

**COMPLETE SYNCRONIZATION OF A PLANNER MAGNETIC BINARIES PROBLEM
WHEN BIGGER PRIMARY IS OBLATE SPHEROID AND SMALLER PRIMARY IS ELLIPSOID**

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ABSTRACT

We study the complete synchronization behavior of the planar magnetic-binaries problem by taking into consideration the bigger primary is oblate spheroids and smaller is ellipsoid evolving from deferent initial conditions using active control technique based on the Lyapunov-stability theory and Routh-Hurwitz criteria. Numerical simulations are performed to plot time series analysis graphs of the master system and the slave system which further illustrate the effectiveness of the proposed control techniques.

Key words: *Space dynamics, magnetic-binaries problem, complete synchronization, Lyapunov stability theory and Routh- Hurwitz criteria.*

1. INTRODUCTION

In the last few years considerable research has been done in non-linear dynamical systems and their various properties. One of the most important properties is synchronization which is an important topic in the nonlinear dynamics. Chaos control and synchronization are especially noteworthy and important research fields leveling to affect dynamics of chaotic systems in order to apply them for different kinds of applications that can be examined within many different scientific research. At present, there are different kinds of control methods and techniques that have been proposed for carrying out chaos control and synchronization of chaotic dynamical systems. Chaotic synchronization did not attract much attention until Pec-ora and Carroll introduced a method to synchronize two identical chaotic systems with deferent initial conditions in 1990 and they demonstrated that chaotic synchronization could be achieved by driving or replacing one of the variables of a chaotic system with a variable of another similar chaotic device. Various techniques have been proposed and implemented successfully for achieving stable synchronization between identical and non-identical systems. Notable among these methods, the active control scheme proposed by E. W. Bai & K. E. Lonngren 1997 has been received and successfully implemented in almost all the field of nonlinear sciences for synchronization for different systems with various techniques E. W. Bai *et al.* 2002, H. K. Chen 2005, U. E. Vincent 2005, A. N. Njah 2006, Y. Lei *et al.* 2006, A. Ucar *et al.* 2007, Y. Lei *et al.* 2007, U. E. Vincent 2008, A. N. Njah & U. E. Vincent 2008, U. E. Vincent & J. A. Laoye 2007, A. Khan & M. Shahzad 2013. M. Shahzad, I. Ahmed. 2013 Israr Ahmad, Azizan Bin Saaban etc. 2015.

Chaos synchronization using active control has recently been widely accepted as an efficient technique for synchronizing chaotic systems. This method has been applied to many practical systems such as spatiotemporal dynamical systems (Codreanu 2003), the Rikitake two-disc dynamo-a geographical system (Vincent 2005), Non-linear Bloch equations modeling "jerk" equation (Ucar *et al.* 2003), Chua's circuits (Tang & Wang 2006), Complex dynamos (Mahmoud 2007), Non-linear equations of acoustic gravity waves (Vincent 2008b), Qi system (Lei *et al.* 2006 ; Lei *et al.* 2007. Active Control technique is based on Lyanupov Stability theory and Routh-Hurwitz criterion. In 2013 Ayub Khan and Priyamvada Tripathi have investigated the synchronization behavior of a restricted three body problem under the effect of radiation pressure. In an another paper the Complete synchronization, anti-synchronization and hybrid synchronization of two identical parabolic restricted three body problem have been studied by Ayub Khan and Rimpipal in 2013. Anti-synchronization between two different hyperchaotic systems systems by using active control have been studied by M. Mossa Al-sawalha and M.S.M. Noorani in 2009.

In 2013 M javid Idrisi and Z. A. Taqvi have been studied the restricted three body problem when one of the primaries is an ellipsoid.

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Stormer (1907) has studied the motion of a charged particle which is moving in the field of a magnetic dipole as a two body problem. This problem in general is quite complicated and is a non integrable. A. Mavraganis (1978 ... 1988) has studied the motion of a charge particle which is moving in the field of two rotating magnetic dipoles instead of one dipole.

In this article, active control techniques base on the Lyapunov stability theory and Routh-Hurwitz criteria have been used to study the complete synchronization behavior of planar magnetic-binaries problem by taking into consideration the small primary is ellipsoid and bigger primary is an oblate spheroid including the effect of the gravitational forces of the primaries on the small body. The system under consideration is chaotic for some values of parameter involved in the system. Here two systems (master and slave) are synchronized and start with deferent initial conditions. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time.

2. EQUATION OF MOTION

In formulating the problem we shall assume that the two dipoles (primaries) one is in the shape of oblate spheroid and other is ellipsoid of magnetic moments M_1 and M_2 respectively participate in the circular motion around their centre of mass O Fig(1). The motion of a charged particle P of charge q and mass m defined by its radius vector $\mathbf{r}$ will be referred to a frame of reference $Oxyz$ that rotates in the same direction and the same angular velocity ω as the dipoles, which in this frame are taken to stay at rest on x -axis. Here we assumed that the distance between the primaries as the unit of distance and the coordinate of one primary is $(\mu, 0, 0)$ then the other is $(\mu-1, 0, 0)$. We also assumed that the sum of their masses as the unit of mass. If mass of the one primaries μ then the mass of the other is $(1-\mu)$. The unit of time in such a way that the gravitational constant G has the value unity and $q=mc$ where c is the velocity of light. $r_1^2 = (x-\mu)^2 + y^2$, $r_2^2 = (x+1-\mu)^2 + y^2$, $\mathcal{A} = \frac{M_2}{M_1}$ (M_1, M_2 are the magnetic moments of the primaries which lies perpendicular to the plane of the motion).

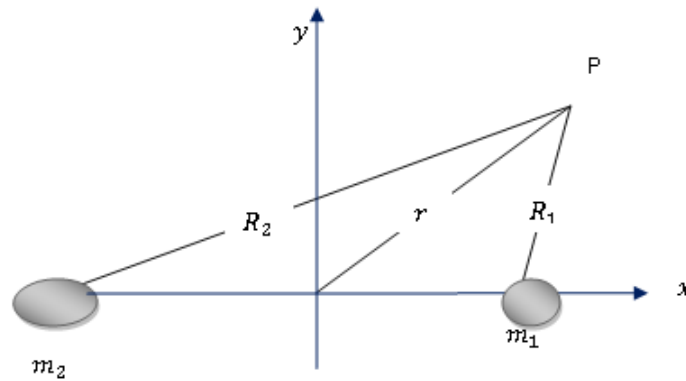


Figure-1

Then the equation of motion of the particle P may be written as:

$$\ddot{x} - \dot{y} f = U_x \quad (1)$$

$$\ddot{y} + \dot{x} f = U_y \quad (2)$$

Where

$$f = 2\omega - \left[\frac{1}{r_1^3} + \frac{I}{2(1-\mu)r_1^5} + \frac{\lambda V}{\mu r_2^2} \right], \quad U_x = \frac{\partial U}{\partial x} \quad \text{and} \quad U_y = \frac{\partial U}{\partial y}$$

$$U = (x^2 + y^2) \frac{\omega^2}{2} + \omega \{ (x^2 + y^2) - x\mu \} \left[\frac{1}{r_1^3} + \frac{I}{2(1-\mu)r_1^5} \right] + \{ (x^2 + y^2) - x(1-\mu) \} \frac{V\omega\lambda}{\mu r_2^2} + \frac{(1-\mu)}{r_1} + \frac{I}{2(1-\mu)r_1^5} + V \quad (3)$$

$$I = \frac{(1-\mu)(R_e^2 - R_p^2)}{5} \quad R_e, R_p \text{ Equatorial and polar radius of oblate spheroid respectively}$$

$$V = \frac{3}{2} \left[\left\{ 1 + \frac{y^2 - (x+1-\mu)^2}{(a^2 - b^2)} \right\} \frac{F(\varphi, k)}{\sqrt{(a^2 - c^2)}} + \left\{ \frac{(x+1-\mu)^2}{(a^2 - b^2)} + \frac{(c^2 - a^2)y^2}{(a^2 - b^2)(b^2 - c^2)} \right\} \frac{E(\varphi, k)}{\sqrt{(a^2 - c^2)}} + \frac{\sqrt{(\gamma + c^2)} y^2}{(b^2 - c^2)\sqrt{(\gamma + a^2)(\gamma + b^2)}} \right]$$

$$F(\varphi, k) = \int_0^\varphi \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}} \quad \text{Elliptic integral of first kind}$$

$$E(\varphi, k) = \int_0^\varphi \sqrt{1 - k^2 \sin^2 \theta} d\theta \quad \text{Elliptic integral of second kind}$$

$$k = \sqrt{\frac{(a^2 - b^2)}{(a^2 - c^2)}} \quad 0 \leq k^2 \leq 1, \quad \varphi = \sin^{-1} \sqrt{\frac{(a^2 - c^2)}{(y + c^2)}} \quad 0 \leq \varphi \leq \frac{\pi}{2},$$

$$\gamma = \frac{1}{2} [(x + 1 - \mu)^2 + y^2 - p_1 + \sqrt{\{(x + 1 - \mu)^2 + y^2 - p_1\}^2 + 4\{p_3(x + 1 - \mu)^2 + p_4 y^2 - p_2\}}],$$

$$p_1 = a^2 + b^2 + c^2, \quad p_2 = a^2 b^2 + b^2 c^2 + a^2 c^2, \quad p_3 = b^2 + c^2, \quad p_4 = a^2 + c^2,$$

$$p_5 = a^2 + b^2. \quad a, b \text{ and } c \text{ are the axes of the ellipsoid.}$$

$$\omega = 1 + \frac{3}{10} \frac{\mu}{(1 - \mu)} [2a^2 - b^2 - c^2] + \frac{3I}{2\mu}$$

3. COMPLETE SYNCHRONIZATION VIA ACTIVE CONTROL

Let

$$x = x_1, \quad \dot{x} = x_2, \quad y = x_3, \quad \dot{y} = x_4$$

Then the equation (1) and (2) can be written as:

$$\dot{x}_1 = x_2 \quad (4)$$

$$\dot{x}_2 = x_4 \left[2\omega - \left\{ \frac{1}{r_1^3} + \frac{I}{2(1-\mu)r_1^5} + \frac{\lambda V}{r_2^2} \right\} \right] + \omega^2 x_1 + A_1 \quad (5)$$

$$\dot{x}_3 = x_4 \quad (6)$$

$$\dot{x}_4 = -x_2 \left[2\omega - \left\{ \frac{1}{r_1^3} + \frac{I}{2(1-\mu)r_1^5} + \frac{\lambda V}{r_2^2} \right\} \right] + \omega^2 x_3 + A_2 \quad (7)$$

Where

$$\begin{aligned} A_1 = & \omega(2x_1 - \mu) \left\{ \frac{1-\mu}{r_1^3} + \frac{1}{2r_1^5} \right\} - \omega \{ (x_1^2 + x_3^2) - x_1 \mu \} (x_1 - \mu) \left\{ 3 \frac{(1-\mu)}{r_1^5} + \frac{5I}{2r_1^7} \right\} - \\ & (x_1 - \mu) \left\{ \frac{(1-\mu)}{r_1^3} + \frac{5I}{2r_1^7} \right\} + (2x_1 - 1 + \mu) \frac{V\omega\lambda}{r_2^2} - 2\{ (x_1^2 + x_3^2) + x_1(1-\mu) \} \\ & \times \frac{(x_1 + 1 - \mu)V\omega\lambda}{r_2^4} + \left[\{ (x_1^2 + x_3^2) + x_1(1-\mu) \} \frac{\omega\lambda}{r_2^2} + 1 \right] \\ & \times \left[\frac{E(\varphi, k) - F(\varphi, k)}{p_6 p_8} - \left\{ 1 - k^2 \sin^2 \varphi \frac{(x_1 + 1 - \mu)^2}{p_6} + \left(\frac{1}{p_6} + \frac{1 - k^2 \sin^2 \varphi}{p_9} \right) x_3^2 \right\} \right. \\ & \times \frac{\gamma_1 + p_3}{2(\gamma_1 + a^2)(2\gamma_1 + p_1 - r_2^2) \sqrt{(\gamma_1 + c^2)} \sqrt{1 - k^2 \sin^2 \varphi}} \\ & \left. - \frac{(2c^2 \gamma_1 + p_{11} + \gamma_1^2)(\gamma_1 + p_3)x_3^2}{2p_7(2\gamma_1 + p_1 - r_2^2) \sqrt{(\gamma_1 + c^2)} (p_{10} + p_5 \gamma_1 + \gamma_1^2)^{\frac{3}{2}}} \right] (-3\mu(x_1 + 1 - \mu)) \end{aligned}$$

$$r_1^2 = (x_1 - \mu)^2 + x_3^2, \quad r_2^2 = (x_1 + 1 - \mu)^2 + x_3^2, \quad p_6 = a^2 - b^2, \quad p_7 = b^2 - c^2, \quad p_8 = \sqrt{a^2 - c^2},$$

$$p_9 = \frac{(a^2 - b^2)(b^2 - c^2)}{(c^2 - a^2)}, \quad p_{10} = b^2 a^2, \quad p_{11} = p_{10} - c^2 p_5,$$

$$\gamma_1 = \frac{1}{2} [(x_1 + 1 - \mu)^2 + x_3^2 - p_1 + \sqrt{\{(x_1 + 1 - \mu)^2 + x_3^2 - p_1\}^2 + 4\{p_3(x_1 + 1 - \mu)^2 + p_4 x_3^2 - p_2\}}]$$

$$\begin{aligned} A_2 = & 2x_3 \omega \left\{ \frac{1-\mu}{r_1^3} + \frac{I}{2r_1^5} \right\} - \omega x_3 \{ (x_1^2 + x_3^2) - x_1 \mu \} \left\{ 3 \frac{(1-\mu)}{r_1^5} + \frac{5I}{2r_1^7} \right\} \\ & - x_3 \left\{ \frac{(1-\mu)}{r_1^3} + \frac{5I}{2r_1^7} \right\} - 2x_3 \frac{V\omega\lambda}{r_2^2} - 2x_3 \{ (x_1^2 + x_3^2) - x_1(1-\mu) \} \frac{V\omega\lambda}{r_2^4} \\ & - 3\mu x_3 \left[\{ (x_1^2 + x_3^2) - x_1(1-\mu) \} \frac{\omega\lambda}{r_2^2} + 1 \right] \end{aligned}$$

$$\times \left[\frac{E(\varphi, k)}{p_9 p_8} + \frac{F(\varphi, k)}{p_6 p_8} - \left\{ 1 - k^2 \sin^2 \varphi \frac{(x_1 + 1 - \mu)^2}{p_6} + \left(\frac{1}{p_6} + \frac{1 - k^2 \sin^2 \varphi}{p_9} \right) x_3^2 \right\} \right. \\ \times \frac{\gamma_1 + p_4}{2(\gamma_1 + a^2)(2\gamma_1 + p_1 - r_2^2)\sqrt{(\gamma_1 + c^2)}\sqrt{1 - k^2 \sin^2 \varphi}} \\ \left. - \frac{(2c^2\gamma_1 + p_{11} + \gamma_1^2)(\gamma_1 + p_4)x_3^2}{2p_7(2\gamma_1 + p_1 - r_2^2)\sqrt{(\gamma_1 + c^2)}(p_{10} + p_5\gamma_1 + \gamma_1^2)^{\frac{3}{2}}} + \frac{\sqrt{(\gamma_1 + c^2)}}{p_7\sqrt{(p_{10} + p_5\gamma_1 + \gamma_1^2)}} \right]$$

Corresponding to master system (4, 5, 6 and 7), the identical slave system is defined as:

$$\dot{y}_1 = y_2 + u_1(t) \quad (8)$$

$$\dot{y}_2 = y_4 \left[2\omega - \left\{ \frac{1}{r_1^3} + \frac{I}{2(1-\mu)r_1^5} + \frac{\lambda V}{r_2^2} \right\} \right] + \omega^2 y_1 + B_1 + u_2(t) \quad (9)$$

$$\dot{y}_3 = y_4 + u_3(t) \quad (10)$$

$$\dot{y}_4 = -y_2 \left[2\omega - \left\{ \frac{1}{r_1^3} + \frac{I}{2(1-\mu)r_1^5} + \frac{\lambda V}{r_2^2} \right\} \right] + \omega^2 y_3 + B_2 + u_4(t) \quad (11)$$

Where

$$B_1 = \omega(2y_1 - \mu) \left\{ \frac{1-\mu}{r_1^3} + \frac{I}{2r_1^5} \right\} - \omega \{ (y_1^2 + y_3^2) - y_1\mu \} (y_1 - \mu) \left\{ 3 \frac{(1-\mu)}{r_1^5} + \frac{5I}{2r_1^7} \right\} \\ - (y_1 - \mu) \left\{ \frac{(1-\mu)}{r_1^3} + \frac{5I}{2r_1^7} \right\} + (2y_1 - 1 + \mu) \frac{V\omega\lambda}{r_2^2} - 2 \{ (y_1^2 + y_3^2) - y_1(1-\mu) \} \\ \times \frac{(y_1 + 1 - \mu)V\omega\lambda}{r_2^4} + \left[\{ (y_1^2 + y_3^2) - y_1(1-\mu) \} \frac{\omega\lambda}{r_2^2} + 1 \right] \\ \times \left[\frac{E(\varphi, k) - F(\varphi, k)}{p_6 p_8} - \left\{ 1 - k^2 \sin^2 \varphi \frac{(y_1 + 1 - \mu)^2}{p_6} + \left(\frac{1}{p_6} + \frac{1 - k^2 \sin^2 \varphi}{p_9} \right) y_3^2 \right\} \right. \\ \times \frac{\gamma_1 + p_3}{2(\gamma_1 + a^2)(2\gamma_1 + p_1 - r_2^2)\sqrt{(\gamma_1 + c^2)}\sqrt{1 - k^2 \sin^2 \varphi}} \\ \left. - \frac{(2c^2\gamma_1 + p_{11} + \gamma_1^2)(\gamma_1 + p_3)x_3^2}{2p_7(2\gamma_1 + p_1 - r_2^2)\sqrt{(\gamma_1 + c^2)}(p_{10} + p_5\gamma_1 + \gamma_1^2)^{\frac{3}{2}}} \right] (-3\mu(y_1 + 1 - \mu))$$

$$r_1^2 = (y_1 - \mu)^2 + y_3^2, \quad r_2^2 = (y_1 + 1 - \mu)^2 + y_3^2, \quad p_6 = a^2 - b^2, \quad p_7 = b^2 - c^2, \quad p_8 = \sqrt{a^2 - c^2},$$

$$p_9 = \frac{(a^2 - b^2)(b^2 - c^2)}{(c^2 - a^2)}, \quad p_{10} = b^2 a^2, \quad p_{11} = p_{10} - c^2 p_5,$$

$$\gamma_1 = \frac{1}{2} \left[(y_1 + 1 - \mu)^2 + y_3^2 - p_1 + \sqrt{\{(y_1 + 1 - \mu)^2 + y_3^2 - p_1\}^2 + 4\{p_3(y_1 + 1 - \mu)^2 + p_4 y_3^2 - p_2\}} \right]$$

$$B_2 = 2y_3\omega \left\{ \frac{1-\mu}{r_1^3} + \frac{I}{2r_1^5} \right\} - \omega y_3 \{ (y_1^2 + y_3^2) - y_1\mu \} \left\{ 3 \frac{(1-\mu)}{r_1^5} + \frac{5I}{2r_1^7} \right\} \\ - y_3 \left\{ \frac{(1-\mu)}{r_1^3} + \frac{5I}{2r_1^7} \right\} - 2y_3 \frac{V\omega\lambda}{r_2^2} - 2y_3 \{ (y_1^2 + y_3^2) - y_1(1-\mu) \} \frac{V\omega\lambda}{r_2^4} + \\ - 3\mu y_3 \left[\{ (y_1^2 + y_3^2) - y_1(1-\mu) \} \frac{\omega\lambda}{r_2^2} + 1 \right] \\ \times \left[\frac{E(\varphi, k)}{p_9 p_8} + \frac{F(\varphi, k)}{p_6 p_8} - \left\{ 1 - k^2 \sin^2 \varphi \frac{(y_1 + 1 - \mu)^2}{p_6} + \left(\frac{1}{p_6} + \frac{1 - k^2 \sin^2 \varphi}{p_9} \right) y_3^2 \right\} \right. \\ \times \frac{\gamma_1 + p_4}{2(\gamma_1 + a^2)(2\gamma_1 + p_1 - r_2^2)\sqrt{(\gamma_1 + c^2)}\sqrt{1 - k^2 \sin^2 \varphi}} \\ \left. - \frac{(2c^2\gamma_1 + p_{11} + \gamma_1^2)(\gamma_1 + p_4)y_3^2}{2p_7(2\gamma_1 + p_1 - r_2^2)\sqrt{(\gamma_1 + c^2)}(p_{10} + p_5\gamma_1 + \gamma_1^2)^{\frac{3}{2}}} + \frac{\sqrt{(\gamma_1 + c^2)}}{p_7\sqrt{(p_{10} + p_5\gamma_1 + \gamma_1^2)}} \right]$$

where $u_i(t)$; $i = 1, 2, 3, 4$ are control functions to be determined. Let $e_i = y_i - x_i$; $i = 1, 2, 3, 4$ be the synchronization errors. From (4) to (11), we obtain the error dynamics as follows:

$$\dot{e}_1 = e_2 + u_1(t) \quad (12)$$

$$\dot{e}_2 = 2\omega e_4 + \omega^2 e_1 + B_1 - A_1 + u_2(t) \quad (13)$$

$$\dot{e}_3 = e_4 + u_3(t) \quad (14)$$

$$\dot{e}_4 = -2\omega e_2 + \omega^2 e_3 + B_2 - A_2 + u_4(t) \quad (15)$$

This error system to be controlled is a linear system with control functions. Thus, let us redefine the control functions so that the terms in (12) to (15) which cannot be expressed as linear terms in e_i 's are eliminated:

$$\begin{aligned} u_1(t) &= v_1(t) \\ u_2(t) &= -B_1 + A_1 + v_2(t) \\ u_3(t) &= v_3(t) \\ u_4(t) &= -B_2 + A_2 + v_4(t) \end{aligned}$$

The new error system can be expressed as:

$$\left. \begin{aligned} \dot{e}_1 &= e_2 + v_1(t) \\ \dot{e}_2 &= 2\omega e_4 + \omega^2 e_1 + v_2(t) \\ \dot{e}_3 &= e_4 + v_3(t) \\ \dot{e}_4 &= -2\omega e_2 + \omega^2 e_3 + v_4(t) \end{aligned} \right\} \quad (16)$$

The above error system to be controlled is a linear system with a control input $v_i(t)$ ($i = 1, \dots, 4$) as function of the error states e_i ($i = 1, \dots, 4$). As long as these feedbacks stabilize the system e_i ($i = 1, \dots, 4$) converge to zero as time t tends to infinity. This implies that master and the slave system are synchronized with active control. We choose.

$$\begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \end{bmatrix} = A \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \quad (17)$$

Here A is a 4×4 coefficient matrix to be determined. As per Lyapunov stability theory and Routh-Hurwitz criterion, in order to make the closed loop system (17) stable, proper choice of elements of A has to be made so that the system (17)

must have all eigen values with negative real parts. Choosing

$$A = \begin{bmatrix} -1 & -1 & 0 & 0 \\ -\omega^2 & -1 & 0 & -2\omega \\ 0 & 0 & -1 & -1 \\ 0 & 2\omega & -\omega^2 & -1 \end{bmatrix} \quad (18)$$

and, defining a matrix B as

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{bmatrix} = B \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \quad (19)$$

Where B is

$$B = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad (20)$$

Clearly, B has eigen values with negative real parts. This implies $\lim_{t \rightarrow \infty} |e_i| = 0$; $i = 1, 2, 3, 4$ and hence, complete synchronization is achieved between the master and slave systems. Time Series Analysis graphs of the above are shown next to each via figures 2 and 9.

4. NUMERICAL SIMULATION

We select the parameters $\mu = .00230437$ and $\lambda = 1$ with the different initial conditions for master and slave systems. Simulation results for uncoupled system are presented in figures.2,4,6,8 and that of controlled system are shown in figures.3,5,7 and 9 for respectively.

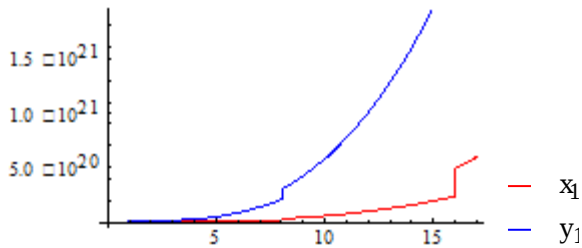


Figure-2

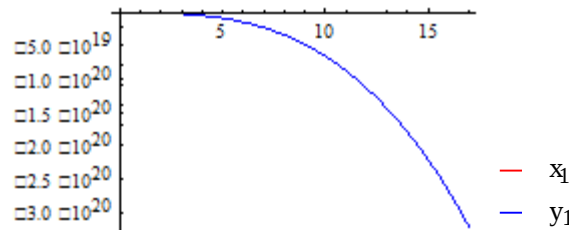


Figure-3

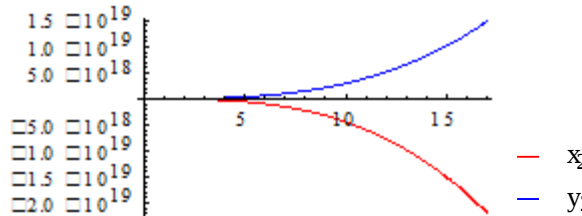


Figure-4

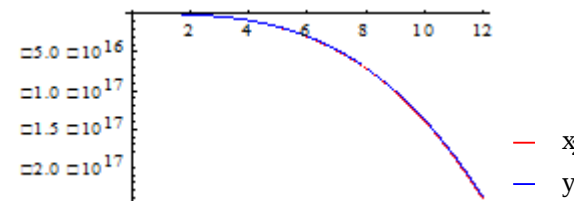


Figure-5

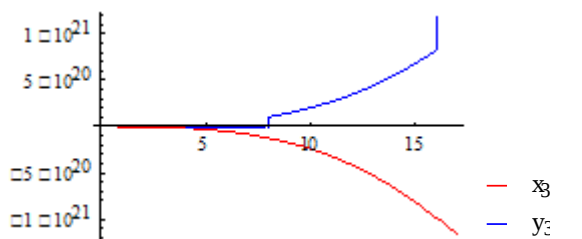


Figure-6

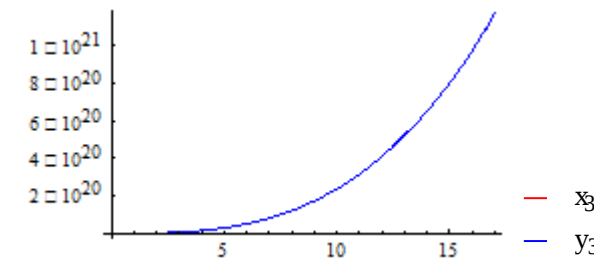


Figure-7

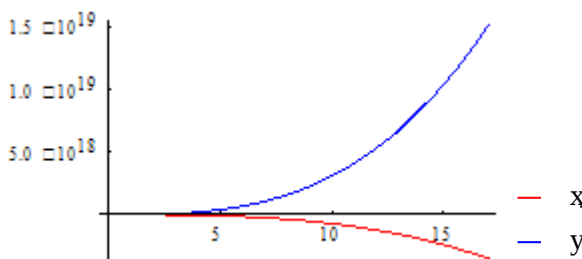


Figure-8

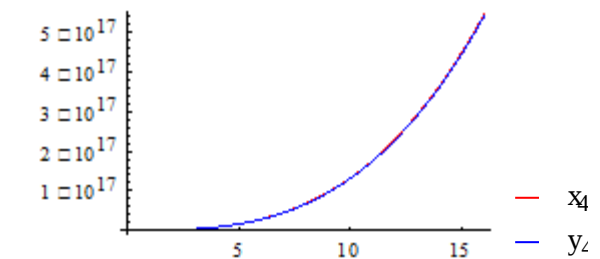


Figure-9

5. CONCLUSION

An investigation on complete synchronization and in the planar magnetic-binaries problem by taking into consideration the small primary is ellipsoid including the effect of the gravitational forces of the primaries on the small body, via active control technique based on Lyapunov stability theory and Routh-Hurwitz criteria have been made. Here two systems (master and slave) are complete synchronized and start with deferent initial conditions. This problem may be treated as the design of control laws for chaotic slave system using the known information of the master system so as to ensure that the controlled receiver synchronizes with master system. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time. The results were validated by numerical simulations using Mathematica.

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