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HARMONIOUS COLORING OF CENTRAL GRAPH OF SOME TYPES OF GRAPHS

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ABSTRACT

Harmonious colouring is a proper vertex colouring such that no two edges share the same colour pair. The harmonious Chromatic number of a graph is the least number of colours in such a colouring. The purpose of this paper is to study the harmonious coloring of central graph of flower graph, belt graph, rose graph and steering graph and harmonious chromatic number for these graphs. Also we give some structural properties of these graphs.

Keywords: *Harmonious coloring, Harmonious chromatic number, central graph of a graph, flower graph, belt graph, rose graph, steering graph.*

INTRODUCTION

Harmonious coloring:

In graph theory, a **harmonious coloring** is a (proper) vertex coloring in which every pair of colors appears on at most one pair of adjacent vertices. The **harmonious chromatic number** $\chi_H(G)$ of a graph G is the minimum number of colors needed for any harmonious coloring of G.

Central graph of a graph:

The central graph of any graph G is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G.

Definition: A **Flower graph** is obtained from a cycle C = v_1, v_2……v_n, v_1 by joining the two consecutive vertices by a new vertices u_1, u_2,….u_n. A flower graph has 2n vertices and 3n edges, where n is the length of the cycle. We denote this graph F_n.

Example:

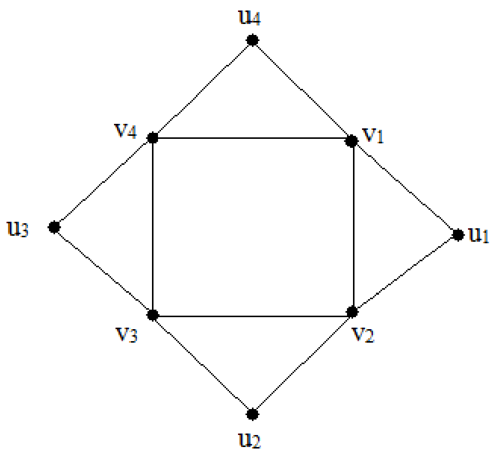

Figure 1: Flower graph F_4

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Structural Properties of central graph of Flower graph:

Maximum degree $\Delta = 4$

Minimum degree $\delta = 2$

Number of vertices in $C(F_n) = 5n$

Number of edges in $C(F_n) = \frac{(2n)(2n-1)}{2} + 3n$

Maximum degree $\Delta[C(F_n)] = 2n-1$

Minimum degree $\delta[C(F_n)] = 2$

Theorem: The Harmonious chromatic number of central graph of Flower graph $C[F_n]$ is $\Delta[C(F_n)] + 3$.
(i.e) $\chi_H C(Fn) = \Delta[C(F_n)] + 3$.

Proof: Let F_n be the Flower graph with 2n vertices and 3n edges.

Let $\{v_1, v_2 \ldots\ldots v_n, u_1, u_2 \ldots\ldots u_n\}$ be the vertices of the Flower graph F_n.

By the definition of Central graph, each edge is subdivided by a new vertex.

Therefore, Assume that each edge (v_i, v_{i+1}) and the line joining v_i and v_{i+1} to a new vertex u_i, i= 1 to n are subdivided by the vertices e_{jj}, x_i, y_i, i & j = 1 to n respectively.

Assign the coloring to vertices as follows:

$c(v_i) = i$ for $1 \leq i \leq n$

$c(u_i) = n+i$ for $1 \leq i \leq n$

$c(e_{jj}) = 2n+1$ for $1 \leq j \leq n$

$c(x_i) = 2n+2$ for $1 \leq i \leq n$

$c(y_i) = n+i$ for $1 \leq i \leq n$

First we claim that, c is a proper coloring.

Since $c(v_i)$,$c(u_i)$ and its neighbours receive distinct colors.

Further $c(v_i) \neq c(u_i)$.

Hence c is a proper coloring.

Next we claim that, c is a Harmonious coloring.

It is clear that each vertex receive a distinct color at a distance atmost 2 from all other vertices.

Thus the coloring is harmonious.

Finally we claim that c is the minimum number of colors in a harmonious coloring.

Suppose not we have the following cases.

Case (i): The color set $c(v_i)$ & $c(u_i)$ containing 2n colors.

If we assign 2n-1 colors, then the color pair will not distinct. which is contradict the definition of harmonious colors.

Case (ii): The neighbours of u_i and v_i are given the same color. Then the color pair repeats which is contradiction to the definition of harmonious coloring.

Illustration for the above theorem:

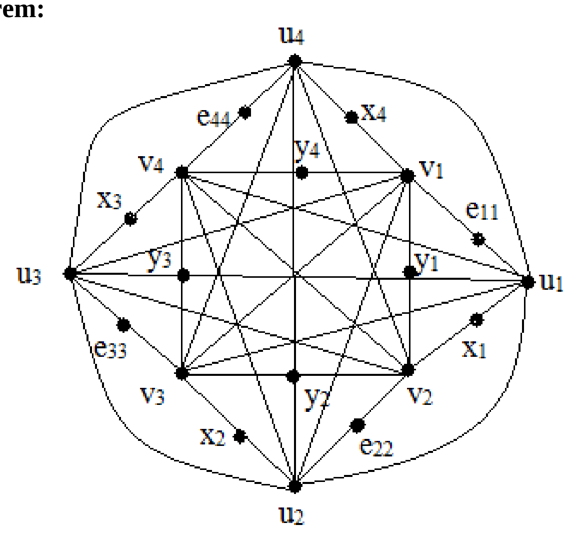


Figure 2: Central graph of Flower graph [$C(F_4)$]

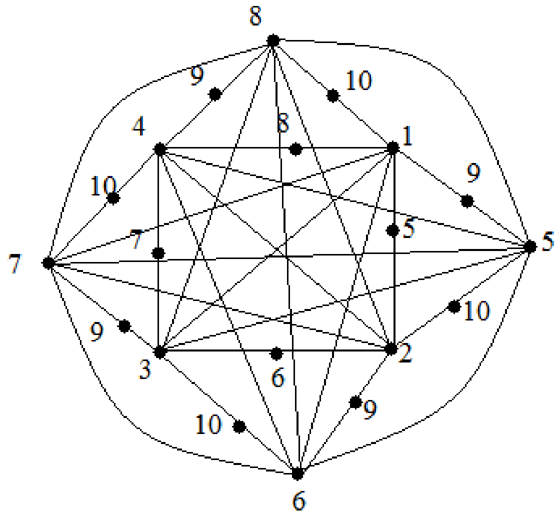
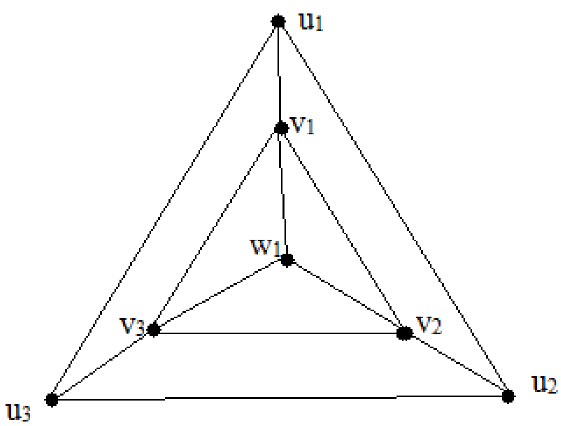


Figure 3: χ_H $C[F_4] = 10$

Definition: A **Belt graph** is obtained from Helm graph by joining its outer vertices. It has 2n+1 vertices 4n edges. It is denoted by B_n.

Example:



Structural Properties of central graph of graph:

Minimum degree $\delta = 3$

Number of vertices in C(G) = 6n+1

Number of edges in C(G) = 8n

Maximum degree Δ[C(G)] = 2n

Minimum degree δ[C(G)] = 2

Theorem: The Harmonious chromatic number of central graph of Belt graph C[B_n] is 4n+1. $(i.e)\chi_H$[C(B_n)] = 4n+1.

Proof: Let {v_1, v_2……v_n, u_1, u_2……u_n, w_1} be the vertices of the Belt graph .

By the definition of Central graph, each edge is subdivided by a new vertex.

The edges (u_i, u_{i+1}) & (u_n, u_1) is obtained from the outer vertices helm graph are joined. Therefore, Assume that each edges(u_i ,u_{i+1}) &(u_n,u_1) are subdivided by the new vertex x_i, e_{jj}, x_i, y_i, $z_{i,}$, i & j = 1 to n respectively.

Assign the coloring to vertices as follows:
c(v_i) = i for 1≤ i ≤ n

c(u_i) = n+i for 1≤ i ≤ n

c(w_1) = 2n+1

c(e_{jj}) = (2n+2)+j

c(x_i) = (2n+1)+i for 1≤ i ≤ n

c(y_i) = (3n+1)+i

c(z_i) = (2n+1)+i

Clearly 4n+1 coloring.

First we claim that, c is a proper coloring.

Since c(v_i), c(u_i), c(w_i) and its neighbours receive distinct colors.

Further c(v_i) ≠ c(u_i) ≠ c(w_i)

Hence c is a proper coloring.

Next we claim that, c is a Harmonious coloring.

Its clear that each vertex receive a distinct color at a distance atmost 2 from all other vertices.

Thus the coloring is harmonious.

Finally we claim that c is the minimum number of colors in a harmonious coloring.

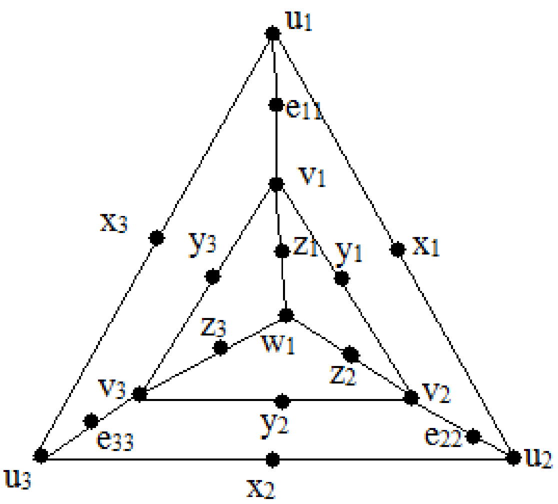
Suppose not we have the following cases.

Case (i): The color set c(v_i), c(u_i) & c(w_1) containing 2n+1.

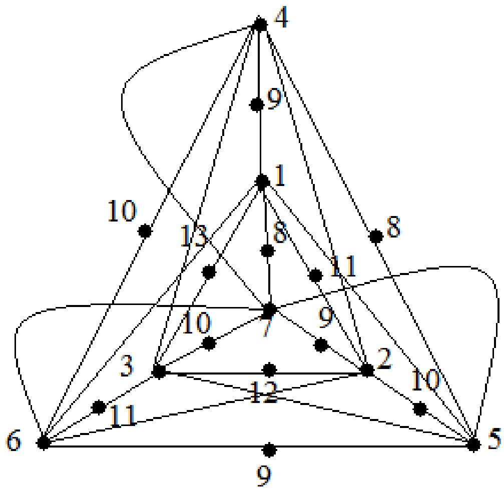
If we assign 2n colors, then the color pair will not distinct. which is contradict the definition of harmonious colors.

Case (ii): The neighbours of u_i, v_i, w_1 are given the same color. then the color pair repeats which is contradiction to the definition of harmonious coloring.

Illustration for the above theorem:



Central graph of Belt graph $C[B_3]$



$\chi_H[C(B_n)] = 4(3)+1=13$

Definition: A **Rose graph** is obtained from a wheel graph w_n with n vertices ($n\geq 4$) by joining the two consecutive vertices $v_1, v_2,\ldots\ldots\ldots v_n$ by a new vertices $u_1,u_2,\ldots.u_n$. A Rose graph has 2n+1 vertices and 4n edges. Where n is length of the cycle .We denote this graph by R_n.

Example:

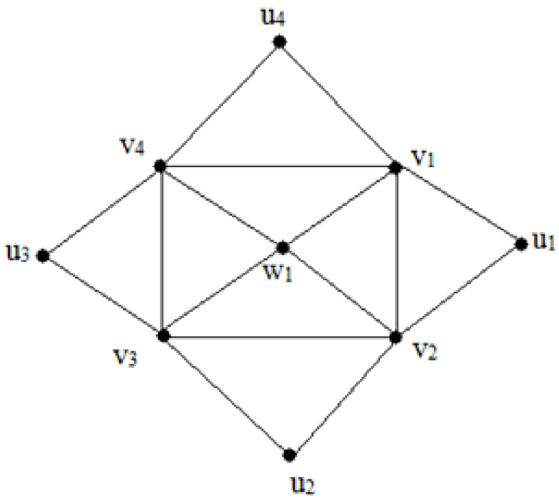


Figure 1: Rose graph R_4

Structural Properties of central graph of Rose graph:

Minimum degree $\delta = 2$

Number of vertices in $C(R_n) = 6n+1$

Number of edges in $C(R_n) = \frac{(2n)(2n+1)}{2} + 4n$

Maximum degree $\Delta[C(R_n)] = p-1$ where p is the point on the orignal graph
(or) 2n.
Minimum degree $\delta[C(R_n)] = 2$

Theorem: The Harmonious chromatic number of central graph of Rose graph $C[R_n]$ is 4n-1.
(i.e) $\chi_H C(R_n) = 4n-1$.

Proof: Let F_n be the Rose graph with 2n+1 vertices and 4n edges.

Let $\{v_1, v_2 \ldots\ldots v_n, u_1, u_2 \ldots\ldots u_n, w_1\}$ be the vertices of the Rose graph R_n.

By the definition of Central graph, each edge is subdivided by a new vertex.

Therefore, Assume that each edge (v_i,v_{i+1}) and the line joining v_i and v_{i+1} to a new vertex u_i, i= 1 to n are subdivided by the vertices e_{jj}, z_i, x_i, y_i, i & j = 1 to n respectively.

Assign the coloring to vertices as follows:
$c(v_i) = i$ for $1\leq i \leq n$

$c(u_i) = n+i$ for $1\leq i \leq n$

$c(w_1) = 2n+1$

$c(e_{jj}) = 2n+2$ for $1\leq j \leq n$

$c(z_i) = 2n+3$ for $1\leq j \leq n$

$c(x_i) = (2n+3)+i$ for $1\leq i \leq n$

$c(y_i) = n+ i$ for $1\leq i \leq n$

Clearly 4n-1 coloring.

First we claim that, c is a proper coloring.

Since $c(v_i)$, $c(u_i)$ and $c(w_1)$ and its neighbors receive distinct colors. Further $c(v_i) \neq c(u_i) \neq c(w_1)$.

Hence c is a proper coloring.

Next we claim that, c is a Harmonious coloring.

Its clear that each vertex receive a distinct color at a distance atmost 2 from all other vertices.

Thus the coloring is harmonious.

Finally we claim that c is the minimum number of colors in a harmonious coloring.

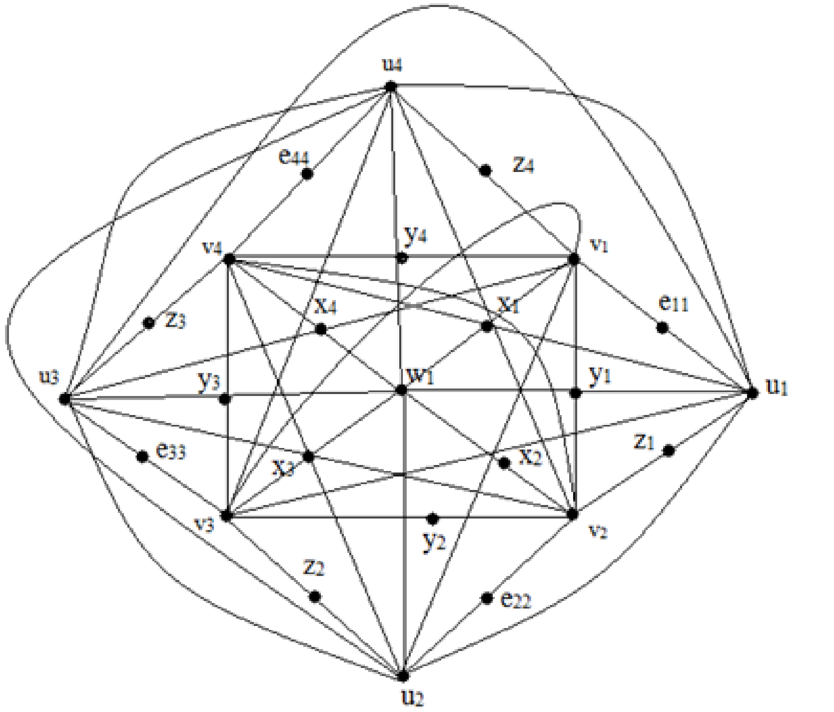
Suppose not we have the following cases.

Case (i): The color set $c(v_i)$, $c(u_i)$ & $c(w_1)$ containing 2n+1 colors.

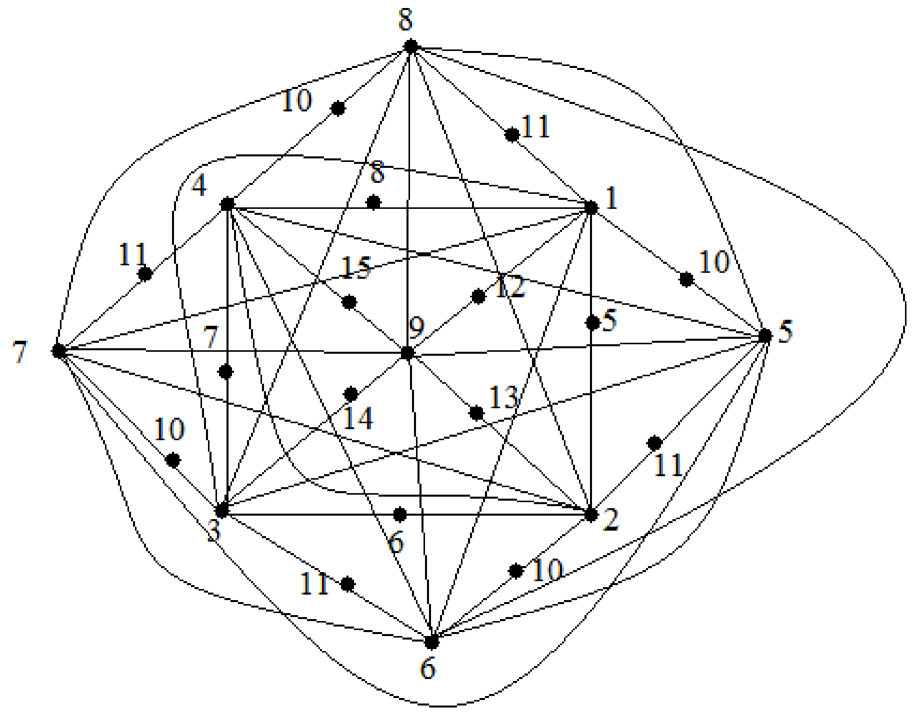
If we assign 2n colors, then the color pair will not distinct. which is contradict the definition of harmonious colors.

Case (ii): The neighbors of u_i, v_i and w_1 are given the same color. then the color pair repeats which is contradiction to the definition of harmonious coloring.

Illustration for the above theorem:



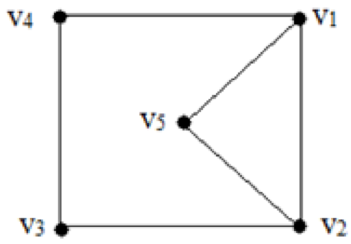
Central graph of Rose graph C[R_4]



χ_H[C(R_n)] = 4(4)-1 = 15

Definition: A **Steering graph** is a graph formed by connecting a any two vertices of a cycle by a single vertex. A Steering graph with n vertices (n > 3). A stearing graph has n+ 1 vertex and n+2 edges. It is denoted by S_n.

Example:



Stearing graph S_4

Structural properties of central graph of stearing graph:

Maximum degree $\Delta[S_n] = 3$

Minimum degree $\delta[S_n] = 2$

Number of vertices in $C[S_n] = 2n+3$

Number of edges in $C[S_n] = \frac{(n+1)n}{2} + (n+2)$

Maximumdegree $\Delta[C(Sn)] = n$(length of the cycle in original graph)

Minimum degree $\delta[C(S_n)] = 2$

Theorem: The harmonious chromatic number of central graph of stearing graph $C[S_n]$ is 2n,where n is the length of the cycle.

(i.e) $\chi_H [C(S_n)] = 2n$.

Proof: Let S_n be the stearing graph with n+1 vertices and n+2 edges.

Let $\{v_1, v_2,\ldots..v_{n+1}\}$ be the vertices of S_n.let the vertices joining v_1 and v_2 we obtained a new vertex u_1. By the definition of Central graph, each edge is subdivided by a new vertex. Assume that each edge (v_i,v_{i+1}) and the line joining v_i and v_{i+1}, v_n and v_1 are subdivided by the vertices f_i, i=1, 2…..n respectively.And the line joining v_1 and u_1, v_2 and u_1 are subdivided the vertices x_1,y_1.

Assign coloring to the vertices as follows:
$c(v_i) = i$ for $1\leq i \leq n+1$

$c(u_i) = n+1$

$c(f_{i,}) = n+i$ for $1\leq i \leq n$

$c(x_1) = 2n-1$

$c(y_1) = 2n$

Clearly c is 2n coloring.

First we claim that c is proper coloring.

Since each $c(v_i)$, $c(u_i)$ and its neighbours receive distinct colors.

Further $c(v_i) \neq c(u_i)$.

Hence c is a proper coloring.

Next we claim that c is harmonious coloring.

It is clear that each vertex receive a distinct color at a distance atmost 2 from all other vertices.

Thus the coloring is harmonious.

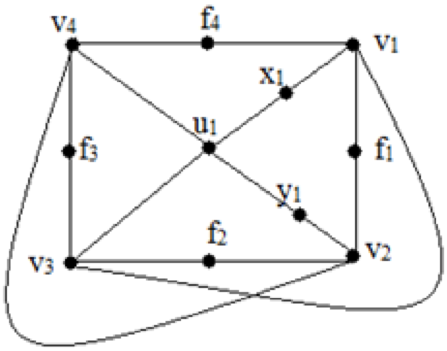
Finally we claim that c is minimum number of colors in a harmonious coloring.

Suppose not we have following cases.

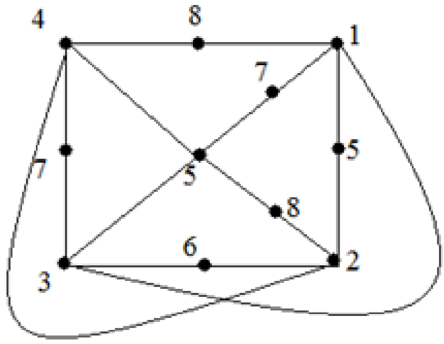
Case (i): The color set $c(v_i)$ and $c(u_i)$ contains n+1 colors. If we assign n colors ,then the will not be distinct which contradict the definition of harmonious coloring.

Case (ii): The neighbour of v_i and u_i are given the same color,then the color pair repeats which is contradiction to the definition of harmonious coloring. Hence the minimum number of colors in a harmonious coloring for central graph of stearing graph is 2n.

Illustration of the above theorem:



Central graph of stearing graph $C[S_4]$



$\chi_H\ [C(S_n)] = 2(4) = 8$

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