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FUZZY QUEUES ANALYSIS

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ABSTRACT

The main aim of this research paper is to illustrate the general approach which can be followed for the queuing systems in a fuzzy environment. The approach that has been presented in this research study is based on the Zadeh's extension principle, the concept of possibility and the Fuzzy Markov chains. In this research study the analysis of the M/F/1 and the FM/FM/1 systems are carried out and the findings are presented. The queue M/F/1 basically denotes a queue that have exponential interarrival time and which have fuzzy service rate. On the other hand the queue FM/FM/1 can be described as queue whose exponential interarrival time is fuzzified along with its service rate. In the practical applications it has been found that the Fuzzy queues are more relevant that the crisp queues. This research study will also provide a numerical example so as to illustrate the approach in a better manner. In order to establish the validity of the approach that has been taken up by the researcher a numerical problem along with its solution has been presented. Though in this research study the researcher has only investigated the M/F/1 and FM/FM/1 queuing systems, but this approach can be utilized to investigate any other queuing system.

***Keywords**: Fuzzy sets, Membership functions, Performance measures, Trapezoidal fuzzy number, Triangular fuzzy number.*

INTRODUCTION

The Zadeh's extension, the concept of possibility and the fuzzy Markov chains are basically used as a proposed approach for the queuing systems in a fuzzy environment. In this research study the approaches are explained through the analytical results for the M/F/1 and the FM/FM/1 systems. In the practical applications it has been found that the Fuzzy queues are more relevant that the crisp queues. This research study will also provide a numerical example so as to illustrate the approach in a better manner.

In the queuing systems, the Poisson arrival can be explained as a fairly reasonable assumption. The rate of service is more of a possibilistic nature than probabilistic. It is not possible to express in exact terms, the parameters λ and μ in the systems of M/M/1 or M/D/1 as they are frequently fuzzy in most of the practical situations. Under such circumstances it can be said that the linguistic expressions of these parameters like "the mean arrival rate is approximately 5" and the "mean service rate is approximately 10", looks more realistic. Through this research paper the researcher has tried to outline a general approach for the queuing system in a fuzzy environment. In order to understand the approach in broad manner, two distinct fuzzy queues which are denoted as M/F/1 and FM/FM/1 have been investigated by the researcher. The queue M/F/1 basically denotes a queue that have exponential inter arrival time and which have fuzzy service rate. On the other hand the queue FM/FM/1 can be described as queue whose exponential inter arrival time is fuzzified along with its service rate. The main approach that has been used by the researcher for carrying out the investigation of these fuzzified stochastic systems is based on the Zadeh's extension principle. Though in this research study the researcher has only investigated the M/F/1 and FM/FM/1 queuing systems, but this approach can be utilized to investigate any other queuing system.

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DEFINITIONS

The fuzzy set was first introduced by Professor Lotfi A. Zadeh in the year 1965. The main intent behind presenting the fuzzy set was to show the impreciseness and indistinctness among the various activities that take place every day.

i) The fuzzy set can be explained as a membership function mapping element for a space in a domain or in a universe of discourse X for the unit of interval [0, 1]. For example $A = \{x, \mu_A(x)); x \in X\}$, in that case $\mu_A : X \to [0, 1]$, can be said to be mapping which can be described as the degree of membership function of the fuzzy set A. Here $\mu_A(x)$ can be said to the membership value that is related to $x \in X$ of the fuzzy set A. These grades of membership are generally represented by real numbers within the range of [0, 1].

ii) The normal fuzzy can be explained as the fuzzy set A that belongs to the universe of discourse X and points out that there is atleast one $x \in X$ in such a manner that $\mu_A(x) = 1$.

iii) The fuzzy set A can be said to be convex fuzzy set only if for any $x_1, x_2 \in X$, the inequality $\mu_A\{\lambda x_1 + (1-\lambda)x_2\} \geq min\ \{\mu_A(x_1), \mu_A(x_2)\}.\ 0 \leq \lambda \leq 1$, has to be satisfied by the membership function A.

iv) A trapezoidal fuzzy number can be represented as A (a, b, c, d; 1) which has the membership function $\mu(x)$ is basically expressed as:

$$\mu(x) = \begin{cases} (x-a)/(b-a), & a \leq x < b \\ 1, & b \leq x < c \\ (d-x)(d-c), & c \leq x < d \\ 0, & otherwise \end{cases}$$

v) Triangular fuzzy number A(x) which is generally depicted as A(a,b,c;1) and have a membership function $\mu(x)$, can be expressed as:

$$\mu(x) = \begin{cases} (x-a)/(b-a), & a \leq x < b \\ 1, & x = b \\ (c-x)(c-b), & b \leq x < c \\ 0, & otherwise \end{cases}$$

vi) The α-cut of a fuzzy number A(x) is generally defined as:

$$A(\alpha) = [x : \mu(x) \geq \alpha,\ \alpha \in [0,1]\}$$

It is possible to add two trapezoidal fuzzy numbers in the following manner:

$$(a_1, b_1, c_1, d_1) + (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2,\ c_1 + c_2, d_1 + d_2).$$

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M/F/I QUEUES

For the purpose of this research study the researcher has taken into consideration a queuing system which has one server along with Poisson arrival. It is considered that λ be the mean of the arrival rate and discipline of the service is based on the first-come-first-serve rule. Let us consider that the time required for the service be S has approximate value and is basically depicted through a possibility distribution $\pi(t) = \mu_s(t)$, which generally keeps the more or less values of the service time restricted. All the system measures have been induced with the possibility distribution based on the possible non-fuzzy values of t. This process is done in such a manner that it leads to the fuzzification of the original queuing system. Thus, in the broader sense it can be said that the queuing system that has been fuzzified is basically a perception of the normal queuing system which can be described as the original of the queuing system. One point that has to be kept in mind that the exact location of the fuzzy queuing system that is original is not known. The only information that is known is that it is located in the set Q of the queuing systems. In the complete research study the fuzzified queuing system will be represented by M/F/1 and the original queuing system will be represented by M/D/1. Among the originals of M/F/1 the set Q can be described as Q= {(M/D/1) t϶ SUP ‘S’}, here t can be said to the service time of the system MJ/D/1 and the SUP ‘S’ = $\{t \in R^+, \mu_s(t) > 0\}$ can be described as the support of ‘S’. The statement “M/D/1 can be said to be the original of M/F/1” is called fuzzy. By fuzzy logic it can be said to have the truth value $\mu_s(t)$.

The Zadeh's expansion principle can be used for the possibility distributions of the system performance measures of M/F/1 on the basis of the solutions of the original problem M/D/1 when the exact value of t is known. Symbolically it can be said to be,

$$\mu_{f(S)}(x) = SUP\left\{\mu_s(t) \,\middle|\, t \epsilon F^{-1}(x)\right\}.$$

Where, 'f' is basically expressed as any entity which has the parameter S. A similar type of approach has been pointed out by Prade (1980) to solve the problems that are related to the fuzzy queuing.

FUZZY MARKOV CHAIN

This research also defines an imbedded fuzzy Markov chain for the queuing system M/F/1. Let us consider that the X(S) describes the stochastic fuzzy process which can be expressed as the fuzzy Markovian by taking a look at the system immediately after the completion of the services of a particular customer and the services for the next customer in the queue is about to begin. Let us assume that the probability of '*i*' arrivals during the time of service 'S' be denoted as P*i*. It is to be taken into consideration that P*i* is basically a fuzzy function, in such a manner that there is a possibility distribution, which is induced by μ_s on each of the points of the fuzzy function. In this research study the arrival process has been considered to have Poisson arrival and have the parameters λ. In such a scenario the fuzzy probability function can be expressed as:

$$\mu_\phi(x) = SUP\left\{\mu_s(t) \,\middle|\, x = \left[exp\ (-\lambda t)(\lambda t)^i\right]/\, i!\right\} \tag{1}$$

After the above process it becomes easy to construct the one step transition matrix for a fuzzy Markov chain in a direct manner. One point that has been accounted is that the arrival of the 'j' customers during the intervals of the service is required for the transition of the state from zero to state j or for the transition from state i to the state j. It has to be noted that in order to complete the transition process of the state i to the state j where (j ≥ i-1) > 0 requires that the (j-i +1) customers have come during the intervals of the service, the extra arrival since necessary has to take into account the customer who is understood to depart from the point of transition. In order to move from i to j where j<i-1 is completely not possible till the time the service takes place only one at a time. The transition probability matrix of the imbedded Markov chain is denoted by $P=[P_{ij}]$. Thus, it is possible that in all the conditions it can be written that for j ≥ i-1, i≥ 1, we have:

$$P = \left[P_{ij}\right] = \begin{Bmatrix} P_0, P_1, P_2, \ldots \\ P_0, P_1, P_2, \ldots \\ 0, 0, P_0, \ldots \end{Bmatrix}$$

Here, P_{ij} can be defined as,

$$V_j \geq i-1,\ i \geq 1;\ \mu_\phi(x_{ij}) = SUP\ \left\{\mu_s(t) \middle| x_{ij} =\ \left[exp\ (-\lambda t)(\lambda t)^{j-i+1}\right]/\ \ (j-i+1)!\right\} \tag{2}$$

On similar basis it can be said that normal Markov chain's perception is the fuzzy Markov chain, thus the usual Markov chain can also be called the original of the fuzzy Markov chain. It is to be noted that the location of the original of the fuzzy Markov chain is not known exactly, just this information is there that it is present in the set U of the Markov chains. On the basis of the queuing theory it is possible to solve the stationary equations for every possible original of the fuzzy Markov chain.

$$V_t,\ \lambda \in R^+,\ t < 1/\lambda;\ \pi_0 = 1-\lambda t,\ \pi_1 = (1-\lambda t)\left[exp(\lambda t)-1\right],$$

$$\pi_n = (1-\ \lambda t)\Sigma(-1)^{n-k} exp(k\lambda t)\ \left[\left\{(k\lambda t)^{n-k}/(n-k)!\right\} + \left\{(k\lambda t)^{n-k-1}/(n-k-1)!\right\}\right]\ \ for\ n\ \geq 2, \tag{3}$$

It can be said that the equations [1, 3] are known to be the steady state equation.

Also going further, the measure of the system performance can be denoted by:

$$L = \left[\lambda t(2-\ \lambda t)/\,2(1-\lambda t)\right],\ \ W = L/\lambda =\ t(2-\ \lambda t)/\,2(1-\lambda t) \tag{4}$$

Here, π_n denotes the steady state probability of n customers in the system at the point of departure, L denotes the expected length of the queue, W describes the expected time for sojourn. Through the concept of the extension principle the outcomes that have been obtained in the case of fuzzy can be explained through their membership functions for the steady-state solutions:

$$V\lambda\epsilon\ R^{+};\ \mu_{\pi}\left(x\right)=SUP\left\{\mu_{s}\left(t\right)\ x=1-\lambda t\right\},$$

$$\mu_{\pi}\left(x\right)=SUP\left\{\ \mu_{s}\left(t\right)\ \middle|x=\left(1-\lambda t\right)\left[e\left(\lambda t\right)-1\right]\right\} \tag{5}$$

$$Vn\geq 2,\ \mu_{\pi}\left(x\right)=SUP\left\{\begin{matrix}\mu_{s}\left(t\right)\middle|x=\left(1-\lambda t\right)\ \Sigma\ \left(-1\right)^{n-k}\times\ exp\ \left(k\lambda t\right)\left[\left(k\lambda t\right)^{n-k}\right]/\\ \left[\left(n-k\right)!\right]\ +\ \left[\left(k\lambda t\right)^{n-k-1}\right]/\ \left[\left(n-k-1\right)!\right]\end{matrix}\right\} \tag{6}$$

The performance measure of the system can be explained as:

$$V\lambda\epsilon\ R^{+};\ \mu_{Lk}\left(x\right)=SUP\ \left\{\mu_{S}\left(t\right)\middle|x=\left[\lambda t\left(2-\lambda t\right)/\,2\left(1-\lambda t\right)\right]\right\}$$

$$\mu_{Wk}\left(x\right)=SUP\ \left\{\mu_{S}\left(t\right)x=\left[t\left(2-\lambda t\right)/\,2\left(1-\lambda t\right)\right]\right\} \tag{7}$$

Here, L basically points to the K-th factorial moment of the size of the system. On the other hand W denotes the regular k-th moment of the waiting time of the system. Hence more importantly the generalization of the Little's formula in the case of the fuzzy can be explained as:

$$V\lambda\epsilon\ R^{+};\ \mu_{Wk}\left(y\right)=SUP\ \left\{\mu_{L}\left(x\right)\middle|y=\left[x/\lambda^{k}\right]\right\}$$

$$=\ SUP_{x\epsilon\ R+}\ SUP_{t\epsilon\ R+}\ \left\{\mu_{L}\left(t\right)\ \middle|x=d^{k}((1-\lambda t)(1-z)/(1-zexp\left[\lambda t\left(1-z\right)\right]\right\} \tag{8}$$

(FM/FM/I) FUZZY QUEUES

Let take into consideration a one server queuing system which is denoted by FM/FM/1. In this queuing system both the arrival and the departure processes are Poisson processes which are based on the fuzzy parameters. The functions of density that are related to the inter arrival times and the service times are basically described as:

$$a\left(t\right)=\lambda\theta\ exp\ \left(-\lambda\theta t\right),\ b\left(t\right)=\mu\theta exp\left(-\mu\theta t\right) \tag{9}$$

here, both λ and μ can be said to be linguistic. It has to be taken into consideration that there is a possibility distribution which is related to two fuzzy parameters, λ and μ, in a FM/FM/1 system of queuing. The usual M/M/1 queue is said to be the original of the fuzzy queue FM/FM/1, which has the membership function μ(M/M/1) = min $\{\mu_\lambda(\lambda), \mu_\mu(\mu)\}$. Thus, it can be said that all the fuzzy functions generally have the parameters λ and μ and thus they can be defined as

$$\mu_{f}\left(z\right)=SUP_{x,y\epsilon R}min\left\{\mu_{\lambda}\left(x\right),\mu_{\mu}\left(y\right)z=\ f\left(x,y\right)\right\}$$

In order to explain the imbedded fuzzy Markov chains in a FM/FM/1 system, the changes in the successive departure period has to be taken into consideration. But, now the time duration between all the set of transition is a fuzzy random variable which has the possibility that possibility distribution of the time of service is a fuzzy function b(t). Since it has been already said earlier that the process of arrival is a Poisson process thus with the fuzzy parameter λ, it can be considered that the conditional possibility distribution for the i arrivals, given the time of service t, is basically expressed by:

$$\mu_{P(A=\ i\,|s=\ t)}$$

$$Vt\epsilon R^{+};\mu_{P(A=\ i\ \ s=\ t)}\left(y\right)=SUP\ \left\{\mu_{\lambda}\left(x\right)\middle|y=\left[exp\ \left(-xt\right)\left(xt\right)^{i}\right]/\ \ i!\right\} \tag{10}$$

In order to get the marginal possibility distribution for the i arrivals in an arbitrary service intervals t, depicted as μ_P, this function has to be weighted by b(t) and then the sum has to be integrated over the complete t. This can be written as:

$$\mu_{P}\left(z\right)=SUP\ min\ \ \mu_{\lambda}\left(x\right),\ \mu_{\mu}\left(y\right)\middle|V_{t}\epsilon\ R^{+},$$

$$z=\int_{t}p\left\{A=n:s=t\right\}b\left(t\right)dt\,=\int_{x}^{\infty}\left\{\left[\,y\left(xt\right)^{i}\exp\left[-t\left(x+y\right)\right]/\,i\,!\right]\right\}dt \tag{11}$$

The square matrix of the imbedded fuzzy Markov chain in FM/FM/1 system is basically denoted by P = $[P_{ij}]$. It is to be noted that P_{ij} = P{x_{n+1} = j x_n = i} = P{A=j-i+1}. Hence the one step transition probabilities can be explained as ∀j ≥ i - 1, i≥ 1, i, j ϵ I.

After solving every possible original of the imbedded fuzzy Markov chain, we understand the steady state solution and system performance measures can be expressed as:

$$\forall\lambda,\ \mu \in R^{+},\ \lambda/\mu<1;\ \pi_{n}=(\lambda/\mu)^{n}(1-\lambda/\mu),\ L=\lambda/(\mu-\lambda),\ W=1/(\mu-\lambda) \quad (12)$$

The subsequent results in the fuzzy case of the queue FM/FM/1 are given as:

$$\mu_{L}(\sigma)=SUP\ min\ \{\mu_{\lambda}(x),\ \mu_{\mu}(y) \mid \sigma=1/(y-x)\}$$

On further, the Little's formula in higher moments can be derived from $\forall k \in$ I;

$$\mu_{wk}(\sigma)=SUP\{\mu_{L}(t) \mid \sigma=t/l^{k}\}=SUP\ min\ \{\mu_{\lambda}(x),\ \mu_{\mu}(y) \mid t=\{d^{k}[(y-k)/(y-zk)]/dz^{k}\}\}$$

NUMERICAL EXAMPLE

In this section a numerical example has been provided by the researcher to illustrate the application of the complete process.

At the end of an assembly line of automotives a special tune up station has been set up so as to make up the adjustments in those vehicles which cannot match up with the standards of exhaust gas emissions that has been set up been the federal government. Since the arrival of the failures are found to completely of the random in nature thus can be justified the Poisson arrival assumption of the process with λ = 0.1 vehicle per minute. Every arrival has to serviced for an adjustment which requires approximately five minutes and can be expressed through a trapezoidal fuzzy number which has the member function:

$$Vt\epsilon R^{+};\ \mu_{s}(t)=0,\ t\le 2,$$

$$\mu_{s}(t)=(1/2t)-1,\ 2\le t\le 4,$$

$$\mu_{s}(t)=1, 4\le t\le 6,$$

$$\mu_{s}(t)=(-1/2t)+4,\ 6\le t\le 8,$$

$$\mu_{s}(t)=0,\ t\ge 8.$$

In order to calculate the space required for storage, the management of the company wants to know the following:

a) The mean number of automobiles present at the station
b) The sojourn time expected for every automobile
c) The probability about the presence of more than two automobiles at the station.

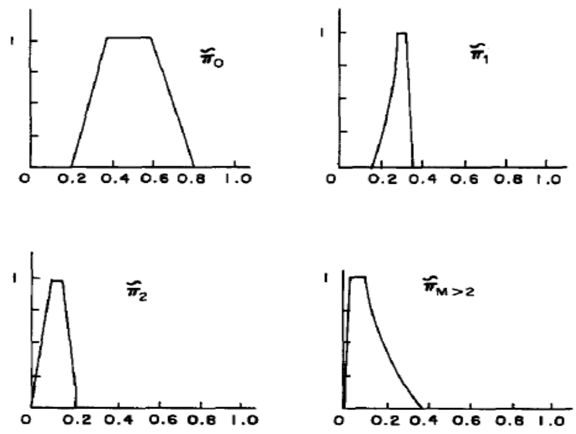

Fig. 1. Stationary fuzzy probabilities.

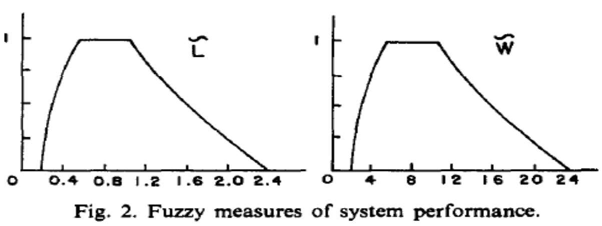

Fig. 2. Fuzzy measures of system performance.

In the above given figures the figures 1 and 2 gives the solution to the problem. The stationary fuzzy probabilities are shown in the figure 1 and the fuzzy measures of the performances of the system are given by the figure 2. One thing that has to be pointed out is that the membership function of the results does not belong to the trapezoidal fuzzy numbers any more except for $\pi_{0.}$

DISCUSSION AND CONCLUSION

In the research study though the researcher has only investigated and analyzed two simple queuing models, the approach that has been shown in this research study can be easily used to analyze more complicate fuzzy queues. For example this system can be easily used to analyze the systems like M/F/C, M/F/C/K etc. and these systems can be easily analyzed by adopting a similar kind of an approach which has been used to analyze the simple fuzzy queues. The main idea behind this approach is to consider the fuzzy queue as the perception of the usual crisp queue. The system performances are measured by the corresponding fuzzy which are then explained by the membership functions which are then obtained from the outcome of the every possible original queues on the basis of the extension principle. The priority model of discipline queues are very crucial in the real life applications especially when treating or giving preferences to certain individuals like giving importance or preference to an emergency situation in a hospital. They are also very important in the modeling and the analysis of the communication networks and the transmission of the internet data. There are various reasons beyond the limit of humans that leads to the imprecise understanding of the parameters of the queuing models in case of the real life situations. Thus, the performance measures of the system and the average total cost because of the inactivity would also become fuzzy. It is a known fact that valuable information are lost if the outcomes are obtained in the form of the crisp values. The validity of the analysis approach that has been taken by the researcher to analyze the queues is shown by the researcher by solving numerical problems that are related to the analysis of the queues. But this approach is not just limited to similar queues and can be followed for more complex queues.

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