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ASCENDING GRAPHOIDAL TREE COVER

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ABSTRACT

Ascending graphoidal tree cover of a graph G is a partition of edges of G into trees $G_1, G_2, \ldots, G_n$ such that $|E(G_i)|<|E(G_{i+1})|$ for all i=1 to n-1 and every vertex of G is an internal vertex of at most one tree. In this paper, we investigate the ascending graphoidal tree cover for some standard graphs.

Keywords: *Graphoidal tree cover, Ascending cover, Ascending graphoidal tree cover.*

AMS Subject Classification: *05C70.*

1. PRELIMINARIES

In this paper we consider only simple graphs G. In [6], we introduce the concept of Ascending cover which is decomposition of G into edge disjoint sub graphs $G_1, G_2, \ldots, G_n$ such that $|E(G_i)|<|E(G_{i+1})|$ for all i=1 to n-1. It is observed that if $\psi = \{G_1, G_2, \ldots, G_n\}$ is an ascending cover of G then $q=\sum_{i=1}^{n}|E(G_i)| \geq 1+2+\ldots+n=\binom{n+1}{2}$ and if $q=\binom{n+1}{2}$ then $|E(G_i)|=i$, $1 \leq i \leq n$. Further if each G_i is connected, it is known as Continuous Monotonic Decomposition of G [6]. If each G_i is isomorphic to a sub graph of G_{i+1} then it is known as Ascending Sub graph Decomposition. The concept of graphoidal cover was introduced by E. Sampath kumar and B. D. Acharya [1]. In [6]; we study Ascending graphoidal cover, which is ascending cover of G into internally disjoint paths, for some standard graphs. In [8], we defined and studied graphoidal tree cover which is partition of E(G) into internally vertex disjoint trees. Definitions which are not seen here can be found in [3] and [4]. In this paper, we propose to study Ascending graphoidal tree cover.

2. MAIN RESULTS

Throughout this paper we consider only connected graphs.

Definition 2.1: Ascending Graphoidal Tree Cover (AGTC) of G is defined as ascending cover of G satisfying the following conditions:

(i) each sub graph is isomorphic to a tree
(ii) every vertex is an internal vertex of at most one tree.

In other words, Ascending Graphoidal Tree Cover is a decomposition of G into edge-disjoint sub graphs $G_1, G_2, \ldots, G_n$ such that

(i) $|E(G_i)|<|E(G_{i+1})|$ for all i=1 to n-1
(ii) each sub graph is isomorphic to a tree
(iii) every vertex is an internal vertex of at most one tree.

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Lemma 2.2: If a (p, q) graph G admits AGTC then $p \geq n+1$.

Proof: As $n \leq |E(G_n)| \leq p-1$, we have $p \geq n+1$.

Theorem 2.3: Any path $P_n (n \geq 2)$ admits AGTC into q parts if and only if $|E(P_n)| = \frac{q(q+1)}{2}$ for some positive integer q.

Proof: Label the vertices of P_n by (0, 1, 2,…, n-1) and suppose $|E(P_n)| = \frac{q(q+1)}{2}$ for some q. Then the Ascending graphoidal tree cover is as follows:

$$T_i = \left(\frac{(i-1)i}{2}, \frac{(i-1)i}{2}+1, \frac{(i-1)i}{2}+2, \ldots, \frac{i(i+1)}{2} \right) \text{ for } 1 \leq i \leq q.$$

Thus P_n admits AGTC into q parts for some positive integer q. The converse is straight forward.

Theorem 2.4: Any cycle $C_n (n \geq 3)$ admits AGTC into q parts if and only if $|E(C_n)| = n = \frac{q(q+1)}{2}$ for some positive integer q.

Proof: Label the vertices of C_n by (0, 1, 2,…, n-1) and suppose $|E(C_n)| = \frac{q(q+1)}{2}$ for some q. Then consider

$$T_i = \left(\frac{(i-1)i}{2}, \frac{(i-1)i}{2}+1, \frac{(i-1)i}{2}+2, \ldots, \frac{i(i+1)}{2} \right) \text{ for } 1 \leq i \leq q-1 \text{ and}$$

$$T_q = \left(\frac{(q-1)q}{2}, \frac{(q-1)q}{2}+1, \ldots, \frac{q(q+1)}{2}-1, 0 \right) \text{ is clearly AGTC of } C_n.$$

Thus C_n admits AGTC.

Theorem 2.5: The complete graph K_p admits AGTC into n parts if and only if p=n+1.

Proof: Let p=n+1. Let $E(G_1) = (v_1, v_2)$ and $E(G_i) = \{(v_{i+1}, v_j): 1 \leq j \leq i, 2 \leq i \leq n-1\}$ and $E(G_n) = \{(v_{n+1}, v_j): 1 \leq j \leq n\}$. Clearly $\{G_1, G_2, \ldots, G_n\}$ is a AGTC with $|E(G_i)|=i$, $1 \leq i \leq n$. Hence it is the required AGTC of G.

Conversely if K_p admits AGTC into n parts, then $|E(K_p)| = \frac{n(n+1)}{2}$ and so p=n+1.

Theorem 2.6: The wheel $W_m = K_1 + C_{m-1}$ admits AGTC into n trees if and only if n=3 and 4.

Proof: Let $V(W_m) = \{v_0, v_1, \ldots, v_{m-1}\}$ where v_0 is the central vertex of W_m. Since v_0 is of maximum degree and by the condition (ii) in the definition of Ascending graphoidal tree cover, we consider G_n as a star with v_0 as a central vertex. Let $G_n = \{(v_0, v_i): 1 \leq i \leq n\}$. Then G_{n-1} should be defined as a path of length n-1, say $\{(v_1, v_2, \ldots, v_n)\}$. Since v_0 is the internal vertex of G_n, at most one of the edges $v_0 v_i (n+1 \leq i \leq m-1)$ say, $v_0 v_{n+1}$ lies in G_{n-2} and the remaining n-3 edges are from C_{m-1} starting from $v_n v_{n+1} v_{n+2} \ldots v_{2n-3}$. If $(v_0 v_{n+2}) = G_1$ then one of the edges $v_0 v_{n+2}, v_0 v_{n+3}, \ldots, v_0 v_{2n-2}$ do not belong to any subgraphs $G_i (2 \leq i \leq n-3)$. Hence there should be at most 2 internal vertices in G_{n-2} so that $|E(G_{n-2})| \leq 4$ or $n \leq 6$. As $|E(G_n)| = \frac{n(n+1)}{2}$, we have $\frac{n(n+1)}{2} = 2(m-1)$.

That is, n(n+1)=4(m-1) and $n \leq 6$. Then we get n=3, 4.

Converse is straight forward.

Theorem 2.7: The complete bipartite graph $K_{m,n}$ admits AGTC if and only if n=2m-1 or n=2m+1.

Proof: Let (V_1, V_2) be the bipartition of $K_{m,n}$ where $V_1 = \{u_1, u_2, \dots, u_m\}$ and $V_2 = \{v_1, v_2, \dots, v_n\}$.

Case (i): If n=2m-1.

Consider $T_1 = (v_n, u_1)$

$$T_2 = \{(v_{n-1}, u_i) / 1 \le i \le 2\}$$
$$T_3 = \{(v_{n-2}, u_i) / 1 \le i \le 3\}$$
$$\dots$$
$$T_m = \{(v_{n-m+1}, u_i) / 1 \le i \le m\}$$
$$T_{m+1} = \{(v_{n-m}, u_i) / 1 \le i \le m\} \cup (u_2, v_n)$$
$$T_{m+2} = \{(v_{n-m-1}, u_i) / 1 \le i \le m\} \cup \{(u_3, v_j) / n-1 \le j \le n\}$$
$$T_{m+3} = \{(v_{n-m-2}, u_i) / 1 \le i \le m\} \cup \{(u_4, v_j) / n-2 \le j \le n\}$$
$$\dots$$
$$T_n = \{(v_1, u_i) / 1 \le i \le m\} \cup \{(u_m, v_j) / n-m+2 \le j \le n\}.$$

Thus $\{T_1, T_2, \dots, T_n\}$ is an AGTC for $K_{m,n}$ into n parts if n=2m-1.

Case (ii): If n=2m+1.

Consider $T_1 = (v_{n-1}, u_1)$

$$T_2 = \{(v_{n-2}, u_i) / 1 \le i \le 2\}$$
$$T_3 = \{(v_{n-3}, u_i) / 1 \le i \le 3\}$$
$$\dots$$
$$T_m = \{(v_{n-m}, u_i) / 1 \le i \le m\}$$
$$T_{m+1} = \{(v_{n-m-1}, u_i) / 1 \le i \le m\} \cup (u_1, v_n)$$
$$T_{m+2} = \{(v_{n-m-2}, u_i) / 1 \le i \le m\} \cup \{(u_2, v_j) / n-1 \le j \le n\}$$
$$T_{m+3} = \{(v_{n-m-3}, u_i) / 1 \le i \le m\} \cup \{(u_3, v_j) / n-2 \le j \le n\}$$
$$\dots$$
$$T_{n-1} = \{(v_1, u_i) / 1 \le i \le m\} \cup \{(u_m, v_j) / n-m+1 \le j \le n\}.$$

Thus $\{T_1, T_2, \dots, T_{n-1}\}$ is an AGTC for $K_{m,n}$ into n-1 parts if n=2m+1. The converse of the above two cases are straight forward.

The following examples illustrate the above theorem 2.7 for n=2m+1 and n=2m-1.

Example 2.8:
(i) Consider $K_{4,9}$

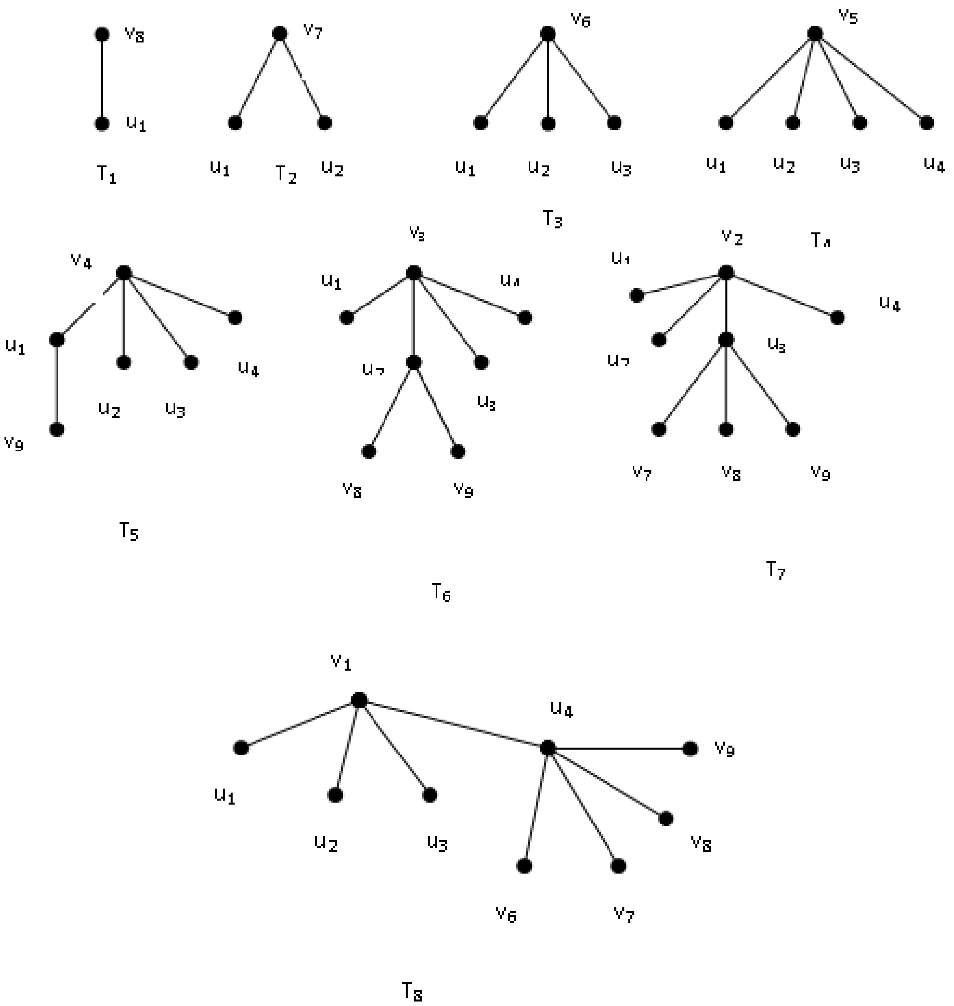

Figure - 1

(ii) Consider $K_{4,7}$

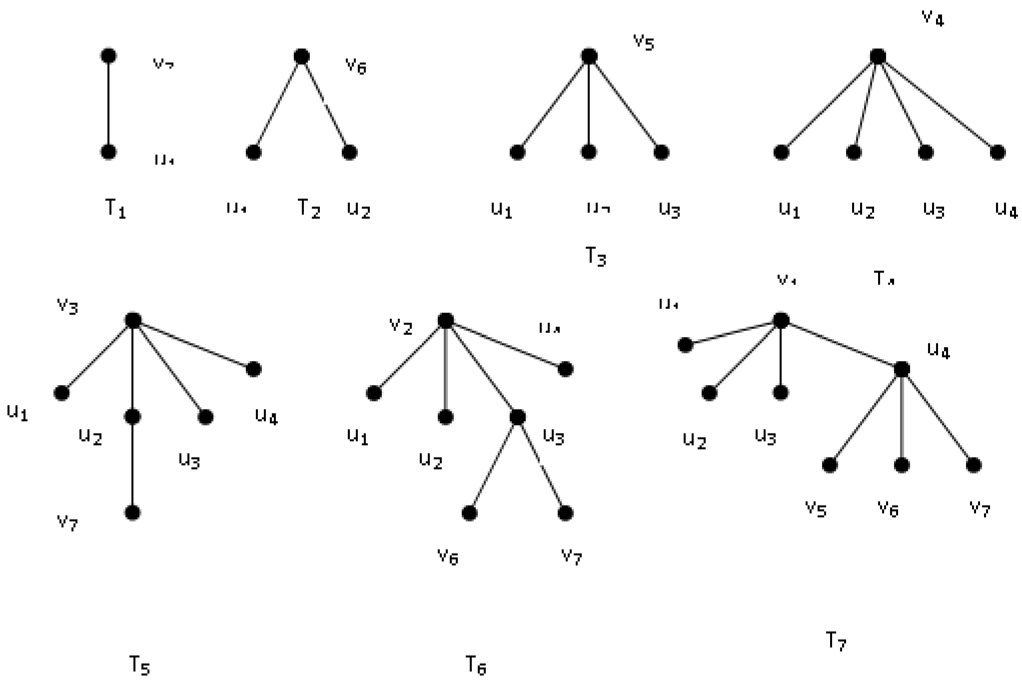

Figure - 2

Theorem 2.9: The Helm H_m admits AGTC into n parts if and only if n=5, 6 and 8 or m=5, 7, 12.

Proof: Let $V(H_m)=\{c,u_1,u_2,\dots,u_m,v_1,v_2,\dots,v_m\}$ having c as the central vertex of H_m. The Helm H_m is shown as in Fig. 3. Since c is of maximum degree and by the definition of AGTC, we consider G_n as a star with c as its central vertex.

Let $G_n=\{(c,u_i):1\le i\le n\}$. Then G_{n-1} should be defined as a tree having n-1 edges with atmost one of the edges from $\{(c,u_i):n+1\le i\le m\}$ say cu_{n+1}; by the definition of AGTC. Now suppose cu_{n+2} lies in G_{n-2} and the remaining edges of G_{n-2} are from C_m and the pendant edges incident to C_m. If u_{n+2},u_{n+3} and u_{n+4} are internal vertices of G_{n-2} then by (ii) of AGTC definition any one of the edges $cu_{n+2},cu_{n+3},cu_{n+4}$ do not belong to any of the sub graphs $G_i, 1\le i\le n-3.$ So there should be at most 2 internal vertices in G_{n-2} such that $|E(G_{n-2})|\le 6$ or $n\le 8$. As $|E(H_m)|=\frac{n(n+1)}{2}$,

We have

$$3m=\frac{n(n+1)}{2}.$$

$$6m=n(n+1), m\ge 3 \text{ and } n\le 8.$$

Then we get n=5, 6 and 8.
Converse is straight forward.

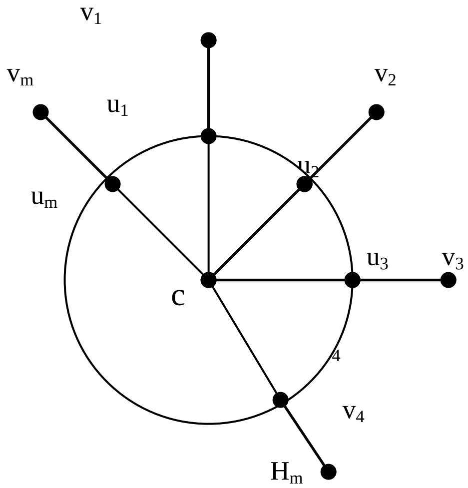

Figure - 3

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