let X = Y= {a, b, c, d}, τ ={ϕ ,{a},{b},{a, b}, X} and σ ={ϕ,{d},{c, d},{a, c, d},{b, c, d}, Y}.
Let f: X→Y be defined f(a) = c, f(b) = d, f(c) = a, f(d) = b. The function f is αg*p-continuous but not continuous.

Theorem 3.5: Every precontinuous (resp.α-continuous) map is αg*p-continuous.

Proof: Let f: X→Y be a precontinuous (resp.α-continuous) map. Let V be a closed set in Y. Since f is pre-continuous (resp.α-continuous), $f^{-1}(V)$ is preclosed (resp.α-closed) in X. Also every preclosed (resp.α-closed) set is αg*p-closed, $f^{-1}(V)$ is αg*p-closed in X. Therefore f is αg*p-continuous map.

The converse of the above theorem need not be true as seen in the following example.

Example 3.6: Let X = Y= {a, b, c, d}, τ ={ϕ,{a},{b},{a, b},{a, c},{a, b, c}, X} and σ ={ϕ,{b},{c},{b, c}, Y}.

Let f: X→Y be defined f(a) = d, f(b) = c, f(c) = b, f(d) = a. The function f is αg*p-continuous but not precontinuous.

Example 3.7: Let X = Y= {a, b, c, d}, τ ={ϕ,{a, b}, X} and σ ={ϕ,{b},{b, c},{b, d},{a, b, d},{b, c, d}, Y}.

Let f: X→Y be defined f(a) = a, f(b) = b, f(c) = c, f(d) = d. The function f is αg*p-continuous but not α-continuous.

Corollary 3.8: Every regular continuous map is αg*p-continuous**.**

The converse of the above corollary need not be true as seen in the following example.

Example 3.9: Let X = Y= {a, b, c, d}, τ ={ϕ ,{a},{b, c},{a, b, c}, X} and σ = {ϕ,{c},{a, c},{c, d},{a, b, c },{a, c, d}, Y}.

Let f: X→Y be defined f(a) = c, f(b) = d, f(c) = a, f(d) = b. The function f is αg*p-continuous but not regular continuous.

Theorem 3.10: If a map f: X→Y is continuous, then the following holds.

(i) If f is αg*p-continuous, then f is g*p-continuous.

(ii) If f is αg*p-continuous, then f is gp-continuous (resp. gpr-continuous, gsp-continuous, mg-continuous, wg-continuous, rwg-continuous).

Proof: (i) Let V be a closed set in Y. Since f is αg*p-continuous, then $f^{-1}(V)$ is αg*p-closed in X. Since every αg*p-closed set is g*p-closed then $f^{-1}(V)$ is g*p-closed in X. Hence f is g*p-continuous.

Similarly we can prove (ii).

The converse of the above theorem need not be true as seen in the following example.

Example 3.11: Let X = Y= {a, b, c, d}, τ = {ϕ,{a},{b},{a, b},{a, b, c}, X} and σ = {ϕ,{c},{a, c},{b, c},{a, b, c}, Y}.
Let f: X→Y be defined f(a) = a, f(b) = c, f(c) = b, f(d) = d. The function f is gp-continuous but not αg*p-continuous.

Example 3.12: Let X = Y= {a, b, c, d}, τ = {ϕ ,{a},{b},{a, b},{a, b, c}, X} and σ = {ϕ,{c},{b, c},{c, d},{a, c, d}, {b, c, d},Y}.
Let f: X→Y be defined f(a) = b, f(b) = a, f(c) = d, f(d) = c The function f is gpr-continuous but not αg*p-continuous.

Example 3.13: Let X = Y= {a, b, c, d}, τ ={ϕ ,{a, b}, X} and σ ={ϕ,{c},{d },{c, d}, Y}.
Let f : X→Y be defined f(a) = d , f(b) = a , f(c) = b, f(d) = c . The function f is gsp-continuous but not αg*p-continuous.

Example 3.14: Let X = Y= {a, b, c, d}, τ = {ϕ ,{a},{b},{a, b},{a, b, c}, X} and σ ={ϕ,{c},{a, c},{b, c},{a, b, c}, Y}.
Let f: X→Y be defined f(a) = c, f(b) = a, f(c) = b, f(d) = d. The function f is g*p-continuous but not αg*p-continuous.

Example 3.15: Let X = Y= {a, b, c, d}, τ = {ϕ ,{a},{b},{a, b},{b, c},{a, b, c}, X} and σ = {ϕ,{c},{a, c}, Y}.
Let f: X→Y be defined f(a) = c, f(b) = b, f(c) = a, f(d) = d. The function f is mg-continuous but not αg*p-continuous.

Example 3.16: Let X = Y= {a, b, c, d}, τ = {ϕ ,{a},{b},{a, b}, X} and σ = {ϕ,{b},{b, c},{b, d},{b, c, d}, Y}.
Let f: X→Y be defined f(a) = b, f(b) = c, f(c) = d, f(d) = a. The function f is wg-continuous but not αg*p-continuous.

Example 3.17: Let X = Y= {a, b, c, d} be given the topologies τ = {ϕ,{a},{b},{d},{a, b},{a, d},{b, d},{a, b, d}, X} and σ ={ϕ,{c},{b, c},{c, d},{a, c, d},{b, c, d }, Y}. Let f: X→Y be defined f(a) = d, f(b) = a, f(c) = b, f(d) = c. The function f is rwg-continuous but not αg*p-continuous.

Theorem 3.18: Every gp*-continuous (resp.αg*-continuous) map is αg*p-continuous.

Proof: Let f: X→Y be a gp*-continuous (resp.αg*-continuous) map. Let V be any closed set in Y. Since f is gp*-continuous (resp.αg*-continuous), $f^{-1}(V)$ is gp*-closed (resp.αg*-closed) in X. Also every gp*-closed (resp.αg*-closed) set is αg*p-closed, $f^{-1}(V)$ is αg*p-closed in X. Therefore f is αg*p-continuous map.

The converse of the above theorem need not be true as seen in the following example.

Example 3.19: Let X = Y= {a, b, c, d}, τ = {ϕ,{a}, X} and σ = {ϕ,{a},{a, b},{a, d},{a, b, d },{a, c, d}, Y}.
Let f : X→Y be defined f(a) = a, f(b) = b, f(c) = d, f(d) = c. The function f is αg*p-continuous but not gp*-continuous.

Example 3.20: Let X = Y= {a, b, c, d}, τ = {ϕ , {c, d}, X} and σ = {ϕ, {b}, {a, b}, {b, d}, {a, b, c}, {a, b, d}, Y}.
Let f: X→Y be defined f(a) = c, f(b) = d, f(c) = b, f(d) = a. The function f is αg*p-continuous but not αg*-continuous.

Remark 3.21: αg*p-continuity is independent of semi-continuity and semi-precontinuity as seen from the following example.

Example 3.22: Let X = Y= {a, b, c, d}, τ = {ϕ,{a},{b},{a, b},{a, c},{a, b, c}, X} and σ = {ϕ,{c},{b, c},{a, c, d}, Y}. Let f: X→Y be defined f(a) = a, f(b) = c, f(c) = b, f(d) = d.

The function f is αg*p-continuous but not semi-continuous and semi-precontinuous, since f^{-1}({a, d}) = {a, d} is αg*p-closed but not semi-closed and semi-preclosed.

Define f(a) = c, f(b) = b, f(c) = a, f(d) = d.

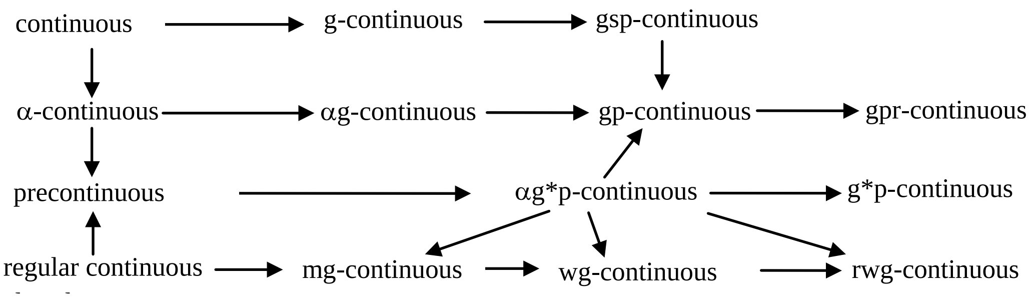
The function f is semi-continuous and semi-precontinuous but not αg*p-continuous, since f^{-1}({b}) = {b} is semi-closed and semi-preclosed but not αg*p-closed.

Remark 3.23: The following examples shows that αg*p-continuous maps are independent of g-continuous, g*-continuous, sg-continuous, gs-continuous and αg-continuous.

Example 3.24: Let X = Y= {a, b, c, d}, τ = {ϕ,{a, b}, X} and σ ={ϕ,{b},{a, b},{a, b, c},{a, b, d}, Y}.
Let f: X→Y be defined f(a) = b, f(b) = d, f(c) = c, f(d) = a. The function f is αg*p-continuous but not g, g*, sg, gs, αg-continuous, since f^{-1}({d}) = {b} is αg*p-closed but not g, g*, sg, gs, αg-closed sets.

Example 3.25: Let X = Y= {a, b, c, d},τ = {ϕ,{a},{b},{a, b},{a, b, c}, X} and σ = {ϕ,{c},{b, c},{a, c},{a, b, c}, Y}.
Let f: X→Y be defined f(a) = b, f(b) = c, f(c) = a, f(d) = d. The function f is g, g*, sg, gs, αg-continuous but not αg*p-continuous, since f^{-1}({b, d}) = {a, d} is not αg*p-closed.

By the above results we have the following diagram:



4.αg*p-CONTINUITY AND ITS CHARACTERISTICS

Theorem 4.1: Let f : (X, τ)→ (Y, σ) be a map .Then the following conditions are equivalent

i) f is αg*p-continuous.
ii) The inverse image of each open set in Y is αg*p-open in X.
iii) f(αg*p-cl(A)) ⊆ cl(f(A)) for each subset A of X.
iv) For each subset B of Y, αg*p-cl(f^{-1}(B)) ⊆ f^{-1}(cl(B)).

Proof: (i) ⇒ (ii): Let G be an open set in Y. Then Y \ G is closed in Y. By hypothesis, f^{-1}(Y \ G) = X \ f^{-1}(G) is αg*p-closed in X. Hence f^{-1}(G) is αg*p-open in X.

(ii) ⇒ (i): Let G be a closed set in Y.Then Y \ G is open in Y. By hypothesis, f^{-1}(Y \ G) = X \ f^{-1}(G) is αg*p-open in X.Therefore f^{-1}(G) is αg*p-closed in X. Hence f is αg*p-continuous.

(i) ⇒ (iii): Let A be a subset of X. Since A ⊆ f^{-1}(f(A)) and f(A) ⊆ cl(f(A)) , we have A ⊆ f^{-1}(f(A)) ⊆ f^{-1}(cl(f(A))).
Therefore by assumption f^{-1}(cl(f(A))) is αg*p-closed set of X. Hence αg*p-cl(A) ⊆ f^{-1}(cl(f(A))).
Thus f(αg*p-cl(A)) ⊆ f(f^{-1}(cl(f(A))) ⊆ cl(f(A)).

(iii) ⇒ (iv): Let B be a subset of Y and f (A) = B.So by assumption, f(αg*p-cl(A)) = f(αg*p-cl(f^{-1}(B))).
Therefore αg*p-cl(f^{-1}(B)) ⊆ f^{-1} (f(αg*p-cl(f^{-1}(B)))) ⊆ f^{-1}(cl(B)).

(iv) ⇒ (i): Let B be a closed set in Y. Then by assumption, αg*p-cl(f^{-1}(B)) ⊆ f^{-1}(cl(B)) = f^{-1}(B).Therefore f^{-1}(B) is αg*p-closed set in X. Hence f is αg*p-continuous.

Theorem 4.2: Let A be a subset of a topological space X. Then x ∈ αg*p-cl(A) if and only if for any αg*p-open set U containing x, A ∩ U ≠ ϕ.

Proof: Let $x \in \alpha g^*p\text{-}cl(A)$ and suppose that there is a αg*p-open set U in X such that $x \in U$ and $A \cap U = \phi$ implies that $A \subseteq X \setminus U$ which is αg*p-closed in X implies $\alpha g^*p\text{-}cl(A) \subseteq \alpha g^*p\text{-}cl(X \setminus U) = X \setminus U$. Since $x \in U$ implies that $x \notin X \setminus U$ implies that $x \notin \alpha g^*p\text{-}cl(A)$, this is a contradiction.

Conversely, Suppose that, for any αg*p-open set U containing x, $A \cap U \neq \phi$. To prove that $x \in \alpha g^*p\text{-}cl(A)$.

Suppose that $x \notin \alpha g^*p\text{-}cl(A)$ then there is a αg*p-closed set F in X such that $x \notin F$ and $A \subseteq F$. Since $x \notin F$ implies that $x \in X \setminus F$ which is αg*p-open in X. Since $A \subseteq F$ implies that $A \cap (X \setminus F) = \phi$, this is a contradiction.
Thus $x \in \alpha g^*p\text{-}cl(A)$.

Theorem 4.3: Let f: X→Y be a function from a topological space X into a topological space Y. If f: X→Y is αg*p-continuous then $f(\alpha g^*p\text{-}cl(A)) \subseteq cl(f(A))$ for every subset A of X.

Proof: Since $f(A) \subseteq cl(f(A))$ then $A \subseteq f^{-1}(cl(f(A)))$. Since cl(f(A)) is a closed set in Y and f is αg*p-continuous then by definition $f^{-1}(cl(f(A)))$ is a αg*p-closed set in X containing A. Hence $\alpha g^*p\text{-}cl(A) \subseteq f^{-1}(cl(f(A)))$.

Therefore $f(\alpha g^*p\text{-}cl(A)) \subseteq cl(f(A))$.

The converse of the above theorem need not be true as seen from the following example

Example 4.4: Let X = Y = {a, b, c, d} with τ = {ϕ, {a},{c, d},{a, c, d}, X} and σ ={ϕ,{b, c},{b, c, d},{a, b, c}, Y}. Define a function f: X→Y by, f(a) = c, f(b) = b, f(c) = d, f(d) = a. For every subset A of X, $f(\alpha g^*p\text{-}cl(A)) \subseteq cl(f(A))$ holds. But f is not αg*p-continuous, since {a, d} is closed in Y, $f^{-1}(\{a, d\}) = \{c, d\}$ which is not αg*p-closed set in X

Theorem 4.5: Let f : X →Y be a function. Then the following statements are equivalent:

(1) For each $x \in X$ and each open set V containing f(x) there exists a αg*p-open set U containing x such that $f(U) \subset V$.
(2) $f(\alpha g^*p\text{-}cl(A)) \subset cl(f(A))$ for every subset A of X.

Proof: (1) ⇒ (2): Let $y \in f(\alpha g^*p\text{-}cl(A))$ then there exists an $x \in \alpha g^*p\text{-}cl(A)$ such that y = f(x). Let V be any open neighbourhood of y. Since $x \in \alpha g^*p\text{-}cl(A)$,there exists an αg*p-open set U such that $x \in U$ and $U \cap A \neq \phi$, $f(U) \subset V$. Since $U \cap A \neq \phi$, $f(A) \cap V \neq \phi$. Therefore $y = f(x) \in cl(f(A))$.Hence $f(\alpha g^*p\text{-}cl(A)) \subset cl(f(A))$.

(2) ⇒ (1): Let $x \in X$ and V be any open set containing f(x). Let $A = f^{-1}(Y \setminus V)$. Since $f(\alpha g^*p\text{-}cl(A)) \subset cl(f(A)) \subset Y \setminus V$ then $(\alpha g^*p\text{-}cl(A)) \subset f^{-1}(Y \setminus V) = A$. Hence $\alpha g^*p\text{-}cl(A) = A$. Since $f(x) \in V \Rightarrow x \in f^{-1}(V) \Rightarrow x \notin A \Rightarrow x \notin \alpha g^*p\text{-}cl(A)$.

Thus there exists an open set U containing x such that $U \cap A = \phi$. Therefore $f(U) \subset V$.

Definition 4.6:

(1) A space (X, τ) is called αg*p-space if every αg*p-closed is closed
(2) A space (X, τ) is called αg*p-$T_{1/2}$ space if every αg*p-closed is preclosed.
(3) A space (X, τ) is called $_{\alpha g^*p}T_{\alpha}$- space if every αg*p-closed set is α-closed set.

Theorem 4.7: Let f: X→Y be a function. Let (X, τ) and (Y, σ) be any two spaces such that $\tau_{\alpha g^*p}$ is a topology on X. Then the following statements are equivalent:

(i) For every subset A of X, $f(\alpha g^*p\text{-}cl(A)) \subseteq cl(f(A))$ holds.
(ii) $f : (X, \tau_{\alpha g^*p}) \to (Y, \sigma)$ is continuous.

Proof: Suppose (i) holds. Let A be closed in Y. By hypothesis $f(\alpha g^*p\text{-}cl(f^{-1}(A))) \subseteq cl(f(f^{-1}(A))) \subseteq (A) = A$.

Also $f^{-1}(A) \subseteq \alpha g^*p\text{-}cl(f^{-1}(A))$. Hence $\alpha g^*p\text{-}cl(f^{-1}(A)) = f^{-1}(A)$. This implies $f^{-1}(A) \in \tau_{\alpha g^*p}$. Thus $f^{-1}(A)$ is closed in $(X, \tau_{\alpha g^*p})$ and so f is continuous. This proves (ii).

Suppose (ii) holds. For every subset A of X, cl(f(A)) is closed in Y. Since f: $(X, \tau_{\alpha g^*p}) \to (Y, \sigma)$ is continuous, $f^{-1}(cl(A))$ is closed in $(X, \tau_{\alpha g^*p})$. By definition, $\alpha g^*p\text{-}cl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A)))$.

Now we have, $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(cl(f(A)))$ and by αg*p-closure, $\alpha g^*p\text{-}cl(A) \subseteq \alpha g^*p\text{-}cl(f^{-1}(cl(f(A))) = f^{-1}(cl(f(A)))$. Therefore $f(\alpha g^*p\text{-}cl(A)) \subseteq cl(f(A))$. This proves (i).

Remark 4.8: The Composition of two αg*p-continuous maps need not be αg*p-continuous map and this can be shown by the following example.

Example 4.9: Let X = Y = Z = {a, b, c, d}, τ = {ϕ,{a},{b},{a, b},{a, b, c}, X}, σ = {ϕ,{a},{b, c},{a, b, c}, Y} and η = {ϕ,{a}, Z}.

Define g: Y→Z by g(a) = b, g(b) = c, g(c) = a, g(d) = d and define f: X→Y by f(a) = b, f(b) = a, f(c) = d, f(d) = c. Both f and g are αg*p-continuous maps. But g ○ f is not αg*p-continuous map, since $(g \circ f)^{-1}(\{b, c, d\}) = f^{-1}[g^{-1}(\{b, c, d\})] = f^{-1}(\{a, b, d\}) = \{a, b, c\}$ is not a αg*p-closed set in X.

Theorem 4.10: Let f: X→Y is αg*p-continuous function and g: Y→Z is continuous function then g ○ f: X→Z is αg*p-continuous.

Proof: Let g be continuous function and V be any open set in Z. Then $g^{-1}(V)$ is open in Y. Since f is αg*p-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is αg*p-open in X. Hence g ○ f is αg*p-continuous.

Theorem 4.11: Let f: X→Y is αg*p-continuous function and g: Y→Z is αg*p-continuous function and Y is αg*p-space then g ○ f: X→Z is αg*p-continuous.

Proof: Let g be αg*p-continuous function and V be any open set in Z then $g^{-1}(V)$ is αg*p-open in Y and Y is αg*p-space, then $g^{-1}(V)$ is open in Y. Since f is αg*p-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is αg*p-open in X. Hence g ○ f is αg*p-continuous.

Definition 4.12: A function f: (X, τ)→ (Y, σ) is called an αg*p-irresolute if $f^{-1}(V)$ is αg*p-closed set in (X, τ) for every αg*p-closed set V in (Y, σ).

Definition 4.13: A function f: (X, τ)→ (Y, σ) is called αg*p-closed if f (V) is αg*p-closed set in (Y, σ) for every αg*p-closed set V in (X, τ).

Theorem 4.14: Every α-irresolute function is αg*p-continuous.

Proof: Suppose that a map f: (X, τ)→ (Y, σ) is α-irresolute. Let V be an open set in Y. Then V is α-open in Y. Since f is α-irresolute, $f^{-1}(V)$ is α-open and hence αg*p-open in X. Thus f is αg*p-continuous.

Theorem 4.15: Every αg*p-irresolute function is αg*p-continuous.

Proof: Let f: X →Y be αg*p-irresolute function. Let V be a closed set in Y then V is αg*p-closed in Y. Since f is αg*p-irresolute, $f^{-1}(V)$ is αg*p-closed in X. Hence f is αg*p-continuous.

The converse of the above theorem need not be true it can be seen from the following example.

Example 4.16: Let X=Y ={a, b, c, d}, τ = {ϕ,{a},{b},{a, b}, X} and σ ={ϕ,{a, b}, Y}.
Define f: X→Y by f(a) = b, f(b) = a, f(c) = d, f(d) = c. Then f is αg*p-continuous but not αg*p-irresolute, since $f^{-1}(\{a\}) = \{b\}$ is not an αg*p-closed set in X.

Theorem 4.17: Let f: X→Y is αg*p-irresolute function and g: Y→Z is αg*p-irresolute function then g ○ f: X→Z is αg*p-irresolute.

Proof: Let g be αg*p-irresolute function and V be any αg*p-open set in Z then $g^{-1}(V)$ is αg*p-open in Y. Since f is αg*p-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is αg*p-open in X. Hence g ○ f is αg*p-irresolute.

Theorem 4.18: If a map f: (X, τ)→ (Y, σ) is αg*p-irresolute, if and only if the inverse image $f^{-1}(V)$ is αg*p-open set in X for every αg*p-open set V in Y.

Proof: Assume that f: X→Y is αg*p-irresolute. Let G be αg*p-open in Y. Then Y \ G is αg*p -closed in Y.

Since f is αg*p-irresolute, $f^{-1}(Y \setminus G)$ is αg*p-closed in X. But $f^{-1}(Y \setminus G) = X \setminus f^{-1}(G)$.Thus $f^{-1}(G)$ is αg*p-open in X.

Conversely, Assume that the inverse image of each αg*p-open set in Y is αg*p-open in X. Let F be any αg*p-closed set in Y. By assumption $f^{-1}(Y \setminus F)$ is αg*p -open in X. But $f^{-1}(Y \setminus F) = X \setminus f^{-1}(F)$. Thus $X \setminus f^{-1}(F)$ is αg*p-open in X and so $f^{-1}(F)$ is αg*p-closed in X. Therefore f is αg*p-irresolute.

Theorem 4.19: If a map f: (X, τ)→ (Y, σ) is αg*p-irresolute, then for every subset A of X, f(αg*pcl(A) ⊆ αcl(f(A)).

Proof: If A ⊆ X then consider αcl(f(A)) which is αg*p-closed in Y. Since f is αg*p-irresolute, f^{-1} (αcl(f(A))) is αg*p-closed in X. Furthermore A ⊆ f^{-1} (f(A)) ⊆ f^{-1} (αcl(f(A))). Therefore by αg*p-closure, αg*p-cl(A) ⊆ f^{-1}(αcl(f(A))), consequently, f(αg*p-cl(A) ⊆ f(f^{-1} (αcl(f(A)))) ⊆ αclf((A)).

Theorem 4.20: Let f: (X, τ)→(Y, σ) and g: (Y, σ)→(Z, η) be any two functions. Then

(i) g ○ f : (X,τ)→(Z,η) is αg*p-continuous if g is r-continuous and f is αg*p-irresolute.
(ii) g ○ f : (X, τ)→ (Z, η) is αg*p-irresolute if g is αg*p-irresolute and f is αg*p-irresolute.
(iii) g ○ f : (X, τ)→ (Z, η) is αg*p -continuous if g is αg*p-continuous and f is αg*p-irresolute.

Proof: (i) Let U be a open set in (Z, η). Since g is r-continuous, g^{-1} (U) is r-open set in (Y, σ). Since every r-open is αg*p-open then g^{-1} (U) is αg*p-open in Y. Since f is αg*p-irresolute then f^{-1} (g^{-1} (U)) is an αg*p-open set in (X, τ).

Thus $(g \circ f)^{-1}$ (U) = f^{-1} (g^{-1} (U)) is an αg*p-open set in (X, τ) and hence g ○ f is αg*p-continuous.

Similarly we can prove (ii) and (iii).

Theorem 4.21: Every αg*p-space is αg*p-$T_{1/2}$ space.

Proof: Let (X, τ) be a αg*p -space and let A ⊆ X be an αg*p-closed set in X. Then A is closed in X. Since every closed set is a preclosed set then A is preclosed. Therefore (X, τ) is a αg*p-$T_{1/2}$ space

Theorem 4.22: Let f : (X, τ)→ (Y, σ)be a function then,

(1) If f is αg*p-irresolute and X is αg*p-$T_{1/2}$ space, then f is pre-irresolute.
(2) If is αg*p-continuous and X is αg*p-$T_{1/2}$ space, then f is precontinuous.

Proof:

(1) Let V be preclosed in Y, then V is αg*p-closed in Y. Since f is αg*p-irresolute, f^{-1}(V) is αg*p-closed in X. Since X is αg*p-$T_{1/2}$ space, f^{-1}(V) is preclosed in X. Hence f is pre-irresolute.
(2) Let V be closed in Y. Since f is αg*p-continuous, f^{-1}(V) is αg*p-closed in X. Since X is αg*p-$T_{1/2}$ space, f^{-1}(V) is preclosed. Therefore f is precontinuous.

Theorem 4.23: A function f: X→Y be a bijection. Then the following are equivalent:

(i) f is αg*p-open,
(ii) f is αg*p-closed,
(iii) f^{-1} is αg*p-irresolute.

Proof: Suppose f is αg*p -open. Let F be αg*p-closed in X. Then X \ F is αg*p-open. By definition, f(X \ F) is αg*p-open. Since f is bijection, Y \ f(F) is αg*p-open in Y. Therefore f is αg*p-closed. This proves (i) ⇒ (ii).

Let g = f^{-1}. Suppose f is αg*p-closed. Let V be αg*p-open in X. Then X \ V is αg*p-closed in X.

Since f is αg*p-closed, f(X \ V) is αg*p-closed. Since f is a bijection, Y \ f(V) is αg*p-closed that implies f(V) is αg*p-open in Y. Thus g^{-1} (V) is αg*p-open in Y. Therefore f^{-1} is αg*p-irresolute. This proves (ii) ⇒ (iii).

Let V be αg*p-open in X. Then X \ V is αg*p-closed in X. Since f^{-1} is αg*p-irresolute and $(f^{-1})^{-1}$(X \ V) = f(X \ V) = Y \ f(V) is αg*p-closed in Y that implies f(V) is αg*p-open in Y. Therefore f is αg*p-open. This proves (iii) ⇒ (i).

Theorem 4.24: Let f: (X, τ)→ (Y, σ) be a bijective, αg-irresolute and preclosed function. Then f(A) is αg*p-closed in Y for every αg*p-closed set A of X **.**

Proof: Let A be αg*p-closed in (X, τ). Let V be αg-open set of (Y, σ) containing f(A). Since f is αg-irresolute, f^{-1}(V) is αg-open in X. Since A ⊆ f^{-1}(V) and A is αg*p- closed, pcl(A) ⊆ f^{-1}(V). Since f is bijective and preclosed function, f(pcl(A)) = pcl(f(pcl(A)) .Now pcl(f(A)) ⊆ pcl(f(pcl(A)) = f(pcl(A)) ⊆ V. Hence f(A) is αg*p-closed set in Y.

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