International Journal of Mathematical Archive-7(8), 2016, 23-29
Available online through www.ijma.info ISSN 2229 – 5046

VAGUE GENERALIZED ALPHA CLOSED SETS IN TOPOLOGICAL SPACES

L. MARIAPRESENTI*[1], I. AROCKIARANI[2]

[1,2]Nirmala college for women, Coimbatore-641018, Tamilnadu, India.

(Received On: 28-07-16; Revised & Accepted On: 12-08-16)

ABSTRACT

The aim of this paper is to explore the notion of vague sets to define a new class of generalized sets namely vague generalized α-closed sets and investigate their properties.

Keywords: *Vague set, Vague topology, Vague generalized alpha closed set.*

1. INTRODUCTION

Classical mathematical methods are not enough to solve the problems of daily life and also are not enough to meet the new requirements. Therefore, some theories such as Fuzzy set theory [19], Rough set theory [17], Soft set theory [15] and Vague set theory [5]. have been developed to solve these problems.

Applications of these theories appear in topology and in many areas of mathematics. Most of these problems were solved by fuzzy set provided by Zadeh [19] and later Atanassov [2] generalized this idea to the new class of intuitionistic fuzzy sets using the notion of fuzzy sets. The theory of vague sets was first proposed by Gau and Buehrer [7]. Vague set theory is actually an extension of fuzzy set theory and vague sets are regarded as a special case of context- dependent fuzzy sets.

2. PRELIMINARIES

Definition 2.1 [4]: A vague set A in the universe of discourse U is characterized by two membership functions given by:

(i) A true membership function $t_A : U \to [0,1]$ and

(ii) A false membership function $f_A : U \to [0,1]$

where $t_A(x)$ is a lower bound on the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence for x", and $t_A(x) + f_A(x) \le 1$. Thus the grade of membership of u in the vague set A is bounded by a subinterval $[t_A(x), 1 - f_A(x)]$ of [0,1]. this indicates that if the actual grade of membership of x is μ(x), then, $t_A(x) \le \mu(x) \le 1 - f_A(x)$.The vague set A is written as $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / u \in U\}$ where the interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A, denoted by $V_A(x)$.

Definition 2.2 [7]: Let A and B be VSs of the form $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / x \in X\}$ and $B = \{\langle x, [t_B(x), 1 - f_B(x)]\rangle / x \in X\}$ Then

(i) $A \subseteq B$ if and only if $t_A(x) \le t_B(x)$ and $1 - f_A(x) \le 1 - f_B(x)$ for all $x \in X$

(ii) A=B if and only if $A \subseteq B$ and $B \subseteq A$

(iii) $A^c = \{\langle x, f_A(x), 1 - t_A(x)\rangle / x \in X\}$

(iv) $A \cap B = \{\langle x, \min(t_A(x), t_B(x)), \min(1 - f_A(x), 1 - f_B(x))\rangle / x \in X\}$

(v) $A \cup B = \{\langle x, \max(t_A(x), t_B(x)), \max(1 - f_A(x), 1 - f_B(x))\rangle / x \in X\}$

Corresponding Author: L. Mariapresenti[1]*

For the sake of simplicity, we shall use the notation $A = \langle x, t_A, 1 - f_A \rangle$ instead of $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / x \in X\}$.

Definition 2.3: A subset A of a topological space (X, τ) is called

(i) a preclosed set [**13**] if $cl(\text{int}(A)) \subseteq A$

(ii) a semi-closed set[**9**] if $\text{int}(cl(A)) \subseteq A$

(iii) a regular closed set[**18]** if $A = cl(\text{int}(A))$

(iv) a α-closed set [**14**] if $cl(\text{int}(cl(A))) \subseteq A$

Definition 2.4: A subset A of a topological space (X, τ) is called

(i) a generalized closed set (briefly g-closed)[**8**] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set in X.

(ii) a semi-generalized closed set (briefly sg-closed) [**3**] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open set in X

(iii) a generalized semi-closed set (briefly gs-closed) [**1**]if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X

(iv) a generalized semi pre closed set (briefly gsp-closed) [**6**]if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X

(v) a generalized pre closed set (briefly gp-closed) [**4**]if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X

(vi) a generalized α-closed set (briefly gα-closed) [**11**] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open set in X.

(vii) a α-generalized closed set (briefly αg-closed) [**10**] $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in X.

3. VAGUE TOPOLOGICAL SPACE

Definition 3.1: A vague topology (VT in short) on X is a family τ of VSs in X satisfying the following axioms.

(i) $0, 1 \in \tau$

(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(iii) $\bigcup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called a Vague topological space (VTS in short) and any VS in τ is known as a Vague open set (VOS in short) in X.

The complement A^c of a VOS A in a VTS (X, τ) is called a vague closed set (VCS in short) in X.

Definition 3.2: Let (X, τ) be a VTS and $A = \langle x, t_A, 1 - f_A \rangle$ be a VS in X. Then the vague interior and a vague closure are defined by

Vint(A)= $\bigcup$ {G/G is an VOS in X and G $\subseteq$ A}

Vcl(A)= $\bigcap$ {K/K is an VCS in X and A $\subseteq$ K}

Note that for any VS A in (X, τ), we have $Vcl(A^c) = (V\text{int}(A))^c$ and $V\text{int}(A^c) = (Vcl(A))^c$.

Example 3.3: We consider the VT Let Let X={a,b} and let $\tau = \{0, G, 1\}$ is an VT on X, where $G = \{\langle x, [0.1, 0.5], [0.1, 0.6]\rangle\}$. Here the only open sets are 0, 1, and G. If $A = \{\langle x, [0.1, 0.6], [0.1, 0.9]\rangle\}$ is a VT on X then,

Vint(A)= $\bigcup$ {G/G is an VOS in X and G $\subseteq$ A}=G

Vcl(A)= $\bigcap$ {K/K is an VCS in X and A $\subseteq$ K}=G^c

Definition 3.4: A vague set A of (X, τ) is said to be a,

(i) a vague pre-closed set if $Vcl(V\text{int}(A)) \subseteq A$

(ii) a vague semi-closed set if $V\text{int}(Vcl(A)) \subseteq A$

(iii) a vague regular-closed set if $A = Vcl(V\text{int}(A))$

(iv) a vague α closed set if $Vcl(V\text{int}(Vcl(A))) \subseteq A$.

Definition 3.5: An vague set A in (X,τ) is said to be a,

(i) vague generalized closed set (briefly VGC) if $Vcl(A) \subseteq U$ whenever $A \subseteq U$ and U is a vague open set in X.
(ii) vague generalized semi-closed set (briefly VGSC) if $Vscl(A) \subseteq U$ whenever $A \subseteq U$ and U is vague open set in X.
(iii) vague generalized pre closed set (briefly VGPC) if $Vpcl(A) \subseteq U$ whenever $A \subseteq U$ and U is a vague open set in X.

Properties 3.6: Let A be any Vague set in (X,τ). then

(i) $V\operatorname{int}(1-A) = 1-(Vcl(A))$ and
(ii) $Vcl(1-A) = 1-(V\operatorname{int}(A))$

Proof: (i) By definition Vcl(A)=$\bigcap$ {K/K is an VCS in X and A$\subseteq$K}.

$1-(Vcl(A))$=1-$\bigcap$ {K/K is an VCS in X and A$\subseteq$K}.
= $\bigcup$ {1 - K/K is an VCS in X and A$\subseteq$K}.
= $\bigcup$ {G/G is an VOS in X and G$\subseteq$1- A}
= $V\operatorname{int}(1-A)$

(ii) The proof is similar to (i).

Theorem 3.7: Let (X,τ) be a VS and let $A \in V(X)$. Then the following properties hold.

(i) $V\operatorname{int}(A) \subset A$
(ii) $A \subset B \Rightarrow V\operatorname{int}(A) \subset V\operatorname{int}(B)$
(iii) $V\operatorname{int}(A) \in \tau$
(iv) A is a vague open set $\Leftrightarrow V\operatorname{int}(A) = A$
(v) $V\operatorname{int}(V\operatorname{int}(A)) = V\operatorname{int}(A)$
(vi) $V\operatorname{int}(0) = 0$, $V\operatorname{int}(1) = 1$
(vii) $V\operatorname{int}(A \bigcap B) = V\operatorname{int}(A) \bigcap V\operatorname{int}(B)$
(viii) $(V\operatorname{int}(A))^c = Vcl(A^c)$

Proof: The proof is obvious.

Theorem 3.8: Let (X,τ) be a VS and Let $A \in V(X)$. Then the following properties hold.

(i) $(A) \subset Vcl(A)$
(ii) $A \subset B \Rightarrow Vcl(A) \subset Vcl(B)$
(iii) $(Vcl(A))^C \in \tau$
(iv) A is a vague Closed set $\Leftrightarrow Vcl(A) = A$
(v) $Vcl(Vcl(A)) = Vcl(A)$
(vi) $Vcl(0) = 0$, $Vcl(1) = 1$
(vii) $Vcl(A \bigcup B) = Vcl(A) \bigcup Vcl(B)$
(viii) $(Vcl(A))^c = V\operatorname{int}(A^c)$

Proof: The proof is obvious.

4. VAGUE GENERALIZED ALPHA CLOSED SETS

Definition 4.3: A vague set A in (X,τ) is said to be a vague generalized alpha closed set (VGαCS in short) if $V\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a VαOS in (X,τ)

Definition 4.4: A vague set A in (X,τ) is said to be a vague alpha generalized closed set (VGαCS in short) if $V\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a VOS in (X,τ).

Example 4.5: Let X={a, b} and let $\tau = \{0, G, 1\}$ is an VT on X, where $G = \{\langle x, [0.4, 0.8], [0.3, 0.7]\rangle\}$. Here the only α open sets are 0, X, and G. Then the VS $A = \{\langle x, [0.4, 0.9], [0.4, 0.8]\rangle\}$ is an VGαCS in (X, τ).

Theorem 4.6: For any VTS (X, τ), we have the following:

(i) Every VCS is a VGαCS in X
(ii) Every VαCS is a VGαCS in X
(iii) Every VRCS is a VGαCS in X
(iv) Every VGαCS is a VαGCS in X
(v) Every VGαCS is a VGPCS in X
(vi) Every VGαCS is a VGSCS in X.

Proof:

(i) Let A be a VCS. Let $A \subseteq U$ and U is a VαOS in (X, τ). Since $V\alpha cl(A) \subseteq Vcl(A)$ and A is a VCS, $V\alpha cl(A) \subseteq Vcl(A) = A \subseteq U$. Therefore A is a VGαCS in X.
(ii) Let $A \subseteq U$ and U is a VαOS in (X, τ).By hypothesis $V\alpha cl(A) = A$. Hence $V\alpha cl(A) \subseteq U$. Therefore A is a VGαCS in X
(iii) (iii), (iv), (v) and (vi)are obvious.

But converse of the above need not be true. It can be shown by the following example.

Example 4.7: Let X ={a, b} and let $\tau = \{0, G, 1\}$ is a VT on X where $G = \{\langle x, [0.2, 0.7], [0.4, 0.6]\rangle\}$. Let $A = \{\langle x, [0.3, 0.8], [0.5, 0.7]\rangle\}$ be any VS in X.Here $V\alpha cl(A) \subseteq G$, whenever $A \subseteq G$, for all VαOS G in X. A is a VGαCS, but not a VCS in X, since $Vcl(A) = 1 \neq A$.

Example 4.8: Let X ={a,b} and let $\tau = \{0, G, 1\}$ is an VT on X where $G = \{\langle x, [0.2, 0.6], [0.3, 0.5]\rangle\}$. Let $A = \{\langle x, [0.4, 0.9], [0.4, 0.8]\rangle\}$ be any VS in X. Clearly $V\alpha cl(A) \subseteq G$, whenever $A \subseteq G$, for all VαOS G in X. Therefore A is an VGαCS, but not a VαCS in X, since $Vcl(V\operatorname{int}(Vcl(A))) = 1 \not\subset A$.

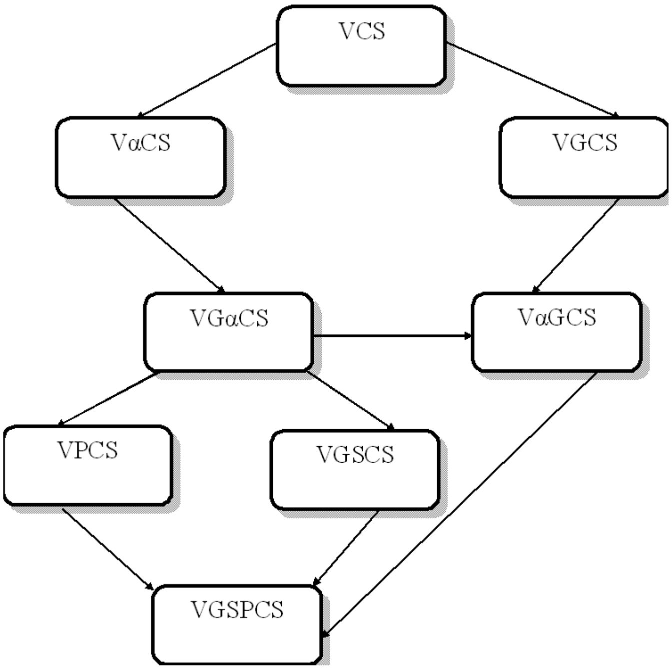
Example: 4.9: Let X ={a,b} and let $\tau = \{0, G, 1\}$ is a VT on X where $G = \{\langle x, [0.1, 0.5], [0.3, 0.7]\rangle\}$. Let $A = \{\langle x, [0.2, 0.6], [0.4, 0.8]\rangle\}$ be any VS in X. A is a VGαCS, but not a VRCS in (X, τ), since $Vcl(V\operatorname{int}(A)) = G^c \neq A$.

Example 4.10: Let X ={a,b} and let $\tau = \{0, G, 1\}$ is a VT on X where $G = \{\langle x, [0.2, 0.7], [0.3, 0.5]\rangle\}$ and let $A = \{\langle x, [0.1, 0.6], [0.2, 0.5]\rangle\}$ be any VS in X. Here A is a VαGCS in X. Consider the VαOS $G_1 = \{\langle x, [0.3, 0.8], [0.4, 0.7]\rangle\}$. Here A is a VαGCS in X.

Example 4.11: Let X ={a, b} and let $\tau = \{0, G, 1\}$ is a VT on X where $G = \{\langle x, [0.3, 0.6], [0.2, 0.8]\rangle\}$ Here the only α open sets are 0,X and G. Let $A = \{\langle x, [0.3, 0.6], [0.3, 0.5]\rangle\}$. be any VS in X. Here $Vpcl(A) = 1 \subseteq 1$. Therefore A is a VGPCS in (X, τ), but not a VGαCS.

Example 4.12: Let X ={a,b} and let $\tau = \{0, G, 1\}$ is a VT on X where $G = \{\langle x, [0.2, 0.4], [0.2, 0.8]\rangle\}$Here the only α open sets are 0,X and G. Let $A = \{\langle x, [0.2, 0.4], [0.3, 0.7]\rangle\}$ be any VS in X. Here $Vscl(A) = 1 \subseteq 1$. Therefore A is a VGSCS in (X, τ), but not a VGαCS.

The diagram gives the implication of the above theorems:



Remark 4.13: VPCS and VGαCS are independent to each other**.**

Example 4.14: Let X ={a,b} and let $\tau = \{0, G, 1\}$ is a VT on X, where $G = \{\langle x, [0.3, 0.6], [0.1, 0.7] \rangle\}$. Here the only α open sets are 0,1 and G. Let $A = \{\langle x, [0.4, 0.7], [0.3, 0.8] \rangle\}$. be a VGαCS(X) but not a VPCS(X), since $Vcl(V\operatorname{int}(A)) = G^c \not\subset A$.

Example 4.15: Let X ={a,b} and let $\tau = \{0, G, 1\}$ is a VT on X, where $G = \{\langle x, [0.1, 0.7], [0.3, 0.8] \rangle\}$. Here the only α open sets are 0, 1 and G. Let $A = \{\langle x, [0.1, 0.6], [0.2, 0.7] \rangle\}$. be a VPCS(X) but not a VGαCS (X).

Remark 4.16: VSCS and VGαCS are independent to each other.

Example 4.17: Let X ={a,b} and let $\tau = \{0, G, X\}$ is a VT on X, where $G = \{\langle x, [0.4, 0.6], [0.3, 0.6] \rangle\}$. Let $A = \{\langle x, [0.5, 0.7], [0.5, 0.8] \rangle\}$ be an VGαCS (X) but not a VSCS (X), since $V\operatorname{int}(Vcl(A)) = 1 \not\subset A$.

Example 4.18: Let X ={a,b} and let $\tau = \{0, G, 1\}$ is a VT on X, where $G = \{\langle x, [0.1, 0.6], [0.2, 0.7] \rangle\}$. Let $A = \{\langle x, [0.1, 0.7], [0.2, 0.7] \rangle\}$ be a VSCS (X) but not a VGαCS (X), since $V\alpha cl(A) = G^c \not\subset G$.

Theorem 4.19: Let (X, τ) be a VTS. Then for every $A \in VG\alpha C(X)$ and for every $B \in VS(X)$, $A \subseteq B \subseteq V\alpha cl(A)$ implies $B \in VG\alpha C(X)$.

Proof: Let a VS $B \subseteq U$ and U be a Vαos in X. Since $A \subseteq B, A \subseteq U$ and A is VGαCS, $V\alpha cl(A) \subseteq U$. By hypothesis, $B \subseteq V\alpha cl(A), V\alpha cl(B) \subseteq V\alpha cl(A) \subseteq U$. Therefore $V\alpha cl(B) \subseteq U$. Hence B is VGαCS of X.

Remark 4.20: The intersection of any two VGαCS is not VGαCS in general as seen from the following example.

Example 4.21: Let X ={a,b} and let $\tau = \{0, G, 1\}$ is a VT on X, where $G = \{\langle x, [0.1, 0.7], [0.3, 0.8] \rangle\}$. Then the VSs $A = \{\langle x, [0.1, 0.8], [0.2, 0.8] \rangle\}$ $B = \{\langle x, [0.2, 0.6], [0.2, 0.6], [0.3, 0.7] \rangle\}$ are VGαCSs but $A \cap B$ is not VGαCS in X.

Theorem 4.22: The union of two VGαCS is an VGαCS in (X,τ), if they are VCS in (X,τ) **Proof:** Assume that A and B are VGαCS in (X,τ). Since A and B are VCS in X. $Vcl(A) = A$ and $Vcl(B) = B$. Let $A \bigcup B \subseteq U$ and U is VαOS in X. Then $Vcl(V\text{int}(Vcl(A \bigcup B))) = Vcl(V\text{int}(A \bigcup B)) \subseteq Vcl(A \bigcup B) = A \bigcup B \subseteq U$, that is $V\alpha cl(A \bigcup B) \subseteq U$ Therefore $A \bigcup B$ is VGαCS.

5. VAGUE GENERALIZED ALPHA OPEN SETS

Definition 5.1: A VS A is said to be a vague generalized alpha open set (VGαOS in short) in (X,τ) if the complement A^c is a VGαOS in X.

The family of all VGαOSs of a VTS (X,τ) is denoted by VGαO(X).

Theorem 5.2: For any VTS (X,τ), we have the following:

(ii) Every VOS is a VGαOS
(iii) Every VαOS is a VGαOS
(iv) Every VROS is a VGαOS. But the converses need not be true.

Proof: (i) Let A be a VOS in X. Then A^c is an VCS in X. Therefore by theorem 3.3 A^c is a VGαCS in X. Hence A is a VGαOS in X.

The proof of (ii) and (iii) are obvious.

The converse of the above theorem is shown by the following example.

Example 5.3: Let X ={a, b} and let $\tau = \{0, G, 1\}$ is a VT on X, where $G = \{\langle x, [0.3, 0.6], [0.2, 0.7]\rangle\}$. Here the only α open sets are 0, X and G. Let $A = \{\langle x, [0.3, 0.7], [0.4, 0.8]\rangle\}$ be any VS in X. A is a VGαOS, but not a VOS in X.

Example 5.4: Let X ={a,b} and let $\tau = \{0, G, 1\}$ is a VT on X, where $G = \{\langle x, [0.3, 0.5], [0.2, 0.6]\rangle\}$ Here the only α open sets are 0, X and G. Let $A = \{\langle x, [0.3, 0.6], [0.3, 0.8]\rangle\}$ be any VS in X. A is a VGαOS, but not a VαOS in X.

Example 5.5: Let X ={a,b} and let $\tau = \{0, G, 1\}$ is an VT on X, where $G = \{\langle x, [0.3, 0.6], [0.2, 0.6]\rangle\}$ Here the only α open sets are 0, X and G. Let $A = \{\langle x, [0.4, 0.7], [0.3, 0.6]\rangle\}$ be any VS in X. A is a VGαOS, but not a VROS in X.

Theorem 5.6: Let (X,τ) be an VTS. If A is an VS of X then the following properties are equivalent:

(i) $A \in VG\alpha O(X)$
(ii) $V \subseteq v\text{int}(Vcl(V\text{int}(A)))$ whenever $V \subseteq A$ and V is a VαCS in X.
(iii) Thereexist VOS G_1 such that $G_1 \subseteq V \subseteq V\text{int}(Vcl(G))$ where G = Vint (A), $V \subseteq A$ and V is a VαCS in X.

Proof: (i) $\Rightarrow$ (ii) Let $A \in VG\alpha O(X)$. Then A^c is a VGαCS in X. Therefore $V\alpha cl(A^c) \subseteq U$ Whenever $A^c \subseteq U$ and U is an VαOS in X. That is $(Vcl(V\text{int}(Vcl(A^c))))^c = V\text{int}(V\text{int}(Vcl(A^c)))^c = V\text{int}(Vcl(Vcl(A^c))^c)$ $= V\text{int}(Vcl(V\text{int}(A^c)^c)) = V\text{int}(Vcl(Vcl(A))) \supseteq U^c$. This implies $U^c \subseteq V\text{int}(Vcl(Vcl(A)))$ whenever $U^c \subseteq A$ and U^c is a VαCS in X. Replace U^c by V, $V \subseteq V\text{int}(Vcl(V\text{int}(A)))$ whenever $V \subseteq A$ and V is a VαCS in X.

(ii) $\Rightarrow$ (iii) Let $V \subseteq V\text{int}(Vcl(V\text{int}(A)))$ whenever $V \subseteq A$ and V is a VαCS in X. Hence $V\text{int}(V) \subseteq V \subseteq V\text{int}(Vcl(V\text{int}(A)))$. Then there exist VOS G_1 in X such that $G_1 \subseteq V \subseteq V\text{int}(Vcl(G))$ where G = Vint(A) and G_1 = Vint(V).

(iii) $\Rightarrow$ (i) Suppose that there exists VOS G_1 such that $G_1 \subseteq V \subseteq V\operatorname{int}(Vcl(G))$ where G= Vint(A); $V \subseteq A$ and V is a VαOs in X. It is clear that $(V\operatorname{int}(Vcl(G)))^c \subseteq V^c$. That is $(V\operatorname{int}(Vcl(V\operatorname{int}(A))))^c \subseteq V^c$. This implies $Vcl(Vcl(V\operatorname{int}(A)))^c \subseteq V^c$. Therefore $Vcl(V\operatorname{int}(Vcl(A^c))) \subseteq V^c$, $A^c \subseteq V^c$ and V^c is VαOS in X. Hence $V\alpha cl(A^c) \subseteq V^c$. That is A^c is a VGαCS in X. This implies $A \in VG\alpha O(X)$.

Theorem 5.7: Let (X,τ) be a VTS. Then for every $A \in VG\alpha O(X)$ and for every $B \in VS(X)$, $V\alpha\operatorname{int}(A) \subseteq B \subseteq A$ implies $B \in VG\alpha O(X)$.

Proof: By hypothesis $V\alpha\operatorname{int}(A) \subseteq B \subseteq A$. Taking complement on both sides, we get $A^c \subseteq B^c \subseteq (V\alpha\operatorname{int}(A))^c$. Let $B^c \subseteq U$ and U is VαOS in X. Since $A^c \subseteq B^c, A^c \subseteq U$ and since A^c is a VGαCS, $V\alpha cl(A^c) \subseteq U$, Also $B^c \subseteq (V\alpha\operatorname{int}(A))^c = (V\alpha cl(A^c))$. Therefore $V\alpha cl(B^c) \subseteq V\alpha cl(A^c) \subseteq U$. Hence B^c is a VGαCS in X. This implies B is an VGαOS in X. That is $B \in VG\alpha O(X)$.

Theorem 5.8: A VS A of a VTS (X,τ) is a VGαOS if and only if $G \subseteq V\alpha cl(A)$, whenever G is a VαCS(X) and $G \subseteq A$.

Proof: Necessity: Assume that A is a VGαOS in X. Also let G be a VαCS in X such that $G \subseteq A$. Then G^c is a VαOS in X such that $A^c \subseteq G^c$. Since A^c is a VGαCS, $V\alpha cl(A^c) \subseteq G^c$. But $V\alpha cl(A^c) = (V\alpha\operatorname{int}(A))^c$. Hence $(V\alpha\operatorname{int}(A))^c \subseteq G$. This implies $G \subseteq V\alpha\operatorname{int}(A)$.

Sufficiency: Assume that $G \subseteq V\alpha\operatorname{int}(A)$, whenever G is a VαCS and $G \subseteq A$. Then $(V\alpha\operatorname{int}(A^c)) \subseteq G^c$, whenever G^c is a VαOS and $V\alpha cl(A^c) \subseteq G^c$. Therefore A^c is a VGαCS. This implies A is a VGαOS.

REFERENCES

1. Arya.S, Nour. T, Characterizations of s-normal spaces, Indian J. Pure Appl. Math. 21 (1990) 717-719.
2. Atanassov. K., Intuitionistic fuzzy Set, Fuzzy set and systems, 20 (1986), 87-96.
3. Bhattacharya. P, Lahiri. B. K, Semi generalized closed sets in topology, Indian J. Math. 29 (1987) 375-382.
4. Borumandsaeid. A and Zarandi. A., Vague set theory applied to BM- Algebras. International journal of algebra, 5, 5 (2011), 207-222.
5. Bustince. H, Burillo. P., Vague sets are intuitionistic fuzzysets, Fuzzy sets and systems, 79 (1996), 403-405.
6. Dontchev. J, On generating semi- preopen sets, Mem. Fac. Sci. Kochi Univ. Ser. A Math. 16 (1995) 35-48.
7. Gau. W. L, Buehrer. D. J., Vague sets, IEEE Trans, Systems Man and Cybernet, 23 (2) (1993), 610-614.
8. Levine. N, Generalized closed sets in topological spaces, Rend. Circ. Mat. Palermo 19 (1970) 89-96.
9. Levine. N, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963) 36-41.
10. Maki. H, Balachandran. K, Devi. R, Associated topologies of generalized α- closed sets and α- generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A Math. 15 (1994)51-63.
11. Maki. H, Devi. R, Balachandran. K, Generalized α- closed sets in topology, Bull. Fukuoka Univ. Ed. Part III 42 (1993) 13-21.
12. Mashhour. A. S, Abd. El-Monsef. M. E and El-Deeb. S. N, On pre continuous mappings and weak pre-continuous mappings, Proc Math, Phys. Soc. Egypt 53 (1982) 47-53.
13. Mashhour A.S., Hasanein I.A. and El-Deeb S.N., α-continuous and α-open mappings., Acta Math. Hung, 41(3-4)(1983) 213-218.
14. Moiodtsov. D., Soft set theory- rst results, Comput. Math. Appl. 37 (4-5) (1999), 19-31.
15. Pawlak. Z., Rough Sets, Int. J. Inf. Comp. Sci. 11 (1982), 341-356.
16. Stone. M, Application of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc. 41(1937) 374-481.
17. Zadeh. L. A., Fuzzy Sets, Information and control, 8 (1965), 338-353.

Source of support: Nil, Conflict of interest: None Declared