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ON GENERALIZED REGULAR INFRA CLOSED SETS

S. DHANALAKSHMI[1], R. MALINI DEVI*[2]

[1,2]Department of Mathematics,
The Standard Fireworks Rajaratnam College for Women, Sivakasi, Tamilnadu-626 123.

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ABSTRACT

In this paper is to introduce the concept of generalized regular infra closed sets and study some of its properties. The corresponding infra topological space formed by the family of these sets is studied. The generalized infra closed set is properly placed between the generalized regular infra closed sets and regular generalized infra closed sets. The infra topological space is discussed in this paper and some applications of this newly defined set are also shown.

Keywords: *Infra topological space, Generalized infra closed set, Regular generalized infra closed sets etc.*

1. INTRODUCTION

Adel.M.AL.Odhari introduced the concept of infra topological space. In this paper we introduced Generalized Regular Infra Closed Set (RICP) and analogue concepts associated with infra topological space. Such as Infra Interior Point, Infra Closure Point, Generalized Regular Infra Closed set of subset A of infra space X. We will be denoted by IIP(A), ICP(A), RICP(A).

2. PRELIMINERIES

Definition 2.1: Let X be any arbitrary set. An**Infra- topological space** on X is a collection τ_{iX} subsets of X such that the following axioms are satisfying:
* Ax-1: ϕ, X $\epsilon\tau_{iX}$.
* Ax-2: The intersection of the elements of any subcollection of τ_{iX} in X.
i.e) If $0_i \epsilon\tau_{iX}$, $1 \leq I \leq n \rightarrow \cap\, 0_i \epsilon\tau_{iX}$.
Terminology, the ordered pair (X, τ_{iX}) is called Infra - Topological Space.

Definition 2.2: Let (X, τ_{iX}) be an (ITS) and A$\subseteq$ X. A is called **infra – open set** (IOS) if A$\epsilon\tau_{iX}$.

Definition 2.3: Let (X, τ_{iX} be infra topological space. A subset C$\subseteq$X is called**infra – closedset** in X if X–C is infra – open set in X. (i.e) C is **infra – closed set** (ICS) iff X–C $\epsilon\tau_{iX}$.

Definition 2.4: Let (X, τ_{iX}) be an infra topological space and AC X. The **InfraClosure Point** (ICP) of A is a set denoted by **icp(A)** and given by icp(A)= $\cap\{C_i: A\subseteq C_i, X–C_i \epsilon\tau_{iX}\}$.
(ie) icp(A) is the intersection of all infra closed set contained the se A.

Definition 2.5: Let (X, τ_{iX}) be an infra topological space and A$\subseteq$ X. The **Infra Interior Points** (IIP) of A is a set denoted by iip(A) and given by:iip(A) = $\cup\{O_i: O_i\subseteq A, O_i \epsilon\tau_{iX}\}$.
(ie) iip(A) is the union of all infra open sets contained in the set A.

Definition 2.6: A subset A of X is a regular infra pen set if A = iipicpA and a regular closed set if A = icpiipA.

Definition 2.7: A subset A is generalized infra closed set if icpA$\subseteq$U whenever A$\subseteq$U and U is an open set.

Definition 2.8: Let A be a subset A of X then ^(A)= $\cap$ {G:A$\subseteq$ G. G is a open subset of X}.

Corresponding Author: R. Malini Devi[2], [1,2]Department of Mathematics,*
The Standard Fireworks Rajaratnam College for Women, Sivakasi, Tamilnadu-626 123.

3. ON GENERALIZED REGULAR CLOSED SETS IN INFRA TOPOLOGICAL SPACE

In this section, the concept of generalized regular infra closed set and generalized regular open sets are introduced. Some properties of these sets are cited. The corresponding infra topological space obtained by these sets is also introduced and some related theorems are studied.

Definition 3.1: A subset A of a infra topological space (X, τ_{iX}) is called a **generalized regular infra closed** if RICP(A) ⊆ U whenever A⊆U and U is an open subset of X.

Example 3.2: Let X={a, b, c, d}and the corresponding infra topological space be τ_{iX}={X, ϕ, {a}, {b},{d}}. Let A={b, c} and U={a, b, c}. Now, regular infra closed set in X containing A= {X, ϕ, {b, c}}. Hence {b, c} is generalized regular infra closed set.

Result 3.3: A subset A of (X, τ_{iX}) is called generalized regular infra closed subset of X iff A^c is a generalized regular infra open subset of X.

Theorem 3.4: A subset A of (X, τ_{iX}) is a generalized regular infra closed set if it is a generalized regular infra closed set.

Proof: Let A be a subset of (X, τ_{iX}). Then by using definition of generalized infra closed set if ICP(A) ⊆ U. Again, by the definition of generalized infra closed set if RICP(A)⊆ U. Hence ICP(A) ⊆ RICP(A). (By above definitions).

Remark 3.5: Converse of the above theorem need not be true.

Example 3.6: Let X= {a, b, c} and the corresponding infra topological space be τ_{iX}={X, ϕ, {a}, {b}}.

Let A= {a} Obviously A is a generalized infra closed subset of X but not a generalized regular infra closed subset of X. (icp (iip (A)) = ϕ ≠ A)

Theorem 3.7: A subset A of (X, τ_{iX}) is a generalized regular infra closed set if it is a generalized regular infraclosed set.

Remark 3.8: Converse of the above theorem need not be true.

Example 3.9: Let X= {a, b, c} and the corresponding infra topological space be τ_{iX}={X, ϕ, {a}}. Let A = {b, c}. A is a generalized regular infra closed set but not a regular infra closed set.

Remark 3.10: RICP(A) is a generalized regularinfra closed set since RICP(A) is a regular infra closed infra set and from theorem (A subset A of (X, τ_{iX}) is a generalized regular infra closed set if it is a regular infraclosed set) it is a generalized regular infraclosed subset of X for any subset A of X.

Theorem 3.11: If A is an infra open subset and a generalized regular infra closed subset of (X, τ_{iX}) then it is a regular infra closed subset of (X, τ_{iX}).

Proof: Assume that a subset A of (X, τ_{iX}) is a generalized regular closed set. We Prove that it is a regular closed set. Let if possible, A is a generalized regular infra closed subset and an infra open subset of X. Therefore A = iip(A) (an infra open subset of X) Hence from definition RICP(A) ⊆ iip(A) = A. (ie) RICP(A) ⊆A. But we know that A ⊆ RICP (A). So, A= RICP(A).(ie) A is a infra regular closed subset of (X, τ_{iX}). Hence proved.

Result 3.12: Φ and X are generalized regular infraclosed subset of X.

Remark 3.13: The finite union or intersection of generalized regularinfra closed set need not be a generalized regular infra closed set.

Example for finite intersection of generalized infraclosed sets need not be generalized infra closed sets: Let X= {a, b, c, d, e} and the corresponding infra topological space be τ_{iX} = {ϕ, X, {a},{b},{d}}. Let A={a, c}. Obviously A is a generalized regular infra closed subset of X. Again let B= {b, c}. B is also a generalized regular infra closed subset of X. But A∩B ={c} is not a generalizedinfra closed subset of X. Since A∩B = c. The infra open sets c, {a, b, c}, X. But RICP (A∩B) does not contained inc. Hence finite intersection of generalized regular infra closed sets need not be a generalized regular infra closed subset of X.

Example of finite union of generalized regularinfra closed sets need not be generalized regular infra closed sets. Let X= {a, b, c} and the corresponding infra topological space be τ_{iX}= {ϕ, X {a}, {c}}. Let A={a, b}. Obviously A is a generalized regularinfraclosed subset of X. Again let B={b, c} B is also a generalized regular infra closed subset of X.

But AUB = {a, b, c} is not a generalized regular infra closed subset of X. Since RICP(A∪B)={ϕ}, an infraopen subset of X. Hence finite union of generalized regular infra open set need not be generalized regular infra open subset of X.

Theorem 3.14: Let A and B be two regular infraclosed subset of (X, τ_{iX}) then A∪B is a generalized regular infraclosed subset of (X, τ_{iX}).

Theorem 3.15: The intersection of a generalized regular infra closed set and a closed set is a generalized infraclosed set.

Proof: Let A be a generalized regular infra closed subset of X. If U is an infraopen subset of X with A∩F ⊆U then A⊆ U∪(X \ F). So, RICP(A)⊆ U∪(X \ F). ICP(A∩F) ⊆ ICP(A) ∩ICP (F)⊆ RICP(A) ∩ICP(F) = RICP(A∩F)⊆ U. So, A∩F is a generalized infra closed set.

Remark 3.16: The intersection of a generalized regular infra closed subset and a regular closed infra set is a generalized regularinfraclosed set. (ie) the intersection of two regular infra closed set is a generalized regular infra closed set.

Theorem 3.17: ICP(A) is a generalized regularinfra closed subset of X in a space where every infraclosed subset of X is also a regular infraclosed set.

Proof: From Remark, RICP(A) is a generalized regular infra closed subset of X and From Theorem, RICP(A)∩ ICP(A) is a generalized regular infra closed subset of X. Since ICP(A) is a infra closed subset of X. Since every infra closed subset of X are also a regular infraclosed subset of X. So, RICP (A) = ICP (A).

(ie) RICP(A) ∩ ICP(A) = ICP(A), a generalized regularinfra closed subset of X.

Theorem 3.18: Let A⊆B⊆RICP(A) and A is a generalized regular infra closed subset of (X, τ_{iX}) then B is also a generalized regular infra closed subset of (X, τ_{iX}).

Proof: Assume that A⊆ B⊆ RICP(A) and A is a generalized regular infra closed subset of (X, τ_{iX}). We prove that B is a generalized regular infraclosed subset of (X, τ_{iX}). Since A is a generalized regular infra closed subset of (X, τ_{iX}). So, RICP(A)⊆U, whenever A⊆ U, U being an infra open subset of X. Let A⊆B⊆ RICP(A). (ie) RICP(A)= RICP(B). Let if possible, there exist an infra open subset V of X such that B⊆V. So, A⊆V and A being a generalized regular infraclosed subset of X, RICP(A)⊆V. (ie) RICP(B)⊆V. Hence B is also a generalized regular infra closed subset of X. Hence the theorem.

Remark 3.19: Let A⊆B⊆ICP(A) and A is a generalized regular infraclosed subset of (X, τ_{iX}) then B is also a generalized regular infraclosed subset of (X, τ_{iX}).

Theorem 3.20: A subset A of X is a generalized regular infra closed subsets iff RICP (A)∩A^c contains a nonzero closed set in X.

Proof: Let if possible A be a generalized regular infra closed subset of X. Also if possible let F be a infra closed subset of X such that F⊆RICP (A) ∩A^c (ie) F ⊆ RICP(A) and F ⊆ A^c . Since F is a infra closed subset of X, F^c is an infra open subset of X containing A.A being generalized regular infra open subset of X, RICP(A)⊆F^c. So, we get a contradiction, which leads to the condition. Conversely, let A⊆G, G being an infra open subset of X. Then G^c⊆A^c, G^c is a infra closed subset of X. Let if possible RClA⊆G. Then RICP(A)∩G^c is a nonzero infra closed subset of RICP(A)∩A^c, which is a contradiction. Hence A is a generalized regular infra closed subset of X.

Remark 3.21: Let A be an open and a generalized regular infra closed subset of X. From theorem, A is a regular infra closed subset of X. Hence, RICP(A)∩A^c = A∩A^c = ϕ. (ie) if A be an infra open and a generalized regular infra closed subset of X.RICP(A)∩A^c = ϕ. On the otherhand RICP(A)∩A^c = ϕ implies that A is a generalized regular infra closed subset of X but need not be an infra open subset of X.

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