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EDGE TRIMAGIC AND SUPER EDGE TRIMAGIC TOTAL LABELING OF SOME GRAPHS

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ABSTRACT

An Edge trimagic total labeling of a graph $G(V,E)$ with p vertices and q edges is a bijection $f: V \cup E \to \{1,2,3 \dots p+q\}$ such that for every edge uv in$E$, $f(u)+f(uv)+f(v)$ is either λ_1 or λ_2 or λ_3. An edge trimagic total graph is called a super edge trimagic total if $f(v)$ = {1, 2, 3,...p}. In this paper, we prove that the graph Double fan P_n+2K_1 is edge trimagic total labeling and the graphs Z-P_n, $B_{n,n}^2$ are super edge trimagic total labeling.

Keywords: *Function, Edge trimagic, Super edge trimagic.*

1. INTRODUCTION

A graph labeling is an assignment of integer to the vertices or edges or both subject to certain conditions. All graphs considered here are finite, simple and undirected. The useful survey on graph labelings by J. A. Gallian (2015) can be found in [1].In 2013 C.Jayasekaran, M.Ragees and C.Devaraj [2] introduces the edge trimagic total labeling of graphs and also C.Jayasekaran and M.Ragees proved edge trimagic and super edge trimagic total labeling[3][5][6]. An edge trimagic total labeling of a (p, q) graph G is a bijection f : V(G) ∪ E(G) ⟶ {1, 2, 3,…, p+q} such that for each edge uv ϵ E, f(u)+f(uv)+f(v) is equal to any of the distinct constants k_1 or k_2 or k_3. A graph G is said to be edge trimagic total if it admits an edge trimagic total labeling. An edge trimagic total labeling is called super edge trimagic total labeling if G has additional property that the vertices are labeled with the smallest positive integers. A.Nellaimurugan and G.Esther [4] proved that double fan P_n+2K_1 and Z-P_n are Mean cordial labeling. S.K.Vaidya and N.H shah proved that splitting graph of star $K_{1,n}$ and $B_{n,n}^2$ are graceful and odd graceful labeling [7]. In this paper, we prove that the graph double fan P_n+2K_1 is edge trimagic total labeling and the graph Z-P_n, $B_{n,n}^2$ are super edge trimagic total labeling.

Definition 1.1: An edge trimagic total labeling of a (p, q) graph G is a bijectionf : V ∪ E⟶ {1,2,3,…,p+q} such that for each edge xy ∈ E(G), the value of f(x)+f(xy)+f(y)) is equal to any of the distinct costants k_1 or k_2 or k_3. A graph G is said to be an edge trimagic total if it admits an edge trimagic total labeling.An edge trimagic total labeling of a graph is called super edge trimagic if $f(v)$ = {1, 2,…,p}. An edge trimagic total labeling of graph is called a superior edge trimagic total labeling if $f(E)$ = {1, 2,…q}.

Definition 1.2: The join G_1+G_2 of G_1 and G_2 consist of $G_1 \cup G_2$ and all the lines joining v_1 with v_2 as vertex set $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and edge set $E(G_1 \cup G_2) = E(G_1) \cup E(G_2) \cup [uv; u \in V(G_1)$ and $v \in V(G_2)]$. The fan P_n+2K_1 is called the double fan.

Definition 1.3: For a simple connected graph G the square of graph G is denoted by G^2 and defined as the graph with the same vertex set as of $G$and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G.

Definition 1.4: In a pair of path P_n is a i^{th} vertex of a path P_1 is joined with $(i+1)^{th}$ vertex of a path P_2. It is denoted by Z-P_n.

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2. EDGE TRIMAGIC TOTAL LABELING

Theorem 2.1: The double fan P_n+2K_1 has edge trimagic total labeling.

Proof: Let w_1,w_2,…,w_n be the vertices of path P_n.Let G be a graph P_n+2K_1.

Let $V(G)$ = $\{u, v, w_i/1 \leq i \ \leq n\}$ and $E(G)$ = $\{w_i w_{i+1},\, uw_i,\, vw_i/1 \leq i \ \leq n\}$.

The order of G is $p = n + 2$ and size $q = 3n - 1$

Let us define the function $f: V \cup E \rightarrow \{1, 2, 3,\ldots,3n + 2\}$ by

Case-1: n is odd

$f(u)$ = 2n+1;

$f(v)$ = 2n+2;

$$f(w_i) = \begin{cases} \frac{i+1}{2}, i \text{ is odd} \\ \frac{n+i+1}{2}, i \text{ is even} \end{cases}$$

$$f(w_i u) = \begin{cases} 2n - \frac{i+1}{2} + 1,\ i \text{ is odd} \\ 2n - \frac{n+i+1}{2} + 1, i \text{ is even} \end{cases}$$

$f(w_i w_{i+1})$= 4n+2-i for all $1 \leq i \ \leq n$

$$f(w_i v) = \begin{cases} 3n + 3 - \frac{i+1}{2}, i \text{ is odd} \\ 3n + 3 - \frac{n+i+1}{2}, i \text{ is even} \end{cases}$$

Now we prove that the double fan P_n+2K_1 has edge trimagic total labeling.

For the edges$w_i w_{i+1}$, $1 \leq i \leq n$

For odd i,

f(w_i)+f($w_i w_{i+1}$) +f(w_{i+1}) = $\frac{i+1}{2}$ + 4n+2-i + $\frac{n+i+1+1}{2}$ = $\frac{9n+7}{2}$ = λ_1

For even i,

f(w_i)+f($w_i w_{i+1}$) +f(w_{i+1}) = $\frac{n+i+1}{2}$ + 4n+2-i + $\frac{i+1+1}{2}$ = $\frac{9n+7}{2}$ = λ_1.

For the edges $w_i u$, $1 \leq i \leq n$

For odd i,

f(w_i)+f($w_i u$)+f(u) = $\frac{i+1}{2}$ + 2n-$\frac{i+1}{2}$+1 + 2n+1 = 4n+2 = λ_2

For even i,

f(w_i)+f($w_i u$)+f(u) = $\frac{n+i+1}{2}$ + 2n-$\frac{n+i+1}{2}$+1 + 2n+1 = 4n+2 = λ_2

For the edges $w_i v$, $1 \leq i \leq n$

For odd i,

f(w_i)+f($w_i v$)+f(v) = $\frac{i+1}{2}$ + 3n+3-$\frac{i+1}{2}$ + 2n+2 = 5n+5 = λ_3

For even i,

f(w_i)+f($w_i v$)+f(v) = $\frac{n+i+1}{2}$ + 3n+3-$\frac{n+i+1}{2}$ + 2n+2 = 5n+5 = λ_3

Hence for each $uv \epsilon E$, f(u) + f(uv) + f(v) has any one of the magic constants $\lambda_1 = \frac{9n+7}{2}$, λ_2 = 4n+2, λ_3 = 5n+5

Hence the double fan P_n+2K_1 has edge trimagic total labeling.

Example 2.2: The double fan P_5+2K_1 given in figure is edge trimagic total labeling.

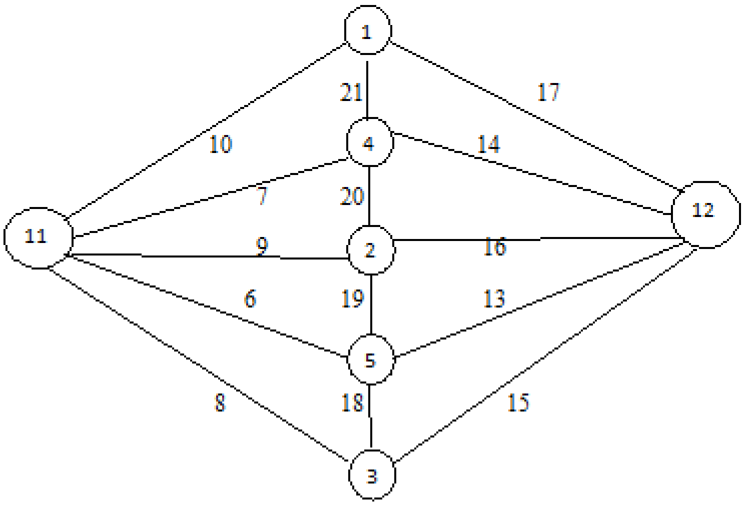

Figure 1: P_5+2K_1 with λ_1 = 26, λ_2 = 22, λ_3=30

Case-1: n is even

$f(u)$= 2n+1;

$f(v)$ = 3n+2;

$$f(w_i) = \begin{cases} \frac{i+1}{2}, i \text{ is odd} \\ \frac{n+i}{2}, i \text{ is even} \end{cases}$$

$$f(w_i u)= \begin{cases} 2n - \frac{i+1}{2} + 1, i \text{ is odd} \\ 2n - \frac{n+i}{2} + 1, i \text{ is even} \end{cases}$$

$f(w_i w_{i+1})$ = 4n+2-i for all $1 \le i \le n$

$$f(w_i v) = \begin{cases} 3n + 2 - \frac{i+1}{2}, i \text{ is odd} \\ 3n + 2 - \frac{n+i}{2}, i \text{ is even} \end{cases}$$

Now we prove that the double fan P_n+2K_1 yields edge trimagic total labeling.

For the edges $w_i w_{i+1}$, $1 \le i \le n$

For odd i,

$f(w_i)$ +$f(w_i w_{i+1})$ +$f(w_{i+1})$ = $\frac{i+1}{2}$ +4n+2-i+$\frac{n+i+1}{2}$ = $\frac{9n+6}{2}$ = λ_1

For even i,

$f(w_i)$ +$f(w_i w_{i+1})$ +$f(w_{i+1})$ = $\frac{n+i}{2}$ +4n+2-i+$\frac{i+1+1}{2}$ = $\frac{9n+6}{2}$ = λ_1

For the edges $w_i u$, $1 \le i \le n$

For odd i,

$f(w_i)$+$f(w_i u)$+$f(u)$ = $\frac{i+1}{2}$+2n-$\frac{i+1}{2}$+1+2n+1 = 4n+2 = λ_2

For even i,

$f(w_i)+f(w_iu)+f(u) = \frac{n+i}{2}+2n-\frac{n+i}{2}+1+2n+1 = 4n+2 = \lambda_2$

For the edges w_iv, $1 \leq i \leq n$

For odd i,

$f(w_i)+f(w_iv)+f(v) = \frac{i+1}{2}+3n+2-\frac{i+1}{2}+3n+2 = 6n+4 = \lambda_3$

For even i,

$f(w_i)+f(w_iv)+f(v) = \frac{n+i}{2}+3n+2-\frac{n+i}{2}+3n+2 = 6n+4 = \lambda_3$

Hence for each $uv \epsilon E$, f(u) + f(uv) + f(v) has any one of the magic constants $\lambda_1 = \frac{9n+6}{2}$, λ_2 = 4n+2,λ_3 = 6n+4

Hence the double fan P_n+2K_1 has edge trimagic total labeling

Example 2.3: The double fan P_4+2K_1 given in figure is edge trimagic total labeling

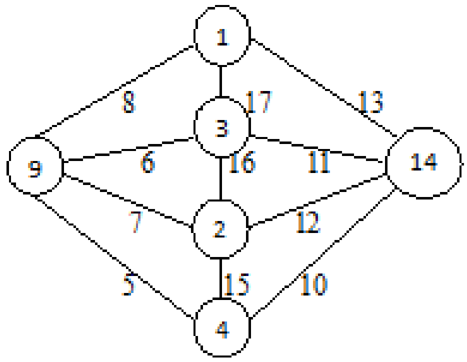

Figure 2: P_4+2K_1 with λ_1 = 21, λ_2 =18, λ_3= 28

3. SUPER EDGE TRIMAGIC TOTAL LABELING

Theorem 3.1: The graph $B^2_{n,n}$ has super edge trimagic total labeling.

Proof: Let G be a restricted $B^2_{n,n}$ graph and let V(G) = {v,u,v_iu_i /$1 \leq i \leq n$ }, where v_iu_i are pendant vertices with $V(G) = V(B_{n,n}) = V(B^2_{n,n})$ and $E(B^2_{n,n}) = E(B_{n,n}) \cup \{uv_i,vu_i/1 \leq i \leq n\}$.The order of $B^2_{n,n}$ is p = 2n + 2 and size is $q = 4n + 1$

Let us define the function $f : V \cup E \longrightarrow$ {1, 2, 3…,6n+3} by

$f(u)$ = 1; $f(v)$ = 2;

$f(u_i)$ = i+2 for all $1 \leq i \leq n$;

$f(v_i)$ = 2n-i+3 for all $1 \leq i \leq n$;

$f(uv_i)$ = 2n+i+2 for all $1 \leq i \leq n$;

$f(uu_i)$ = 4n-i+3 for all $1 \leq i \leq n$;

$f(uv)$ = 4n+3;

$f(uu_i)$ = 5n-i+4 for all $1 \leq i \leq n$;

$f(vv_i)$ = 5n+i+3 for all $1 \leq i \leq n$;

Now we prove that the graph $B^2_{n,n}$ admits super edge trimagic total labeling.

For the edge,
$f(u) + f(uv) + f(v)$ = 1+4n+3+2 = 4n+6 = λ_1

For the edges uu_i, $1 \le i \le n$
$f(u) + f(uu_i) + f(u_i)$ = 1+4n-i+3+i+2 = 4n+6 = λ_1

For the edges vu_i, $1 \le i \le n$
$f(v) + f(vu_i) + f(u_i)$ = 2+5n-i+4+i+2 = 5n+8 = λ_2

For the edges vv_i, $1 \le i \le n$
$f(v) + f(vv_i)$+f(v_i) = 2+5n+i+3+2n-i+3 = 7n+8 = λ_3

For the edges uv_i, $1 \le i \le n$
$f(u) + f(uv_i) + f(v_i)$ = 1+2n+i+2+2n-i+3 = 4n+6 = λ_1

Hence for each $uv \in E$, $f(u) + f(uv) + f(v)$ has any one of the magic constants λ_1 = 4n+6, λ_2 = 5n+8, λ_3 = 7n+8.

Hence the graph $B^2_{n,n}$ is a super edge trimagic total labeling.

Example 3.2: The graph $B^2_{5,5}$ is given in figure super edge trimagic total labeling

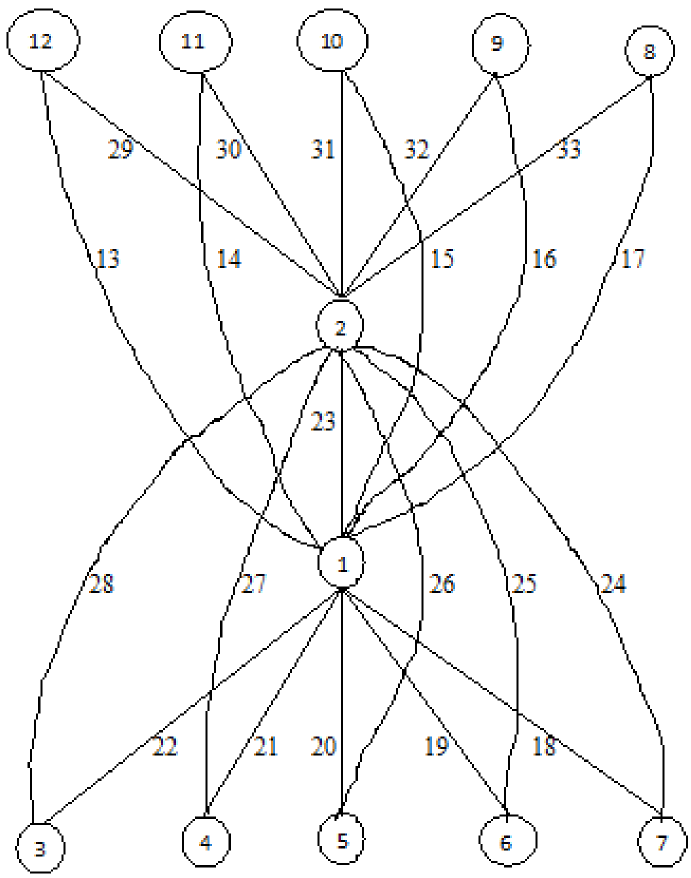

Figure 3: $B^2_{5,5}$ with λ_1 = 26, λ_2 = 33, λ_3=43

Theorem 3.3: Z – (P_n) has a super edge trimagic total labeling.

Proof: Let G be a Z – (P_n).P_n is the i^{th}vertex of a path P_1 is joined with $(i+1)^{th}$ vertex of a path P_2.

Let $V(G)$= $\{u_{i,} v_i/$ 1≤ i ≤ n} and $E(G)$= $\{u_i u_{i+1}, v_i v_{i+1}, u_{i+1} v_i/$1≤ i ≤ n}.

The order of Z – (P_n) is $p = 2n$ and size is $q = 3n - 3$. Let us define the function $f: V \cup E \to$ {1, 2, 3,…5n-3} by

Case-1: n is odd

$$f(u_i) = \begin{cases} \frac{i+1}{2}, i \text{ is odd} \\ \frac{n+i+1}{2}, i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{2n+i+1}{2}, i \text{ is odd} \\ \frac{3n+i+1}{2}, i \text{ is even} \end{cases}$$

$f(u_i u_{i+1})$ = 3n-i for all 1≤ i ≤ n

$f(v_i v_{i+1})$ = 4n-1-i for all 1≤ i ≤ n

$f(u_{i+1} v_i)$ = 5n-3-i+1 for all 1≤ i ≤ n

Now we prove that the graph Z – (P_n) has edge trimagic total labeling.

For the edges $u_i u_{i+1}, 1 \leq i \leq n$

For odd i,

$$f(u_i) + f(u_i u_{i+1}) + f(u_{i+1}) = \frac{i+1}{2} + 3n - i + \frac{n+i+1+1}{2} = \frac{7n+3}{2} = \lambda_1$$

For even i,

$$f(u_i) + f(u_i u_{i+1}) + f(u_{i+1}) = \frac{n+i+1}{2} + 3n - i + \frac{i+1+1}{2} = \frac{7n+3}{2} = \lambda_1$$

For the edges $v_i v_{i+1}, 1 \leq i \leq n$

For odd i,

$$f(v_i) + f(v_i v_{i+1}) + f(v_{i+1}) = \frac{2n+i+1}{2} + 4n - 1 - i + \frac{3n+i+1+1}{2} = \frac{13n+1}{2} = \lambda_2$$

For even i,

$$f(v_i) + f(v_i v_{i+1}) + f(v_{i+1}) = \frac{3n+i+1}{2} + 4n - 1 - i + \frac{2n+i+1+1}{2} = \frac{13n+1}{2} = \lambda_2$$

For the edges $u_{i+1} v_i, 1 \leq i \leq n$

For odd i,

$$f(u_{i+1}) + f(u_{i+1} v_i) + f(v_i) = \frac{n+i+1+1}{2} + 5n - 3 - i + 1 + \frac{2n+i+1}{2} = \frac{13n-1}{2} = \lambda_3$$

For even i,

$$f(u_{i+1}) + f(u_{i+1} v_i) + f(v_i) = \frac{i+1+1}{2} + 5n - 3 - i + 1 + \frac{3n+i+1}{2} = \frac{13n-1}{2} = \lambda_3$$

Hence for each $uv \in E, f(u) + f(uv) + f(v)$ has any one of the magic constants $\lambda_1 = \frac{7n+3}{2}$, $\lambda_2 = \frac{13n+1}{2}$, $\lambda_3 = \frac{13n-1}{2}$.

Case-2: n is even

$$f(u_i) = \begin{cases} \frac{i+1}{2}, i \text{ is odd} \\ \frac{n+i}{2}, i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} \frac{2n+i+1}{2}, i \text{ is odd} \\ \frac{3n+i}{2}, i \text{ is even} \end{cases}$$

$f(u_i u_{i+1})$ = 3n-i for all 1≤ i ≤ n

$f(v_i v_{i+1})$ = 4n-1-i for all 1≤ i ≤ n

$f(u_{i+1} v_i)$ = 5n-3-i+1 for all 1≤ i ≤ n

Now we prove that the graph Z – (P_n) has edge trimagic total labeling.

For the edges $u_iu_{i+1}, 1 \leq i \leq n$

For odd i,

$$f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = \frac{i+1}{2} + 3n - i + \frac{n+i+1}{2} = \frac{7n+2}{2} = \lambda_1$$

For even i,

$$f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = \frac{n+i}{2} + 3n - i + \frac{i+1+1}{2} = \frac{7n+2}{2} = \lambda_1$$

For the edges $v_iv_{i+1}, 1 \leq i \leq n$

For odd i,

$$f(v_i) + f(v_iv_{i+1}) + f(v_{i+1}) = \frac{2n+i+1}{2} + 4n - 1 - i + \frac{3n+i+1}{2} = \frac{13n}{2} = \lambda_2$$

For even i,

$$f(v_i) + f(v_iv_{i+1}) + f(v_{i+1}) = \frac{3n+i}{2} + 4n - 1 - i + \frac{2n+i+1+1}{2} = \frac{13n}{2} = \lambda_2$$

For the edges $u_{i+1}v_i, 1 \leq i \leq n$

For odd i,

$$f(u_{i+1}) + f(u_{i+1}v_i) + f(v_i) = \frac{n+i+1}{2} + 5n - 3 - i + 1 + \frac{2n+i+1}{2} = \frac{13n-2}{2} = \lambda_3$$

For even i,

$$f(u_{i+1}) + f(u_{i+1}v_i) + f(v_i) = \frac{i+1+1}{2} + 5n - 3 - i + 1 + \frac{3n+i}{2} = \frac{13n-2}{2} = \lambda_3$$

Hence for each $uv \in E$, $f(u) + f(uv) + f(v)$ has any one of the magic constants $\lambda_1 = \frac{7n+2}{2}$, $\lambda_2 = \frac{13n}{2}$, $\lambda_3 = \frac{13n-2}{2}$.

Hence the graph Z – (P_n) admits super edge trimagic total labeling

Example 3.4: The graphs Z – (P_3), Z – (P_4) given in figure is super edge trimagic total labeling

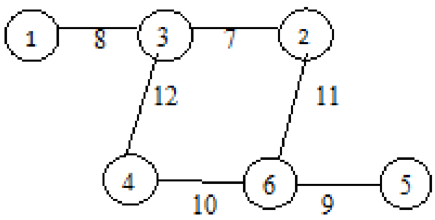

Figure 4: Z – (P_3) with λ_1 = 12, λ_2= 20, λ_3=19

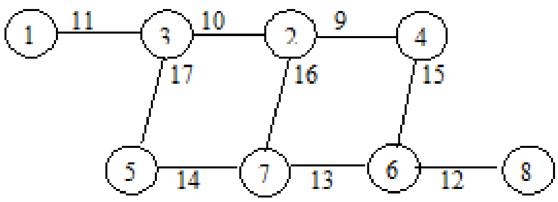

Figure 5: Z – (P_4) with λ_1 = 15, λ_2= 26, λ_3=25

CONCLUSION

In this paper we proved that the Double fan P_n+2K_1 is edge trimagic total labeling and the graphs Z-P_n, $B^2_{n,n}$ are super edge trimagic total labeling.

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