

## NANO SEMI-GENERALIZED HOMEOMORPHISMS IN NANO TOPOLOGICAL SPACE

K. BHUVANESWARI<sup>\*1</sup>, A. EZHILARASI<sup>2</sup>

<sup>1</sup>Professor and Head, Department of Mathematics,  
Mother Teresa Women's University, Kodaikanal, Tamilnadu, India.

<sup>2</sup>Research Scholar, Department of Mathematics,  
Karpagam University, Coimbatore, Tamilnadu, India.

(Received On: 21-06-16; Revised & Accepted On: 22-07-16)

---

### ABSTRACT

*In this paper, we introduce nano sg-closed maps and nano sg – open maps in nano topological spaces and then we introduce and study nano sg - homeomorphisms. We obtain certain characterizations of these maps.*

**Keywords:** Nano closed map, Nano open map, Nano sg – irresolute, Nano sg-closed map, Nano sg-open map, nano sg-homeomorphism.

---

### 1. INTRODUCTION

The concepts of semi-generalized mappings and generalized- semi mappings were introduced by R.Devi *et.al* [7] in 1993. Maki *et.al* [10] introduced g-homeomorphisms and gc-homeomorphisms in topological spaces. Devi [7] introduced and studied sg- closed functions and gs-closed functions. A study on semi-generalized homeomorphism was done by Devi *et. al* [6]. The concept of nano topology was introduced by Lellis Thivagar [7] and he analysed nano closed maps, nano open maps and nano homeomorphisms. In this paper, nano generalized–semi closed functions, nano semi-generalized open functions are introduced and nano semi-generalized homeomorphisms are analysed.

### 2. PREMILINARIES

**Definition 2.1:** [11] A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called semi-open if the image of every open set in  $(X, \tau)$  is semi-open in  $(Y, \sigma)$ .

**Definition 2.2:** [13] A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called g-open if the image of every open set in  $(X, \tau)$  is g-open in  $(Y, \sigma)$ .

**Definition 2.3:** [5] A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called pre-semi open if the image of every semi-open set in  $(X, \tau)$  is semi-open in  $(Y, \sigma)$ .

**Definition 2.4:** [12] A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called sg-closed if the image of every closed set in  $(X, \tau)$  is sg-closed in  $(Y, \sigma)$ .

**Definition: 2.5:** [13] A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is semi-generalized continuous (sg-continuous) if  $f^{-1}(V)$  is sg-closed set in  $X$  for every closed set  $V$  of  $Y$ .

**Definition 2.6:** [10] A bijective function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called a generalized homeomorphism if  $f$  is both g-continuous and g-open.

---

**Corresponding Author: K. Bhuvaneswari<sup>\*1</sup>**  
**<sup>1</sup>Professor and Head, Department of Mathematics,**  
**Mother Teresa Women's University, Kodaikanal, Tamilnadu, India.**

**Definition 2.7:** [6] A bijective function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called a semi-generalized homeomorphism if  $f$  is both sg-continuous and sg-open.

**Definition 2.8:** [8] Let  $U$  be a non-empty finite set of objects called the universe and  $R$  be an equivalence relation on  $U$  named as indiscernibility relation. Then  $U$  is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair  $(U, R)$  is said to be the approximation space. Let  $X \subseteq U$ . Then,

(i) The lower approximation of  $X$  with respect to  $R$  is the set of all objects which can be for certain classified as  $X$  with respect to  $R$  and is denoted by  $L_R(X)$ .  $L_R(X) = U\{R(x): R(x) \subseteq X, x \in U\}$  where  $R(x)$  denotes the equivalence class determined by  $x \in U$ .

(ii) The upper approximation of  $X$  with respect to  $R$  is the set of all objects which can be possibly classified as  $X$  with respect to  $R$  and is denoted by  $U_R(X)$ .  $U_R(X) = U\{R(x): R(x) \cap X \neq \Phi, x \in U\}$ .

(iii) The boundary region of  $X$  with respect to  $R$  is the set of all objects which can be classified neither as  $X$  nor as not- $X$  with respect to  $R$  and it is denoted by  $B_R(X)$ .  $B_R(X) = U_R(X) - L_R(X)$ .

**Property 2.9** [8]: If  $(U, R)$  is an approximation space and  $X, Y \subseteq U$ , then

1.  $L_R(X) \subseteq X \subseteq U_R(X)$
2.  $L_R(\Phi) = U_R(\Phi) = \Phi$
3.  $L_R(U) = U_R(U) = U$
4.  $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5.  $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
6.  $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
7.  $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8.  $L_R(X) \subseteq L_R(Y)$  and  $U_R(X) \subseteq U_R(Y)$  whenever  $X \subseteq Y$
9.  $U_R(X^c) = [L_R(X)]^c$  and  $L_R(X^c) = [U_R(X)]^c$
10.  $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
11.  $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$ .

**Definition 2.10** [8]: Let  $U$  be the universe,  $R$  be an equivalence relation on  $U$  and the nano topology  $\tau_R(X) = \{U, \Phi, L_R(X), U_R(X), B_R(X)\}$  where  $X \subseteq U$ . Then by property 2.5,  $\tau_R(X)$  satisfies the following axioms:

- (i)  $U$  and  $\Phi \in \tau_R(X)$ .
- (ii) The union of the elements of any sub-collection of  $\tau_R(X)$  is in  $\tau_R(X)$ .
- (iii) The intersection of the elements of any finite subcollection of  $\tau_R(X)$  is in  $\tau_R(X)$ .

Then  $\tau_R(X)$  is a topology on  $U$  called the Nano topology on  $U$  with respect to  $X$ .  $(U, \tau_R(X))$  is called the Nano topological space. Elements of the Nano topology are known as nano open sets in  $U$ . Elements of  $[\tau_R(X)]^c$  are called nano closed sets with  $[\tau_R(X)]^c$  being called Dual Nano topology of  $\tau_R(X)$ . If  $\tau_R(X)$  is the Nano topology on  $U$  with respect to  $X$ , then the set  $B = \{U, L_R(X), B_R(X)\}$  is the basis for  $\tau_R(X)$ .

**Definition 2.11** [8]: If  $(U, \tau_R(X))$  is a Nano topological space with respect to  $X$  where  $X \subseteq U$  and if  $A \subseteq U$ , then

- (i) The nano interior of the set  $A$  is defined as the union of all nano open subsets contained in  $A$  and is denoted by  $NInt(A)$ .  $NInt(A)$  is the largest nano open subset of  $A$ .
- (ii) The nano closure of the set  $A$  is defined as the intersection of all nano closed sets containing  $A$  and is denoted by  $NCl(A)$ .  $NCl(A)$  is the smallest nano closed set containing  $A$ .

**Remark 2.12** [8]: Throughout this paper,  $U$  and  $V$  are non-empty, finite universes;  $X \subseteq U$  and  $Y \subseteq V$ ;  $U/R$  and  $V/R'$  denote the families of equivalence classes by equivalence relations  $R$  and  $R'$  respectively on  $U$  and  $V$ .  $(U, \tau_R(X))$  and  $(V, \tau_{R'}(Y))$  are the Nano topological spaces with respect to  $X$  and  $Y$  respectively.

**Definition 2.13** [1]: If  $(U, \tau_R(X))$  is a Nano topological space with respect to  $X$  where  $X \subseteq U$  and if  $A \subseteq U$ , then

(i) The nano semi-closure of  $A$  is defined as the intersection of all nano semi-closed sets containing  $A$  and is denoted by  $NsCl(A)$ .  $NsCl(A)$  is the smallest nano semi-closed set containing  $A$  and  $NsCl(A) \subseteq A$ .

(ii) The nano semi-interior of  $A$  is defined as the union of all nano semi-open subsets of  $A$  and is denoted by  $NsInt(A)$ .  $NsInt(A)$  is the largest nano semi open subset of  $A$  and  $NsInt(A) \subseteq A$ .

**Definition 2.14** [1]: A subset  $A$  of  $(U, \tau_R(X))$  is called nano semi-generalized closed set (Nsg-closed) if  $NsCl(A) \subseteq V$  and  $A \subseteq V$  and  $V$  is nano semi-open in  $(U, \tau_R(X))$ .

**Definition 2.15 [1]:** If  $(U, \tau_R(X))$  is a nano topological space with respect to  $X$  where  $X \subseteq U$  and if  $A \subseteq U$ , then

- (i) The nano semi-generalized closure of  $A$  is defined as the intersection of all nano semi-generalized closed sets containing  $A$  and is denoted by  $NsgCl(A)$ .
- (ii) The nano semi-generalized interior of  $A$  is defined as the union of all nano semi-generalized open subsets of  $A$  and is denoted by  $NsgInt(A)$ .

**Definition 2.16 [9]:** Let  $(U, \tau_R(X))$  and  $(V, \tau_{R'}(Y))$  be two nano topological spaces. Then a mapping  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is nano continuous on  $U$  if the inverse image of every nano open set in  $V$  is nano open in  $U$ .

**Definition 2.17[9]:** A function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is called nano open if the image of every nano open set in  $(U, \tau_R(X))$  is nano open in  $(V, \tau_{R'}(Y))$ .

**Definition 2.18 [9]:** A function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is called nano closed if the image of every nano closed set in  $(U, \tau_R(X))$  is nano closed in  $(V, \tau_{R'}(Y))$ .

**Definition 2.19 [4]:** A function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is called nano g-closed if the image of every nano g-closed set in  $(U, \tau_R(X))$  is nano g-closed in  $(V, \tau_{R'}(Y))$ .

**Definition 2.20 [4]:** A mapping  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is nano semi-generalized continuous on  $U$  if the inverse image of every nano open set in  $V$  is nano sg-open in  $U$ .

**Definition 2.21 [9]:** A bijection  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is called nano homeomorphism if  $f$  is both nano continuous and nano open.

**Definition 2.22 [4]:** A bijection  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is called nano g-homeomorphism if  $f$  is both nano g-continuous and nano g-open.

### 3. NANO SG-CLOSED MAPS

In this section, nano semi-generalized closed functions and nano semi-generalized open functions in nano topological spaces are introduced. We discuss certain characterizations of these functions.

**Definition 3.1:** Let  $(U, \tau_R(X))$  and  $(V, \tau_{R'}(Y))$  be the two nano topological spaces. Then a function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is said to be nano semi-generalized closed function (briefly  $Nsg$ -closed function) if the image of every nano closed set in  $(U, \tau_R(X))$  is  $Nsg$ -closed in  $(V, \tau_{R'}(Y))$ .

**Example 3.2:** Let  $U = \{a, b, c, d\}$  be the universe with  $U/R = \{\{a\}, \{c\}, \{b, d\}\}$  and let  $X = \{a, b\} \subseteq U$ . Then the nano open sets are  $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$  and the nano closed sets are  $\tau_R^c(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$ .  $\{U, \phi, \{a\}, \{a, c\}, \{b, d\}, \{a, b, d\}\}$  are the nano semi-open sets.  $Nsg$ -open sets are  $\{U, \phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ .  $Nsg$ -closed sets are  $\{U, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{b, c\}, \{c, d\}, \{a, c\}, \{b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c\}\}$ . Let  $V = \{x, y, z, w\}$  be the another universe with  $V/R' = \{\{x\}, \{w\}, \{y, z\}\}$  and let  $Y = \{x, z\} \subseteq V$ . Then the nano open sets are  $\tau_{R'}(Y) = \{V, \phi, \{x\}, \{x, y, z\}, \{y, z\}\}$  and the nano closed sets are  $\tau_{R'}^c(Y) = \{V, \phi, \{w\}, \{x, w\}, \{y, z, w\}\}$ .  $\{V, \phi, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{x, w\}, \{y, z\}, \{x, y, z\}, \{x, y, w\}, \{y, z, w\}, \{x, z, w\}\}$  are the  $Nsg$ -open sets.  $Nsg$ -closed sets are  $\{V, \phi, \{x\}, \{y\}, \{z\}, \{w\}, \{y, w\}, \{y, z\}, \{x, w\}, \{z, w\}, \{y, z, w\}, \{x, y, w\}, \{x, z, w\}\}$ . Define the function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  as  $f(a) = y, f(b) = x, f(c) = w, f(d) = z$ . Now  $f(U) = V, f(\phi) = \phi, f(\{b, c, d\}) = \{x, w, z\}, f(\{c\}) = \{w\}, f(\{a, c\}) = \{y, w\}$  are the images of nano closed sets of  $U$  which are  $Nsg$ -closed in  $V$ . Thus the function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is a  $Nsg$ -closed function.

**Theorem 3.3:** A function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$ -closed function if and only if  $NsgClf(A) \subseteq f(NCl(A))$  for every subset  $A$  of  $(U, \tau_R(X))$ .

**Proof:** Let  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  be a  $Nsg$ -closed function and  $A \subseteq U$ . Then  $NCl(A)$  is nano closed in  $(U, \tau_R(X))$  and hence  $f(NCl(A))$  is  $Nsg$ -closed function in  $(V, \tau_{R'}(Y))$ . Since  $A \subseteq NCl(A)$ , it implies that  $f(A) \subseteq f(NCl(A))$ . As  $NsgCl(f(NCl(A)))$  is the  $Nsg$ -closed set containing  $f(A)$ , it follows that  $NsgCl(f(A)) \subseteq NsgCl(f(NCl(A))) = f(NCl(A))$ .

Conversely, let  $A$  be any nano closed set in  $(U, \tau_R(X))$ . Then  $A = NCl(A)$  and so  $f(A) = f(NCl(A)) \supseteq NsgCl(f(A))$  by the given hypothesis. Also, it follows that  $f(A) \subseteq NsgCl(f(A))$ . Hence  $f(A) = NsgCl(f(A))$ . i.e.,  $f(A)$  is  $Nsg$ -closed and hence  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is a  $Nsg$ -closed function.

**Theorem 3.4:** A function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$ -closed if and only if for each subset  $S$  of  $(V, \tau_{R'}(Y))$  and for each nano open set  $A$  of  $(U, \tau_R(X))$  containing  $f^{-1}(S)$ , there is a  $Nsg$ -open set  $B$  of  $(V, \tau_{R'}(Y))$  such that  $S \subseteq B$  and  $f^{-1}(B) \subseteq A$ .

**Proof:** Let  $S$  be the subset of  $(V, \tau_{R'}(Y))$  and  $A$  be a nano open set of  $(U, \tau_R(X))$  such that  $f^{-1}(S) \subset A$ . Now  $V - f(U - A)$ , say  $B$ , is a  $Nsg$ -open set containing  $S$  in  $V$  such that  $f^{-1}(B) \subseteq A$ .

Conversely, let  $F$  be a nano closed set of  $(U, \tau_R(X))$ , then  $f^{-1}(V - f(F)) \subset U - F$  and  $U - F$  is nano open. Now, there is a  $Nsg$ -open set  $B$  of  $(V, \tau_{R'}(Y))$  such that  $V - f(F) \subset B$  and  $f^{-1}(B) \subset U - F$ . Hence  $F \subset U - f^{-1}(B)$  and thus  $V - B \subset f(F) \subset f(U - f^{-1}(B)) \subset V - B$  which implies  $f(F) = V - B$ . Since  $V - B$  is  $Nsg$ -closed,  $f(F)$  is a  $Nsg$ -closed set in  $(V, \tau_{R'}(Y))$  for every nano closed set  $F$  in  $(U, \tau_R(X))$ . Hence  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is a  $Nsg$ -closed function.

The following example shows that the composition of two  $Nsg$ -closed functions need not be  $Nsg$ -closed.

**Example 3.5:** Let  $(U, \tau_R(X)), (V, \tau_{R'}(Y))$  and  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  be as in Example 3.2. Let  $W = \{p, q, r, s\}$  with  $W/R'' = \{\{p\}, \{q\}, \{r, s\}\}$ . Let  $Z = \{p, r\} \subseteq W$ . The nano open sets are  $\tau_{R'}(Z) = \{W, \phi, \{p\}, \{p, r, s\}, \{r, s\}\}$  and nano closed sets are  $\tau_{R'}^C(Z) = \{W, \phi, \{q, r, s\}, \{q\}, \{p, q\}\}$ . The  $Nsg$ -closed sets are  $\{W, \phi, \{p\}, \{r\}, \{q\}, \{s\}, \{p, q\}, \{r, q\}, \{r, s\}, \{q, s\}, \{p, r, q\}, \{p, q, s\}, \{q, r, s\}\}$ . Define a map  $g : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R'}(Z))$  as  $g(x) = r, g(y) = p, g(z) = q, g(w) = s$ . Then both  $f$  and  $g$  are  $Nsg$ -closed maps but their composition  $g \circ f : (U, \tau_R(X)) \rightarrow (W, \tau_{R'}(Z))$  is not a  $Nsg$ -closed map since for the nano closed set  $\{a, c\}$  in  $(U, \tau_R(X))$ ,  $(g \circ f)(\{a, c\}) = g[f(\{a, c\})] = g(\{y, w\}) = \{p, s\}$  which is not a  $Nsg$ -closed set in  $(W, \tau_{R'}(Z))$ . Thus the composition of two  $Nsg$ -closed maps need not be  $Nsg$ -closed.

**Theorem 3.6:** Let  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  and  $g : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R''}(Z))$  be two mappings such that their composition  $g \circ f : (U, \tau_R(X)) \rightarrow (W, \tau_{R''}(Z))$  is a  $Nsg$ -closed mapping. Then the following statements are true.

- (i) If  $f$  is nano continuous and surjective, then  $g$  is  $Nsg$ -closed.
- (ii) If  $g$  is  $Nsg$ -irresolute and injective, then  $f$  is  $Nsg$ -closed.
- (iii) If  $g$  is strongly  $Nsg$ -continuous and injective, then  $f$  is nano closed.

**Proof:**

(i) Let  $A$  be a nano closed set in  $(V, \tau_{R'}(Y))$ . Since  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is nano continuous, it follows that  $f^{-1}(A)$  is nano closed in  $(U, \tau_R(X))$ . Since  $g \circ f : (U, \tau_R(X)) \rightarrow (W, \tau_{R''}(Z))$  is  $Nsg$ -closed,  $(g \circ f)[f^{-1}(A)]$  is  $Nsg$ -closed in  $(W, \tau_{R''}(Z))$ . i.e.,  $g(A)$  is  $Nsg$ -closed in  $(W, \tau_{R''}(Z))$  as the function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is surjective. Hence the image of a nano closed set in  $(V, \tau_{R'}(Y))$  is  $Nsg$ -closed in  $(W, \tau_{R''}(Z))$ . Thus  $g : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R''}(Z))$  is  $Nsg$ -closed.

(ii) Let  $B$  be a nano closed set in  $(U, \tau_R(X))$ . Since the function  $g \circ f : (U, \tau_R(X)) \rightarrow (W, \tau_{R''}(Z))$  is  $Nsg$ -closed, then  $(g \circ f)(B)$  is  $Nsg$ -closed in  $(W, \tau_{R''}(Z))$ . Since  $g$  is  $Nsg$ -irresolute,  $g^{-1}[(g \circ f)(B)]$  is  $Nsg$ -closed in  $(V, \tau_{R'}(Y))$  i.e., the set  $f(B)$  is  $Nsg$ -closed in  $(V, \tau_{R'}(Y))$  since the function  $g : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R''}(Z))$  is injective. Thus the image of a nano closed set in  $(U, \tau_R(X))$  is  $Nsg$ -closed in  $(V, \tau_{R'}(Y))$ . Thus  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$ -closed.

(iii) Let  $D$  be a nano closed set in  $(U, \tau_R(X))$ . Since the function  $g \circ f : (U, \tau_R(X)) \rightarrow (W, \tau_{R''}(Z))$  is  $Nsg$ -closed in  $(W, \tau_{R''}(Z))$ ,  $(g \circ f)(D)$  is  $Nsg$ -closed in  $(W, \tau_{R''}(Z))$ . Since the function  $g : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R''}(Z))$  is strongly  $Nsg$ -continuous,  $g^{-1}[(g \circ f)(D)]$  is nano closed in  $(V, \tau_{R'}(Y))$ . i.e., since the functions  $g : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R''}(Z))$  is injective,  $f(D)$  is nano closed in  $(V, \tau_{R'}(Y))$  for every nano closed set  $D$  in  $(U, \tau_R(X))$ . Thus  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is nano closed.

**Definition 3.7:** A function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is said to be  $Nsg$ -open function if the image  $f(A)$  is  $Nsg$ -open in  $(V, \tau_{R'}(Y))$  for each nano open set  $A$  in  $(U, \tau_R(X))$ .

**Definition 3.8:** A function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$ -open function if and only if  $f(NInt(A)) \subseteq NsgInt(f(A))$  for each subset  $A$  of  $(U, \tau_R(X))$ .

**Proof:** Suppose that  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  be a  $Nsg$ -open function. Let  $A \subseteq U$ . Then  $NInt(A)$  is nano open in  $(U, \tau_R(X))$ . Since  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$ -open,  $f(NInt(A))$  is  $Nsg$ -open in  $(V, \tau_{R'}(Y))$ . Also, as  $NInt(A) \subseteq A$ , it follows that  $f(NInt(A)) \subseteq f(A)$ . Thus  $f(NInt(A))$  is  $Nsg$ -open contained in  $f(A)$ . Hence  $f(NInt(A)) \subseteq NsgInt(f(A))$ .

Conversely, let  $f(NInt(A)) \subseteq NsgInt(f(A))$  for every subset  $A$  of  $(U, \tau_R(X))$ . Let  $F$  be a nano open set in  $(U, \tau_R(X))$ , then  $NInt(F) = F$ . Now  $f(F) \subseteq f(NInt(F)) \subseteq NsgInt(f(F)) = f(F)$ . Hence  $f(F) = NsgInt(f(F))$ . Thus  $f(F)$  is  $Nsg$ -open in  $(V, \tau_{R'}(Y))$  for every nano open set  $F$  in  $(U, \tau_R(X))$ . Hence  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is a  $Nsg$ -open function.

#### 4. NANO SG-HOMEOMORPHISMS

In this section, a new form of homeomorphisms namely,  $Nsg$ -homeomorphism is introduced and some of the properties are studied.

**Definition 4.1** A function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is said to be a  $Nsg$ -homeomorphism if

- (i)  $f$  is one to one and onto
- (ii)  $f$  is a  $Nsg$ -continuous
- (iii)  $f$  is a  $Nsg$ -open

**Theorem 4.2:** Let  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  be an one to one onto mapping. Then  $f$  is  $Nsg$ -homeomorphism if and only if  $f$  is  $Nsg$ -closed and  $Nsg$ -continuous.

**Proof:** Let  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  be a  $Nsg$  –homeomorphism. Then  $f$  is  $Nsg$  –continuous. Let  $A$  be an arbitrary nano closed set in  $(U, \tau_R(X))$ . Then  $U - A$  is nano open. Since  $f$  is  $Nsg$  –open,  $f(U - A)$  is  $Nsg$  –open in  $(V, \tau_{R'}(Y))$ . That is,  $V - f(A)$  is  $Nsg$  –open in  $(V, \tau_{R'}(Y))$ . Hence  $f(A)$  is  $Nsg$  –closed in  $(V, \tau_{R'}(Y))$  for every nano closed set  $A$  in  $(U, \tau_R(X))$ . Hence  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$  –closed. Conversely, let  $f$  be  $Nsg$  –closed and  $Nsg$  –continuous function. Let  $G$  be a nano open set in  $(U, \tau_R(X))$ . Then  $U - G$  is nano closed in  $(U, \tau_R(X))$ . Since  $f$  is  $Nsg$  –closed,  $f(U - G)$  is  $Nsg$  –closed in  $(V, \tau_{R'}(Y))$ . That is,  $f(U - G) = V - f(G)$  is  $Nsg$  –closed in  $(V, \tau_{R'}(Y))$ . Hence  $f(G)$  is  $Nsg$  –open in  $(V, \tau_{R'}(Y))$  for every nano open set  $G$  in  $(U, \tau_R(X))$ . Thus  $f$  is  $Nsg$  –open and hence  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$  –homeomorphism.

**Theorem 4.3:** A one to one map  $f$  of  $(U, \tau_R(X))$  onto  $(V, \tau_{R'}(Y))$  is a  $Nsg$  –homeomorphism if and only if  $f(NsgCl(A) = NCl(f(A))$  for every subset  $A$  of  $(U, \tau_R(X))$ .

**Proof:** If  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$  –homeomorphism, then  $f$  is  $Nsg$  –continuous and  $Nsg$  –closed. If  $A \subseteq U$ , it follows that  $f(NsgCl(A) \subseteq NCl(f(A))$  since  $f$  is  $Nsg$  –continuous. Since  $NsgCl(A)$  is nano closed in  $(U, \tau_R(X))$  and  $f$  is  $Nsg$  –closed map,  $f(NsgCl(A))$  is  $Nsg$  –closed in  $(V, \tau_{R'}(Y))$ . Also  $NsgCl(f(NsgCl(A))) = f(NsgCl(f(A)))$ . Since  $A \subseteq NsgCl(A)$ ,  $f(A) \subseteq f(NsgCl(A))$  and hence  $NCl(f(A)) \subseteq NCl(f(NsgCl(A))) = f(NsgCl(A))$ . Thus  $NCl(f(A)) \subseteq f(NsgCl(A))$ . Therefore,  $f(NsgCl(A) = NCl(f(A))$  if  $f$  is  $Nsg$  –homeomorphism.

Conversely, if  $f(NsgCl(A) = NCl(f(A))$  for every subset  $A$  of  $(U, \tau_R(X))$ , then  $f$  is  $Nsg$  –continuous. If  $A$  is nano closed in  $(U, \tau_R(X))$ , then  $A$  is  $Nsg$  –closed in  $(U, \tau_R(X))$ . Then  $NsgCl(A) = A$  which implies  $f(NsgCl(A)) = f(A)$ . Hence, by the given hypothesis, it follows that  $NCl(f(A)) = f(A)$ . Thus  $f(A)$  is nano closed in  $V$  and hence  $Nsg$  –closed in  $V$  for every nano closed set  $A$  in  $U$ . That is,  $f$  is  $Nsg$  –closed. Thus  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$  –homeomorphism.

**Example 4.4:** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a\}, \{c\}, \{b, d\}\}$  and let  $X = \{a, b\} \subseteq U$ . The nano open sets are  $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$ . Let  $V = \{x, y, z, w\}$  with  $V/R' = \{\{x\}, \{w\}, \{y, z\}\}$  and let  $Y = \{x, z\} \subseteq V$ . The nano open sets are  $\tau_{R'}(Y) = \{V, \phi, \{x\}, \{x, y, z\}, \{y, z\}\}$ . Define a function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  as  $f(a) = y, f(b) = x, f(c) = w, f(d) = z$ . Then  $f$  is bijective,  $Nsg$  –continuous and  $Nsg$  –open and so the function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$  –homeomorphism.

**Theorem 4.5:** If a function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is nano homeomorphism, then it is  $Nsg$  –homeomorphism but not conversely.

**Proof:** As the function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is nano homeomorphism, by definition,  $f$  is bijective, nano continuous and nano open. And hence  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$  –continuous. Since  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is nano open, the image of every nano open set in  $(U, \tau_R(X))$  is nano open in  $(V, \tau_{R'}(Y))$  and hence  $Nsg$  –open in  $(V, \tau_{R'}(Y))$  every nano open set is  $Nsg$  –open. Thus the function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$  –open. Therefore, every nano homeomorphism  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$  –homeomorphism.

The converse of the Theorem 4.5 need not be true as seen from the following example.

**Example 4.6:** In Example 4.4, the map  $f$  is a  $Nsg$  –homeomorphism. Now  $f^{-1}(V) = U$ ,  $f^{-1}(\phi) = \phi$ ,  $f^{-1}(x) = \{b\}$ ,  $f^{-1}(\{y, z\}) = \{a, d\}$ ,  $f^{-1}(\{x, y, z\}) = \{a, b, d\}$ . Hence the inverse images of nano open sets in  $(V, \tau_{R'}(Y))$  are not nano open in  $(U, \tau_R(X))$  and hence  $f$  is not nano continuous. Thus the map  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is not nano homeomorphism.

**Theorem 4.7:** Let  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  be a  $Nsg$  -continuous function. Then the following statement are equivalent.

- (i)  $f$  is a  $Nsg$  -open function
- (ii)  $f$  is a  $Nsg$  -homeomorphism
- (iii)  $f$  is a  $Nsg$  -closed function.

**Proof:**

**(i)⇒(ii):** By the given hypothesis, the function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is bijective,  $Nsg$  -continuous and  $Nsg$  -open. Hence the function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$  -homeomorphism.

**(ii)⇒(iii):** By the given hypothesis, the function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$  -homeomorphism and hence  $Nsg$  -open. Let  $A$  be the nano closed set in  $(U, \tau_R(X))$ . Then  $A^C$  is nano open in  $(U, \tau_R(X))$

By assumption,  $f(A^C)$  is  $Nsg$  -open in  $(V, \tau_{R'}(Y))$  i.e.,  $f(A^C) = (f(A))^C$  is  $Nsg$  -open in  $(V, \tau_{R'}(Y))$  and hence  $f(A)$  is  $Nsg$  -closed in  $(V, \tau_{R'}(Y))$  for every nano closed set  $A$  in  $(U, \tau_R(X))$ . Hence the function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$  -closed function.

**(iii)⇒(i):** Let  $F$  be a nano open set in  $(U, \tau_R(X))$ . Then  $F^C$  is nano closed set in  $(U, \tau_R(X))$ . By the given hypothesis,  $f(F^C)$  is  $Nsg$  -closed in  $(V, \tau_{R'}(Y))$ . Now,  $f(F^C) = (f(F))^C$  is  $Nsg$  -closed, i.e.,  $f(F)$  is  $Nsg$  -open in  $(V, \tau_{R'}(Y))$  for every nano open set  $F$  in  $(U, \tau_R(X))$ . Hence  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$  -open function.

The composition of two  $Nsg$  -homeomorphisms need not always be a  $Nsg$  -homeomorphism as seen from the following example.

**Example 4.8:** Let  $(U, \tau_R(X))$ ,  $(V, \tau_{R'}(Y))$  and  $(W, \tau_{R''}(Z))$  be three nano topological spaces and let  $U = V = W = \{a, b, c, d\}$ , then the nano open sets are  $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$ ,  $\tau_{R'}(Y) = \{V, \phi, \{b\}, \{a, b, c\}, \{a, c\}\}$  and  $\tau_{R''}(Z) = \{W, \phi, \{c\}, \{a, b, c\}, \{a, b\}\}$ . Define two functions  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  and  $g : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R''}(Z))$  as  $f(a) = c, f(b) = b, f(c) = a, f(d) = d$  and  $g(a) = a, g(b) = b, g(c) = c, g(d) = d$ . Here the functions  $f$  and  $g$  are  $Nsg$  -continuous and bijective. Also the image of every nano open set in  $(U, \tau_R(X))$  is  $Nsg$  -open in  $(V, \tau_{R'}(Y))$ . i.e.,  $f(\{a, b, d\}) = \{a, b, c\}$ ,  $f(\{b, d\}) = \{a, c\}$ ,  $f(\{a\}) = \{b\}$ . Thus the function  $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is  $Nsg$  -open and thus  $Nsg$  -homeomorphism. The map  $g : (V, \tau_{R'}(Y)) \rightarrow (W, \tau_{R''}(Z))$  is also  $Nsg$  -continuous, bijective and  $Nsg$  -open. Hence  $g$  is also  $Nsg$  -homeomorphism. But their composition  $g \circ f : (U, \tau_R(X)) \rightarrow (W, \tau_{R''}(Z))$  is not a  $Nsg$  -homeomorphism because for the nano open set  $F = \{a, b\}$  in  $(W, \tau_{R''}(Z))$ ,  $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(\{a, b\})) = f^{-1}(\{a, b\}) = \{a, d\}$  is not  $Nsg$  -open in  $(U, \tau_R(X))$ . Hence the composition  $g \circ f : (U, \tau_R(X)) \rightarrow (W, \tau_{R''}(Z))$  is not  $Nsg$  -continuous and thus not a  $Nsg$  -homeomorphism. Thus the composition of two  $Nsg$  -homeomorphisms need not be a  $Nsg$  -homeomorphism.

## REFERENCES

1. K.Bhuvaneswari and A.Ezhilarasi, On Nano semi-generalized and Nano generalized-semi closed sets in Nano Topological Spaces, International Journal of Mathematics and Computer Applications Research, (2014), 117-124.
2. K.Bhuvaneswari and A.Ezhilarasi, On Nano Semi-Generalized Continuous Maps in Nano Topological Spaces, International Research Journal of Pure Algebra – 5(9), 2015, 149 – 155.
3. K.Bhuvaneswari and K. Mythili Gnanapriya, 2014. Nano generalized closed sets in nano topological spaces. International Journal of Scientific and Research Publications, 4(5): 1 – 3.
4. K.Bhuvaneswari and K. Mythili Gnanapriya, 2015. On Nano generalized continuous function in nano topological space. International Journal of mathematical Archive, 6(6): 182 – 186.

5. S.G. Crossley and S. K. Hilderbrand, 1972. Semi-topological properties. Fund. Math., 74: 233 – 254.
6. R.Devi, K. Balachandran and H. Maki, 1995. Semi-generalized homeomorphisms and generalized-semi homeomorphisms in topological spaces. Indian J. Pure Appl. Math., 26: 271 – 284.
7. R.Devi, H. Maki and K. Balachandran, 1993. Semi-generalized closed maps and generalized-semi closed maps. Mem. Fac. Kochi Univ. Ser. A. Math., 14: 41 – 54.
8. M.Lellis Thivagar, and Carmel Richard, On Nano Forms of Weakly Open sets, International Journal Mathematics and Statistics Invention, (2013),31-37.
9. M.Lellis Thivagar, and Carmel Richard, On Nano Continuity, Mathematical Theory and Modeling, (2013), no.7, 32-37.
10. H.Maki, P. Sundaram and K. Balachandran, 1991. On generalized homeomorphisms in topological spaces. Bull. Fukuoka Univ. Ed. Part III, 40: 13 – 21.
11. A.S.Mashhour, M.E. Abd. El – Monsef and S.N. El – Deeb, 1982. On pre – continuous mappings and weak pre-continuous mappings. Proc. Math. Phys. Soc. Egypt., 53: 47 – 53.
12. Miguel Caldas Cueva, 1995. Semi-generalized continuous maps in topological spaces. Portugaliae. Mathematica., 52 : 399 – 407.
13. Sundaram P., H. Maki and K. Balachandran, 1991. Semi – generalized continuous maps and semi- $T_{1/2}$  spaces. Bull. Fukuoka Univ. Ed., Part III, 40: 33 – 40.
14. P.Sundaram, 1991. Studies on generalizations of continuous maps in topological spaces. Ph.D., Thesis, Bharathiar University, Coimbatore.

**Source of support: Nil, Conflict of interest: None Declared**

**[Copy right © 2016. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]**