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COMPLETE SYNCRONIZATION OF A PLANNER MAGNETIC BINARIES PROBLEM WHEN BIGGER PRIMARY IS OBLATE SPHEROID AND SMALLER PRIMARY IS ELLIPSOID

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ABSTRACT

We study the complete synchronization behavior of the planar magnetic-binaries problem by taking into consideration the bigger primary is oblate spheroids and smaller is ellipsoid evolving from deferent initial conditions using active control technique based on the Lyapunov-stability theory and Routh-Hurwitz criteria. Numerical simulations are performed to plot time series analysis graphs of the master system and the slave system which further illustrate the effectiveness of the proposed control techniques.
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Key words: *Space dynamics, magnetic-binaries problem, complete synchronization, Lyapunov stability theory and Routh- Hurwitz criteria.*

1. INTRODUCTION

In the last few years considerable research has been done in non-linear dynamical systems and their various properties. One of the most important properties is synchronization which is an important topic in the nonlinear dynamics. Chaos control and synchronization are especially noteworthy and important research fields leveling to affect dynamics of chaotic systems in order to apply them for different kinds of applications that can be examined within many different scientific research. At present, there are different kinds of control methods and techniques that have been proposed for carrying out chaos control and synchronization of chaotic dynamical systems. Chaotic synchronization did not attract much attention until Pec-ora and Carroll introduced a method to synchronize two identical chaotic systems with deferent initial conditions in 1990 and they demonstrated that chaotic synchronization could be achieved by driving or replacing one of the variables of a chaotic system with a variable of another similar chaotic device. Various techniques have been proposed and implemented successfully for achieving stable synchronization between identical and non-identical systems. Notable among these methods, the active control scheme proposed by E. W. Bai & K. E. Lonngren 1997 has been received and successfully implemented in almost all the field of nonlinear sciences for synchronization for different systems with various techniques E. W. Bai *et al.* 2002, H. K. Chen 2005, U. E. Vincent 2005, A. N. Njah 2006,Y. Lei *et al.* 2006, A. Ucar *et al.* 2007, Y. Lei *et al.* 2007, U. E. Vincent 2008, A. N. Njah & U. E. Vincent 2008, U. E. Vincent & J. A. Laoye 2007, A. Khan & M. Shahzad 2013. M. Shahzad, I. Ahmed. 2013 Israr Ahmad, Azizan Bin Saaban etc. 2015.

Chaos synchronization using active control has recently been widely accepted as an efficient technique for synchronizing chaotic systems. This method has been applied to many practical systems such as spatiotemporal dynamical systems (Codreanu 2003), the Rikitake two-disc dynamo-a geographical system (Vincent 2005), Non-linear Bloch equations modeling "jerk" equation (Ucar *et al.* 2003), Chua's circuits (Tang & Wang 2006), Complex dynamos (Mahmoud 2007), Non-linear equations of acoustic gravity waves (Vincent 2008b), Qi system (Lei *et al.* 2006 ; Lei *et al.* 2007. Active Control technique is based on Lyanupov Stability theory and Routh-Hurwitz criterion. In 2013 Ayub Khan and Priyamvada Tripathi have investigated the synchronization behavior of a restricted three body problem under the effect of radiation pressure. In an another paper the Complete synchronization, anti-synchronization and hybrid synchronization of two identical parabolic restricted three body problem have been studied by Ayub Khan and Rimpi pal in 2013. Anti-synchronization between two different hyperchaotic systems systems by using active control have been studied by M. Mossa Al-sawalha and M.S.M. Noorani in 2009.

In 2013 M javid Idrisi and Z. A. Taqvi have been studied the restricted three body problem when one of the primaries is an ellipsoid.

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Stormer (1907) has studied the motion of a charged particle which is moving in the field of a magnetic diploe as a two body problem. This problem in general is quite complicated and is a non integrable. A. Mavragnais (1978 … 1988) has studied the motion of a charge particle which is moving in the field of two rotating magnetic dipoles instead of one dipole.

In this article, active control techniques base on the Lyapunov stability theory and Routh-Hurwitz criteria have been used to study the complete synchronization behavior of planar magnetic-binaries problem by taking into consideration the small primary is ellipsoid and bigger primary is an oblate spheroid including the effect of the gravitational forces of the primaries on the small body. The system under consideration is chaotic for some values of parameter involved in the system. Here two systems (master and slave) are synchronized and start with deferent initial conditions. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time.

2. EQUATION OF MOTION

In formulating the problem we shall assume that the two dipoles (primaries) one is in the shape of oblate spheroid and other is ellipsoid of magnetic moments M_1 and M_2 respectively participate in the circular motion around their centre of mass O Fig(1). The motion of a charged particle P of charge q and mass m defined by its radius vector $\boldsymbol{r}$ will be referred to a frame of reference $Oxyz$ that rotates in the same direction and the same angular velocity $\boldsymbol{\omega}$ as the dipoles, which in this frame are taken to stay at rest on x-axis. Here we assumed that the distance between the primaries as the unit of distance and the coordinate of one primary is (μ, 0, 0) then the other is ($\mu-1$, 0, 0). We also assumed that the sum of their masses as the unit of mass. If mass of the one primaries μ then the mass of the other is ($1-\mu$). The unit of time in such a way that the gravitational constant G has the value unity and q= mc where c is the velocity of light. $r_1^2 = (x-\mu)^2 + y^2$, $r_2^2 = (x+1-\mu)^2 + y^2$, $\boldsymbol{\lambda} = \frac{M_2}{M_1}$ (M_1, M_2 are the magnetic moments of the primaries which lies perpendicular to the plane of the motion).

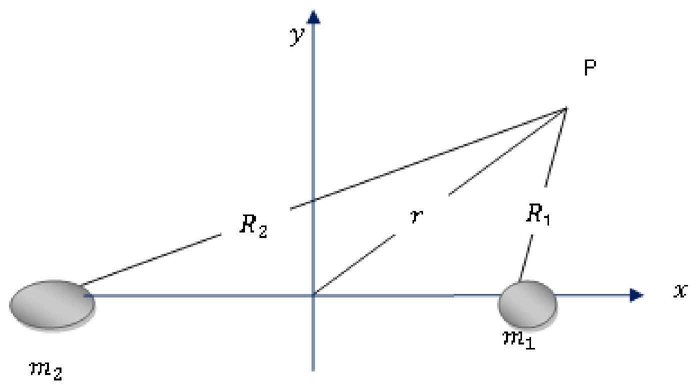


Figure-1

Then the equation of motion of the particle P may be written as:

$$\ddot{x} - \dot{y}\, f = U_x \qquad (1)$$

$$\ddot{y} + \dot{x}\, f = U_y \qquad (2)$$

Where

$$f = 2\,\omega - \left[\frac{1}{r_1^3} + \frac{I}{2(1-\mu)r_1^5} + \frac{\lambda V}{\mu r_2^2}\right],\ U_x = \frac{\partial U}{\partial x} \text{ and } U_y = \frac{\partial U}{\partial y}$$

$$U = (x^2+y^2)\frac{\omega^2}{2} + \omega\left\{(x^2+y^2) - x\mu\right\}\left[\frac{1}{r_1^3} + \frac{I}{2(1-\mu)r_1^5}\right] + \left\{(x^2+y^2) - x(1-\mu)\right\}\frac{V\omega\lambda}{\mu r_2^2} + \frac{(1-\mu)}{r_1} + \frac{I}{2(1-\mu)r_1^5} + V \qquad (3)$$

$I = \frac{(1-\mu)(R_e^2 - R_p^2)}{5}$ R_e, R_p Equatorial and polar radius of oblate spheroid respectively

$$V = \frac{3}{2}\left[\left\{1 + \frac{y^2 - (x+1-\mu)^2}{(a^2-b^2)}\right\}\frac{F(\varphi,k)}{\sqrt{(a^2-c^2)}} + \left\{\frac{(x+1-\mu)^2}{(a^2-b^2)} + \frac{(c^2-a^2)y^2}{(a^2-b^2)(b^2-c^2)}\right\}\frac{E(\varphi,k)}{\sqrt{(a^2-c^2)}} + \frac{\sqrt{(\gamma+c^2)}\,y^2}{(b^2-c^2)\sqrt{(\gamma+a^2)(\gamma+b^2)}}\right]$$

$F(\varphi,k) = \int_0^{\varphi} \frac{d\theta}{\sqrt{1-k^2 sin^2\theta}}$ Elliptic integral of first kind

$E(\varphi, k) = \int_0^{\varphi} \sqrt{1 - k^2 sin^2\theta}\, d\theta$ Elliptic integral of second kind

$k = \sqrt{\frac{(a^2 - b^2)}{(a^2 - c^2)}}$ $\quad 0 \le k^2 \le 1, \quad \varphi = \sin^{-1}\sqrt{\frac{(a^2 - c^2)}{(\gamma + c^2)}} \quad 0 \le \varphi \le \frac{\pi}{2}$,

$\gamma = \frac{1}{2}\left[(x + 1 - \mu)^2 + y^2 - p_1 + \sqrt{\{(x + 1 - \mu)^2 + y^2 - p_1\}^2 + 4\{p_3(x + 1 - \mu)^2 + p_4 y^2 - p_2\}}\right]$,

$p_1 = a^2 + b^2 + c^2,\ p_2 = a^2b^2 + b^2c^2 + a^2c^2,\ p_3 = b^2 + c^2,\ p_4 = a^2 + c^2$,

$p_5 = a^2 + b^2$. a, b and c are the axes of the ellipsoid.

$$\omega = 1 + \frac{3}{10}\frac{\mu}{(1 - \mu)}[2a^2 - b^2 - c^2] + \frac{3I}{2\mu}$$

3. COMPLETE SYNCHRONIZATION VIA ACTIVE CONTROL

Let

$x = x_1,\ \dot{x} = x_2,\ y = x_3,\ \dot{y} = x_4$

Then the equation (1) and (2) can be written as:

$$\dot{x_1} = x_2 \tag{4}$$

$$\dot{x_2} = x_4\left[2\,\omega - \left\{\frac{1}{r_1^3} + \frac{I}{2(1-\mu)r_1^5} + \frac{\lambda V}{r_2^2}\right\}\right] + \omega^2 x_1 + A_1 \tag{5}$$

$$\dot{x_3} = x_4 \tag{6}$$

$$\dot{x_4} = -x_2\left[2\,\omega - \left\{\frac{1}{r_1^3} + \frac{I}{2(1-\mu)r_1^5} + \frac{\lambda V}{r_2^2}\right\}\right] + \omega^2 x_3 + A_2 \tag{7}$$

Where

$$\begin{aligned}
A_1 = {} & \omega(2x_1 - \mu)\left\{\frac{1-\mu}{r_1^3} + \frac{I}{2r_1^5}\right\} - \omega\{(x_1^2 + x_3^2) - x_1\mu\}(x_1 - \mu)\left\{3\frac{(1-\mu)}{r_1^5} + \frac{5I}{2r_1^7}\right\} - \\
& (x_1 - \mu)\left\{\frac{(1-\mu)}{r_1^3} + \frac{5I}{2r_1^7}\right\} + (2x_1 - 1 + \mu)\frac{V\omega\lambda}{r_2^2} - 2\{(x_1^2 + x_3^2) + x_1(1-\mu)\} \\
& \times\frac{(x_1 + 1 - \mu)V\omega\lambda}{r_2^4} + \left[\{(x_1^2 + x_3^2) + x_1(1-\mu)\}\frac{\omega\lambda}{r_2^2} + 1\right] \\
& \times\left[\frac{E(\varphi,k) - F(\varphi,k)}{p_6 p_8} - \left\{1 - k^2 sin^2\varphi\frac{(x_1 + 1 - \mu)^2}{p_6} + \left(\frac{1}{p_6} + \frac{1 - k^2 sin^2\varphi}{p_9}\right)x_3^2\right\}\right. \\
& \times\frac{\gamma_1 + p_3}{2(\gamma_1 + a^2)(2\gamma_1 + p_1 - r_2^2)\sqrt{(\gamma_1 + c^2)}\sqrt{1 - k^2 sin^2\varphi}} \\
& \left. - \frac{(2c^2\gamma_1 + p_{11} + \gamma_1^2)(\gamma_1 + p_3)x_3^2}{2p_7(2\gamma_1 + p_1 - r_2^2)\sqrt{(\gamma_1 + c^2)}(p_{10} + p_5\gamma_1 + \gamma_1^2)^{\frac{3}{2}}}\right]\left(-3\mu\,(x_1 + 1 - \mu)\right)
\end{aligned}$$

$r_1^2 = (x_1 - \mu)^2 + x_3^2,\ \ r_2^2 = (x_1 + 1 - \mu)^2 + x_3^2, p_6 = a^2 - b^2, p_7 = b^2 - c^2, p_8 = \sqrt{a^2 - c^2}$,

$p_9 = \frac{(a^2 - b^2)(b^2 - c^2)}{(c^2 - a^2)},\ p_{10} = b^2a^2, p_{11} = p_{10} - c^2p_5$,

$$\gamma_1 = \frac{1}{2}\left[(x_1 + 1 - \mu)^2 + x_3^2 - p_1 + \sqrt{\{(x_1 + 1 - \mu)^2 + x_3^2 - p_1\}^2 + 4\{p_3(x_1 + 1 - \mu)^2 + p_4x_3^2 - p_2\}}\right]$$

$$\begin{aligned}
A_2 = {} & 2x_3\omega\left\{\frac{1-\mu}{r_1^3} + \frac{I)}{2r_1^5}\right\} - \omega\, x_3\{(x_1^2 + x_3^2) - x_1\mu\}\left\{3\frac{(1-\mu)}{r_1^5} + \frac{5I}{2r_1^7}\right\} \\
& - x_3\left\{\frac{(1-\mu)}{r_1^3} + \frac{5I}{2r_1^7}\right\} - 2x_3\frac{V\omega\lambda}{r_2^2} - 2x_3\{(x_1^2 + x_3^2) - x_1(1-\mu)\}\frac{V\omega\lambda}{r_2^4} \\
& - 3\mu x_3\left[\{(x_1^2 + x_3^2) - x_1(1-\mu)\}\frac{\omega\lambda}{r_2^2} + 1\right]
\end{aligned}$$

$$\times\left[\frac{E(\varphi,k)}{p_9p_8}+\frac{F(\varphi,k)}{p_6p_8}-\left\{1-k^2sin^2\varphi\frac{(x_1+1-\mu)^2}{p_6}+\left(\frac{1}{p_6}+\frac{1-k^2sin^2\varphi}{p_9}\right)x_3{}^2\right\}\right.$$
$$\times\frac{\gamma_1+p_4}{2(\gamma_1+a^2)(2\gamma_1+p_1-r_2^2)\sqrt{(\gamma_1+c^2)}\sqrt{1-k^2sin^2\varphi}}$$
$$\left.-\frac{(2c^2\gamma_1+p_{11}+\gamma_1{}^2)(\gamma_1+p_4)x_3{}^2}{2p_7(2\gamma_1+p_1-r_2^2)\sqrt{(\gamma_1+c^2)}(p_{10}+p_5\gamma_1+\gamma_1{}^2)^{\frac{3}{2}}}+\frac{\sqrt{(\gamma_1+c^2)}}{p_7\sqrt{(p_{10}+p_5\gamma_1+\gamma_1{}^2)}}\right]$$

Corresponding to master system (4, 5, 6 and 7), the identical slave system is defined as:

$$\dot{y_1}=y_2+u_1(t) \tag{8}$$

$$\dot{y_2}=y_4\left[2\,\omega-\left\{\frac{1}{r_1^3}+\frac{I}{2(1-\mu)r_1^5}+\frac{\lambda V}{r_2^2}\right\}\right]+\omega^2y_1+B_1+u_2(t) \tag{9}$$

$$\dot{y_3}=y_4+u_3(t) \tag{10}$$

$$\dot{y_4}=-y_2\left[2\,\omega-\left\{\frac{1}{r_1^3}+\frac{I}{2(1-\mu)r_1^5}+\frac{\lambda V}{r_2^2}\right\}\right]+\omega^2y_3+B_2++u_4(t) \tag{11}$$

Where

$$B_1=\omega(2y_1-\mu)\left\{\frac{1-\mu}{r_1^3}+\frac{I)}{2r_1^5}\right\}-\omega\{(y_1{}^2+y_3{}^2)-y_1\mu\}(y_1-\mu)\left\{3\frac{(1-\mu)}{r_1^5}+\frac{5I}{2r_1^7}\right\}$$
$$-(y_1-\mu)\left\{\frac{(1-\mu)}{r_1^3}+\frac{5I}{2r_1^7}\right\}+(2y_1-1+\mu)\frac{V\omega\lambda}{r_2^2}-2\{(y_1{}^2+y_3{}^2)-y_1(1-\mu)\}$$
$$\times\frac{(y_1+1-\mu)V\omega\lambda}{r_2^4}+\left[\{(y_1{}^2+y_3{}^2)-y_1(1-\mu)\}\frac{\omega\lambda}{r_2^2}+1\right]$$
$$\times\left[\frac{E(\varphi,k)-F(\varphi,k)}{p_6p_8}-\left\{1-k^2sin^2\varphi\frac{(y_1+1-\mu)^2}{p_6}+\left(\frac{1}{p_6}+\frac{1-k^2sin^2\varphi}{p_9}\right)y_3{}^2\right\}\right.$$
$$\times\frac{\gamma_1+p_3}{2(\gamma_1+a^2)(2\gamma_1+p_1-r_2^2)\sqrt{(\gamma_1+c^2)}\sqrt{1-k^2sin^2\varphi}}$$
$$\left.-\frac{(2c^2\gamma_1+p_{11}+\gamma_2{}^2)(\gamma_1+p_3)x_3{}^2}{2p_7(2\gamma_1+p_1-r_2^2)\sqrt{(\gamma_1+c^2)}(p_{10}+p_5\gamma_1+\gamma_1{}^2)^{\frac{3}{2}}}\right]\big(-3\mu\,(y_1+1-\mu)\big)$$

$r_1^2=(y_1-\mu)^2+y_3{}^2$, $r_2^2=(y_1+1-\mu)^2+y_3{}^2$, $p_6=a^2-b^2$, $p_7=b^2-c^2$, $p_8=\sqrt{a^2-c^2}$,

$p_9=\frac{(a^2-b^2)(b^2-c^2)}{(c^2-a^2)}$, $p_{10}=b^2a^2$, $p_{11}=p_{10}-c^2p_5$,

$$\gamma_1=\frac{1}{2}\left[(y_1+1-\mu)^2+y_3{}^2-p_1+\sqrt{\{(y_1+1-\mu)^2+y_3{}^2-p_1\}^2+4\{p_3(y_1+1-\mu)^2+p_4y_3{}^2-p_2\}}\right]$$

$$B_2=2y_3\omega\left\{\frac{1-\mu}{r_1^3}+\frac{I)}{2r_1^5}\right\}-\omega\,y_3\{(y_1{}^2+y_3{}^2)-y_1\mu\}\left\{3\frac{(1-\mu)}{r_1^5}+\frac{5I}{2r_1^7}\right\}$$
$$-y_3\left\{\frac{(1-\mu)}{r_1^3}+\frac{5I}{2r_1^7}\right\}-2y_3\frac{V\omega\lambda}{r_2^2}-2y_3\{(y_1{}^2+y_3{}^2)-y_1(1-\mu)\}\frac{V\omega\lambda}{r_2^4}+$$
$$-3\mu y_3\left[\{(y_1{}^2+y_3{}^2)-y_1(1-\mu)\}\frac{\omega\lambda}{r_2^2}+1\right]$$
$$\times\left[\frac{E(\varphi,k)}{p_9p_8}+\frac{F(\varphi,k)}{p_6p_8}-\left\{1-k^2sin^2\varphi\frac{(y_1+1-\mu)^2}{p_6}+\left(\frac{1}{p_6}+\frac{1-k^2sin^2\varphi}{p_9}\right)y_3{}^2\right\}\right.$$
$$\times\frac{\gamma_1+p_4}{2(\gamma_1+a^2)(2\gamma_1+p_1-r_2^2)\sqrt{(\gamma_1+c^2)}\sqrt{1-k^2sin^2\varphi}}$$
$$\left.-\frac{(2c^2\gamma_1+p_{11}+\gamma_1{}^2)(\gamma_1+p_4)y_3{}^2}{2p_7(2\gamma_1+p_1-r_2^2)\sqrt{(\gamma_1+c^2)}(p_{10}+p_5\gamma_1+\gamma_1{}^2)^{\frac{3}{2}}}+\frac{\sqrt{(\gamma_1+c^2)}}{p_7\sqrt{(p_{10}+p_5\gamma_1+\gamma_1{}^2)}}\right]$$

where $u_i(t)$; i =1 ,2,3,4 are control functions to be determined. Let $e_i = y_i - x_i$; i = 1, 2, 3, 4 be the synchronization errors. From (4) to (11), we obtain the error dynamics as follows:

$$\dot{e_1} = e_2 + u_1(t) \tag{12}$$

$$\dot{e_2} = 2\omega e_4 + \omega^2 e_1 + B_1 - A_1 + u_2(t) \tag{13}$$

$$\dot{e_3} = e_4 + u_3(t) \tag{14}$$

$$\dot{e_4} = -2\omega e_2 + \omega^2 e_3 + B_2 - A_2 + u_4(t) \tag{15}$$

This error system to be controlled is a linear system with control functions. Thus, let us redefine the control functions so that the terms in (12) to (15) which cannot be expressed as linear terms in e_i 's are eliminated:

$$u_1(t) = v_1(t)$$
$$u_2(t) = -B_1 + A_1 + v_2(t)$$
$$u_3(t) = v_3(t)$$
$$u_4(t) = -B_2 + A_2 + v_4(t)$$

The new error system can be expressed as**:**

$$\left.\begin{aligned} \dot{e_1} &= e_2 + v_1(t) \\ \dot{e_2} &= 2\omega e_4 + \omega^2 e_1 + v_2(t) \\ \dot{e_3} &= e_4 + v_3(t) \\ \dot{e_4} &= -2\omega e_2 + \omega^2 e_3 + v_4(t) \end{aligned}\right\} \tag{16}$$

The above error system to be controlled is a linear system with a control input $v_i(t)$ ($i = 1, \dots 4$) as function of the error states e_i ($i = 1, \dots 4$). As long as these feedbacks stabilize the system e_i ($i = 1, \dots 4$) converge to zero as time t tends to infinity. This implies that master and the slave system are synchronized with active control. We choose**.**

$$\begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \end{bmatrix} = A \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \tag{17}$$

Here A is a 4×4 coefficient matrix to be determined. As per Lyapunov stability theory and Routh-Hurwitz criterion, in order to make the closed loop system (17) stable, proper choice of elements of A has to be made so that the system (17)

must have all eigen values with negative real parts. Choosing

$$A = \begin{bmatrix} -1 & -1 & 0 & 0 \\ -\omega^2 & -1 & 0 & -2\omega \\ 0 & 0 & -1 & -1 \\ 0 & 2\omega & -\omega^2 & -1 \end{bmatrix} \tag{18}$$

and, defining a matrix B as

$$\begin{bmatrix} \dot{e_1} \\ \dot{e_2} \\ \dot{e_3} \\ \dot{e_4} \end{bmatrix} = B \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \tag{19}$$

Where B is

$$B = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{20}$$

Clearly, B has eigen values with negative real parts. This implies $\lim_{t\to\infty}|e_i| = 0$; i = 1, 2, 3, 4 and hence, complete synchronization is achieved between the master and slave systems. Time Series Analysis graphs of the above are shown next to each via figures 2 and 9.

4. NUMERICAL SIMULATION

We select the parameters $\mu = .00230437$ and $\lambda = 1$ with the different initial conditions for master and slave systems. Simulation results for uncoupled system are presented in figures.2,4,6,8 and that of controlled system are shown in figures.3,5,7 and 9 for respectively.

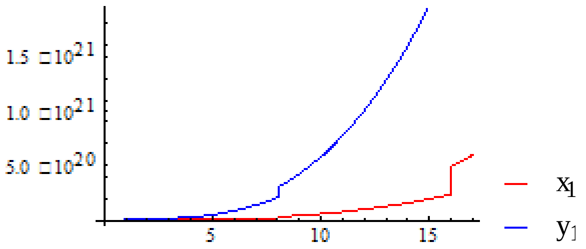


Figure-2

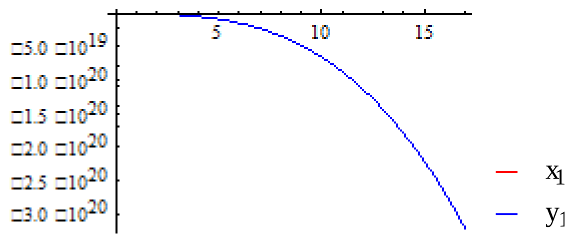


Figure-3

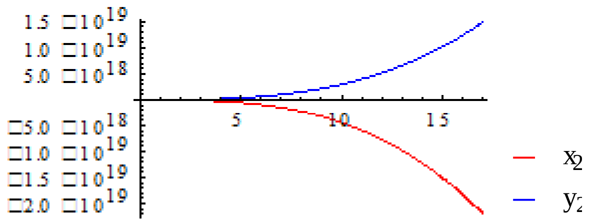
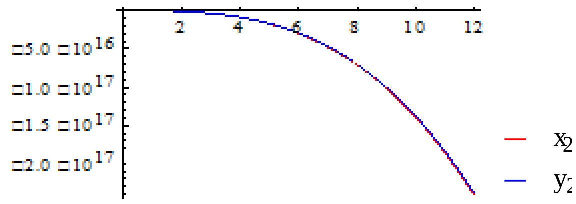


Figure-4



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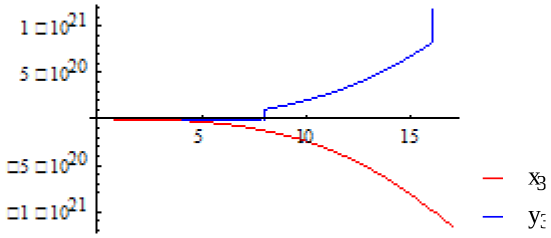

Figure-6

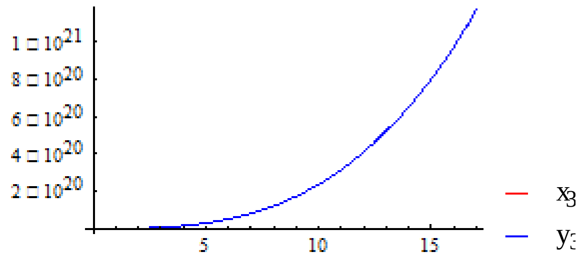

Figure-7

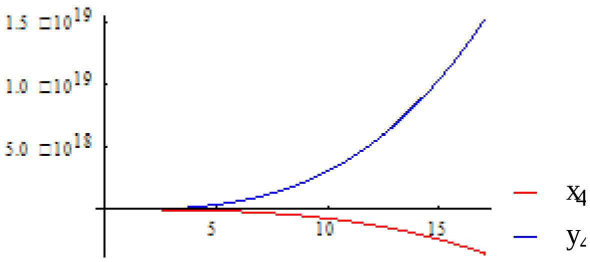

Figure-8

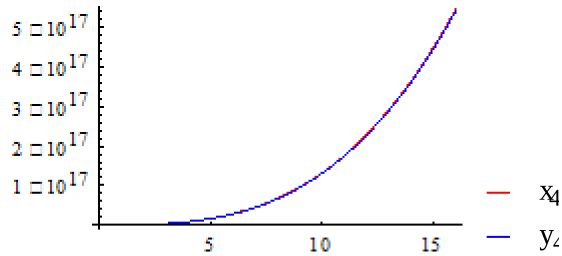

Figure-9

5. CONCLUSION

An investigation on complete synchronization and in the planar magnetic-binaries problem by taking into consideration the small primary is ellipsoid including the effect of the gravitational forces of the primaries on the small body, via active control technique based on Lyapunov stability theory and Routh-Hurwitz criteria have been made. Here two systems (master and slave) are compete synchronized and start with deferent initial conditions. This problem may be treated as the design of control laws for chaotic slave system using the known information of the master system so as to ensure that the controlled receiver synchronizes with master system. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time. The results were validated by numerical simulations using Mathematica.

REFRENCES

1. A. N. NJAH, K. S. OJO. (2010) Backstepping control and synchronization of parametrically and externally excited Φ6 van der pol oscillators with application to secure communications International Journal of Modern Physics B Vol. 24, No. 23 (2010) 4581–4593.
2. Ahmad Taher Azar. (2012) Modeling and Control of Dialysis Systems: Volume 2: Biofeedback Systems and Soft Computing Techniques of Dialysis, Springer-Verlag GmbH, Heidelberg Germany.
3. Arif. Mohd. (2010). Motion of a charged particle when the primaries are oblate spheroids international journal of applied math and Mechanics. 6(4), pp.94-106.
4. Arif. Mohd. (2013). Motion around the equilibrium points in the planar magnetic binaries problem international journal of applied math and Mechanics. 9(20), pp.98-107.

5. Arif.Mohd., Ravi kumar Sagar (2015). Syncronization of a planner magnetic binaries problem when the primaries are oblate spheroid. International journal of pure and applied mathematics Volume 107 No.2 2016, 317-330 ISSN: 1311-8080.
6. Ayub Khan, Priyamyada Tripathi. (2013) Synchronization, Anti-Synchronization and Hybrid-Synchronization of a Double Pendulum under the Effect of External Forces International Journal of Computational Engineering Research (ijceronline.com) Vol. 3 Issue. 1 pp.166-176.
7. Ayub Khan, Rimpipal, (2013) Complete synchronization, anti-synchronization and hybrid synchronization of two identical parabolic restricted three body problem Asian Journal of Current Engineering and Maths 2:2 118 - 126.
8. A. Khan, M. Shahzad. (2013) Synchronization of a circular restricted three body problem with Lorenz hyper chaotic system using robust adaptive sliding mode controller. *Complexity*, 18(6), 58-64.
9. Bai E. W., Lonngren K. E. (1997). Synchronization of two Lorenz systems using active control, Chaos, Solitons and Fractals, Vol. 8.
10. C. Liu, T. Liu, L. Liu and K. Liu, (2004) A new chaotic attractor, Chaos, Solitons and Fractals, Vol. 22, pp. 1031-1038pp. 51-58.
11. G. Leonov, H. Nijmeijer, A. Pogromsky and A. Fradkov, (2010). Dynamics and Control of Hybrid Mechanical Systems, World Scientific, Singapore.
12. Henon, M., (1965), Ann. Astrophys. 28, 499.
13. H. K. Chen. (2005). Synchronization of Two Different Chaotic System: A New System and Each of the Dynamical Systems Lorenz, Chen and Lu, Chaos Solitons & Fractals 25, 1049– 1056.
14. Hill, G. W. (1878). Reasearches in the lunar theory, Am.J.Math. 1, 5-26, pp. 129-147, 245-261.
15. Israr Ahmad, Azizan Bin Saaban, Adyda Binti Ibrahim, Said Al-Hadhrami, Mohammad Shahzad, Sharifa Hilal Al- Mahrouqi, (2015) A research on adaptive control to stabilize and synchronize a hyperchaotic system with uncertain parameters. An International Journal of Optimization and Control: Theories & Applications Vol.5, No.2, pp.51-62.
16. J.H. Park, (2006).Chaos synchronization of nonlinear Bloch equations," Chaos, Solitons and Fractals, vol. 27, pp. 357-361.
17. *L. Lu*, *C. Zhang* and Z.A. Guo, (*2007).*Synchronization between two different chaotic systems with nonlinear feedback control, Chinese Physics, 16, pp.1603-1607.
18. Lei Y., Xu W., Xie W.(2007) Synchronization of two chaotic four-dimensional systems using active control, Chaos, Solitons and Fractals, Vol. 32, pp. 1823- 1829.
19. M. C. Ho and Y.C. Hung. (2002). Synchronization two Different Systems by using Generalized Active Control, Phys. Lett. A 301, 424–428.
20. M. Haeri and A. Emadzadeh, (2007). Synchronizing diferent chaotic systems using active sliding mode control, Chaos, Solitons and Fractals. 31 pp.119-129.
21. M. Mossa Al-sawalha and M.S.M. Noorani. (2009). Anti-synchronization Between Two Different Hyperchaotic Systems. Journal of Uncertain Systems Vol.3, No.3, pp.192-200.
22. Mavraganis A (1978). Motion of a charged particle in the region of a magnetic-binary system. Astroph. and spa. Sci. 54, pp. 305-313.
23. Mavraganis A (1978). The areas of the motion in the planar magnetic-binary problem. Astroph. and spa. Sci. 57, pp. 3-7.
24. Mavraganis A (1979). Stationary solutions and their stability in the magnetic -binary problem when the primaries are oblate spheroids. Astron Astrophys. 80, pp. 130-133.
25. Mavraganis A and Pangalos. C. A. (1983) Parametric variations of the magnetic -binary problem Indian J. pure appl, Math. 14(3), pp. 297- 306.
26. Moulton, F. R., (1920), An Introduction to Celestial Mechanics, Macmillan, New York.
27. M. Shahzad, I. Ahmed. (2013) Experimental study of synchronization & Anti-synchronization for spin orbit problem of Enceladus. *International Journal of control science & Engineering*, 2(3), 41-47.
28. Pecora L. M.,Carroll T.L., (1990) Synchronization in chaotic systems Rev Lett., Vol. 64, pp. 821-824.
29. Papadakis. K.E. (2005), Motion Around The Triangular Equilibrium Points Of The Restricted Three-Body Problem Under Angular Velocity Variation, Astroph. and spa. Sci. 2 pp. 129-148.
30. S. H. Chen and J. Lu. (2002). Synchronization of an Uncertain Unified System via Adaptive Control, Chaos Solitons & Fractals 14, 643–647.
31. Stormer, CF (1907). Arch. Sci. Phys.et Nat. Geneva, pp. 24-350.
32. Szebehely. V. (1967) Theory of orbit Academic press New York.
33. Tang F., Wang L. (2006) An adaptive active control for the modified Chua's circuit, Phys. Lett. A, Vol. 346, pp. 342-346.
34. U. E. Vincent.(2008). Synchronization of Identical and Non-identical 4-D Chaotic Systems Using Active Control, Chaos Solitons Fractals 37, no. 4, 1065–1075.
35. Ucar A., Lonngren K. E., Bai E. W. (2007) Chaos synchronization in RCL-shunted Josephson junction via active control, Chaos, Solitons and Fractals, Vol. 31, pp. 105-111.
36. W. Hongwu and M. Junhai,, (2012).Chaos Controland Synchronization of a Fractional-order Autonomous System, WSEAS Trans. on Mathematics. 11, pp. 700-711.

37. Y. Wang, Z.H. Guan and H.O. Wang, (2003).Feedback an adaptive control for the synchronization of Chen system via a single variable, Phys. Lett A. 312 pp.34-40.
38. Z. M. Ge and C. C. Chen. (2004) Phase Synchronization of Coupled Chaotic Multiple Time Scale Systems, Chaos Solitons & Fractals 20, 639–647.
39. Zhang Xuebing, Zhu Honglan (2008) Anti- synchronization of two different hyper- chaotic systems via Active and Adaptive Control", International Journal of Nonliear Sciences, Vol. 6, No. 3, pp. 216-223.

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