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NANO SEMI–GENERALIZED HOMEOMORPHISMS IN NANO TOPOLOGICAL SPACE

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ABSTRACT

In this paper, we introduce nano sg-closed maps and nano sg – open maps in nano topological spaces and then we introduce and study nano sg - homeomorphisms. We obtain certain characterizations of these maps.

Keywords: *Nano closed map, Nano open map, Nano sg – irresolute, Nano sg-closed map, Nano sg-open map, nano sg-homeomorphism.*

1. INTRODUCTION

The concepts of semi–generalized mappings and generalized- semi mappings were introduced by R.Devi *et.al* [7] in 1993.Maki *et.al* [10] introduced g-homeomorphisms and gc-homeomorphisms in topological spaces. Devi [7] introduced and studied sg- closed functions and gs-closed functions. A study on semi-generalized homeomorphism was done by Devi *et. al* [6]. The concept of nano topology was introduced by Lellis Thivagar [7] and he analysed nano closed maps, nano open maps and nano homeomorphisms. In this paper, nano generalized–semi closed functions, nano semi-generalized open functions are introduced and nano semi-generalized homeomorphisms are analysed.

2. PREMILINARIES

Definition 2.1: [11] A function $f:(X,\tau)\to(Y,\sigma)$ is called semi-open if the image of every open set in (X,τ) is semi-open in (Y,σ).

Definition 2.2: [13] A function $f:(X,\tau)\to(Y,\sigma)$ is called g-open if the image of every open set in (X,τ) is g-open in (Y,σ).

Definition 2.3: [5] A function $f:(X,\tau)\to(Y,\sigma)$ is called pre-semi open if the image of every semi–open set in (X,τ) is semi-open in (Y,σ).

Definition 2.4: [12] A function $f:(X,\tau)\to(Y,\sigma)$ is called sg-closed if the image of every closed set in (X,τ) is sg–closed in (Y,σ).

Definition: 2.5: [13] A function $f:(X,\tau)\to(Y,\sigma)$ is semi-generalized continuous (sg-continuous) if f^{-1}(V) is sg-closed set in X for every closed set V of Y.

Definition 2.6: [10] A bijective function $f:(X,\tau)\to(Y,\sigma)$ is called a generalized homeomorphism if f is both g-continuous and g-open.

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Definition 2.7: [6] A bijective function $f:(X,\tau)\rightarrow(Y,\sigma)$ is called a semi-generalized homeomorphism if f is both sg-continuous and sg-open.

Definition 2.8: [8] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆ U. Then,
(i) The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. $L_R(X) = U\{R(x): R(x) \subseteq X, x\epsilon U\}$ where R(x) denotes the equivalence class determined by xϵU.

(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. $U_R(X) = U\{R(x): R(x) \cap X \neq \Phi, x\epsilon U\}$.

(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not–X with respect to R and it is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.

Property 2.9 [8]: If (U, R) is an approximation space and X, Y⊆U, then

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\Phi) = U_R(\Phi) = \Phi$
3. $L_R(U) = U_R(U) = U$
4. $U_R(X\cup Y) = U_R(X) \cup U_R(Y)$
5. $U_R(X\cap Y) \subseteq U_R(X) \cap U_R(Y)$
6. $L_R(X\cup Y) \supseteq L_R(X) \cup L_R(Y)$
7. $L_R(X\cap Y) = L_R(X) \cap L_R(Y)$
8. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
9. 9. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
10. $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
11. $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$.

Definition 2.10 [8]: Let U be the universe, R be an equivalence relation on U and the nano topology $\tau_R(X) = \{U, \Phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 2.5, $\tau_R(X)$ satisfies the following axioms:

(i) U and $\Phi \in \tau_R(X)$.
(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. (U, $\tau_R(X)$) is called the Nano topological space. Elements of the Nano topology are known as nano open sets in U. Elements of $[\tau_R(X)]^c$ are called nano closed sets with $[\tau_R(X)]^c$ being called Dual Nano topology of $\tau_R(X)$. If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set B = $\{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.11 [8]: If $(U,\tau_R(X))$ is a Nano topological space with respect to X where X ⊆ U and if A⊆ U, then

(i) The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by NInt(A). NInt(A) is the largest nano open subset of A.
(ii) The nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by NCl(A). NCl(A) is the smallest nano closed set containing A.

Remark 2.12 [8]: Throughout this paper, U and V are non-empty, finite universes; X ⊆ U and Y ⊆ V; U/R and V/R' denote the families of equivalence classes by equivalence relations R and R' respectively on U and V. $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ are the Nano topological spaces with respect to X and Y respectively.

Definition 2.13 [1]: If $(U,\tau_R(X))$ is a Nano topological space with respect to X where X ⊆U and if A ⊆ U, then
(i) The nano semi-closure of A is defined as the intersection of all nano semi-closed sets containing A and is denoted by NsCl(A). NsCl(A) is the smallest nano semi-closed set containing A and NsCl(A) ⊆A.

(ii) The nano semi-interior of A is defined as the union of all nano semi-open subsets of A and is denoted by NsInt(A). NsInt(A) is the largest nano semi open subset of A and NsInt(A) ⊆A.

Definition 2.14 [1]: A subset A of $(U,\tau_R(X))$ is called nano semi-generalized closed set (Nsg-closed) if NSCl(A)$\subseteq$ V and A $\subseteq$V and V is nano semi-open in $(U,\tau_R(X))$.

Definition 2.15 [1]: If $(U, \tau_R(X))$ is a nano topological space with respect to X where X ⊆ U and if A ⊆ U, then

(i) The nano semi- generalized closure of A is defined as the intersection of all nano semi-generalized closed sets containing A and is denoted by NsgCl(A).

(ii) The nano semi-generalized interior of A is defined as the union of all nano semi-generalized open subsets of A and is denoted by NsgInt(A).

Definition 2.16 [9]: Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano continuous on U if the inverse image of every nano open set in V is nano open in U.

Definition 2.17[9]: A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called nano open if the image of every nano open set in $(U, \tau_R(X))$ is nano open in $(V, \tau_{R'}(Y))$.

Definition 2.18 [9]: A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called nano closed if the image of every nano closed set in $(U, \tau_R(X))$ is nano closed in $(V, \tau_{R'}(Y))$.

Definition 2.19 [4]: A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called nano g–closed if the image of every nano g–closed set in $(U, \tau_R(X))$ is nano g–closed in $(V, \tau_{R'}(Y))$.

Definition 2.20 [4]: A mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano semi – generalized continuous on U if the inverse image of every nano open set in V is nano sg–open in U.

Definition 2.21 [9]: A bijection $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called nano homeomorphism if f is both nano continuous and nano open.

Definition 2.22 [4]: A bijection $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called nano g–homeomorphism if f is both nano g–continuous and nano g–open.

3. NANO SG-CLOSED MAPS

In this section, nano semi-generalized closed functions and nano semi-generalized open functions in nano topological spaces are introduced. We discuss certain characterizations of these functions.

Definition 3.1: Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be the two nano topological spaces. Then a function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be nano semi–generalized closed function (briefly Nsg –closed function) if the image of every nano closed set in $(U, \tau_R(X))$ is Nsg –closed in $(V, \tau_{R'}(Y))$.

Example 3.2: Let $U = \{a,b,c,d\}$ be the universe with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and let $X = \{a,b\} \subseteq U$. Then the nano open sets are $\tau_R(X) = \{U, \phi, \{a\}, \{a,b,d\}, \{b,d\}\}$ and the nano closed sets are $\tau_R{}^C(X) = \{U, \phi, \{a\}, \{a,b,d\}, \{b,d\}\}$. $\{U, \phi, \{a\}, \{a,c\}, \{b,d\}, \{a,b,d\}\}$ are the nano semi–open sets. Nsg –open sets are $\{U, \phi, \{a\}, \{b\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$. Nsg –closed sets are $\{U, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{b,c\}, \{c,d\}, \{a,c\}, \{b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c\}\}$. Let $V = \{x,y,z,w\}$ be the anotheruniverse with $V/R' = \{\{x\},\{w\},\{y,z\}\}$ and let $Y = \{x,z\} \subseteq V$. Then the nano open sets are $\tau_{R'}(Y) = \{V, \phi, \{x\}, \{x,y,z\}, \{y,z\}\}$ and the nano closed sets are $\tau_{R'}{}^C(Y) = \{V, \phi, \{w\}, \{x,w\}, \{y,z,w\}\}$ $\{V, \phi, \{x\}, \{y\}, \{z\}, \{x,y\}, \{x,z\}, \{x,w\}, \{y,z\}, \{x,y,z\}, \{x,y,w\}, \{y,z,w\}, \{x,z,w\}\}$ are the Nsg –open sets. Nsg –closed sets are $\{V, \phi, \{x\}, \{y\}, \{z\}, \{w\}, \{y,w\}, \{y,z\}, \{x,w\}, \{z,w\}, \{y,z,w\}, \{x,y,w\}, \{x,z,w\}\}$. Define the function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as $f(a) = y, f(b) = x, f(c) = w, f(d) = z$ .Now $f(U) = V, f(\phi) = \phi, f(\{b,c,d\}) = \{x,w,z\}, f(\{c\}) = \{w\}, f(\{a,c\}) = \{y,w\}$ are the images of nano closed sets of U which are Nsg –closed in V. Thus the function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is a Nsg – closed function.

Theorem 3.3: A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg –closed function if and only if $NsgClf(A))\subseteq f(NCl(A))$ for every subset A of $(U,\tau_R(X))$.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a Nsg –closed function and $A\subseteq U$. Then $NCl(A)$ is nano closed in $(U,\tau_R(X))$ and hence $f(NCl(A))$ is Nsg –closed function in $(V,\tau_{R'}(Y))$. Since $A\subseteq NCl(A)$, it implies that $f(A)\subseteq f(NCl(A))$. As $NsgCl(f(NCl(A))$ is the Nsg –closed set containing $f(A)$, it follows that $NsgCl(f(A))\subseteq NsgCl(f(NCl(A)))=f(NCl(A))$.

Conversely, let A be any nano closed set in $(U,\tau_R(X))$. Then $A=NCl(A)$ and so $f(A)=f(NCl(A))\supseteq NsgCl(f(A))$ by the given hypothesis. Also, it follows that $f(A)\subseteq NsgCl(f(A))$. Hence $f(A)=NsgCl(f(A))$. i.e., $f(A)$ is Nsg –closed and hence $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is a Nsg –closed function.

Theorem 3.4: A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg –closed if and only if for each subset S of $(V,\tau_{R'}(Y))$ and for each nano open set A of $(U,\tau_R(X))$ containing $f^{-1}(S)$, there is a Nsg –open set B of $(V,\tau_{R'}(Y))$ such that $S\subseteq B$ and $f^{-1}(B)\subseteq A$.

Proof: Let S be the subset of $(V,\tau_{R'}(Y))$ and A be a nano open set of $(U,\tau_R(X))$ such that $f^{-1}(S)\subset A$. Now $V-f(U-A)$, say B, is a Nsg –open set containing S in V such that $f^{-1}(B)\subseteq A$.

Conversely, let F be a nano closed set of $(U,\tau_R(X))$, then $f^{-1}(V-f(F))\subset U-F$ and $U-F$ is nano open. Now, there is a Nsg –open set B of $(V,\tau_{R'}(Y))$ such that $V-f(F)\subset B$ and $f^{-1}(B)\subset U-F$. Hence $F\subset U-f^{-1}(B)$ and thus $V-B\subset f(F)\subset f(U-f^{-1}(B))\subset V-B$ which implies $f(F)=V-B$. Since $V-B$ is Nsg –closed, $f(F)$ is a Nsg –closed set in $(V,\tau_{R'}(Y))$ for every nano closed set F in $(U,\tau_R(X))$. Hence $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is a Nsg –closed function.

The following example shows that the composition of two Nsg –closed functions need not be Nsg –closed.

Example 3.5: Let $(U,\tau_R(X))$, $(V,\tau_{R'}(Y))$ and $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be as in Example 3.2. Let $W=\{p,q,r,s\}$ with $W/R''=\{\{p\},\{q\},\{r,s\}\}$. Let $Z=\{p,r\}\subseteq W$. The nano open sets are $\tau_{R''}(Z)=\{W,\varphi,\{p\},\{p,r,s\},\{r,s\}\}$ and nano closed sets are $\tau_{R''}{}^{C}(Z)=\{W,\phi,\{q,r,s\},\{q\},\{p,q\}\}$. The Nsg –closed sets are $\{W,\phi,\{p\},\{r\},\{q\},\{s\},\{p,q\},\{r,q\},\{r,s\},\{q,s\},\{p,r,q\},\{p,q,s\},\{q,r,s\}\}$. Define a map $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ as $g(x)=r, g(y)=p, g(z)=q, g(w)=s$. Then both f and g are Nsg –closed maps but their composition $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is not a Nsg –closed map since for the nano closed set $\{a,c\}$ in $(U,\tau_R(X))$, $(g\circ f)(\{a,c\})=g[f(\{a,c\})]=g(\{y,w\})=\{p,s\}$ which is not a Nsg –closed set in $(W,\tau_{R''}(Z))$. Thus the composition of two Nsg –closed maps need not be Nsg –closed.

Theorem 3.6: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ and $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ be two mappings such that their composition $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is a Nsg –closed mapping. Then the following statements are true.

(i) If f is nano continuous and surjective, then g is Nsg –closed.
(ii) If g is Nsg –irresolute and injective, then f is Nsg –closed.
(iii) If g is strongly Nsg –continuous and injective, then f is nano closed.

Proof:

(i) Let A be a nano closed set in $(V,\tau_{R'}(Y))$. Since $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano continuous, it follows that $f^{-1}(A)$ is nano closed in $(U,\tau_R(X))$. Since $g\circ f:(U,\tau_{R'}(X))\to(W,\tau_{R''}(Z))$ is Nsg –closed, $(g\circ f)[f^{-1}(A)]$ is Nsg –closed in $(W,\tau_{R''}(Z))$. i.e., $g(A)$ is Nsg –closed in $(W,\tau_{R''}(Z))$ as the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is surjective. Hence the image of a nano closed set in $(V,\tau_{R'}(Y))$ is Nsg –closed in $(W,\tau_{R''}(Z))$ .Thus $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is Nsg –closed.

(ii) Let B be a nano closed set in $(U,\tau_R(X))$. Since the function $g\circ f:(U,\tau_{R'}(X))\to(W,\tau_{R''}(Z))$ is Nsg –closed, then $(g\circ f)(B)$ is Nsg –closed in $(W,\tau_{R''}(Z))$. Since g is Nsg –irresolute, $g^{-1}[(g\circ f)(B)]$ is Nsg –closed in $(V,\tau_{R'}(Y))$ i.e., the set $f(B)$ is Nsg – closed in $(V,\tau_{R'}(Y))$ since the function $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is injective. Thus the image of a nano closed set in $(U,\tau_R(X))$ is Nsg –closed in $(V,\tau_{R'}(Y))$. Thus $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg –closed.

(iii) Let D be a nano closed set in $(U,\tau_R(X))$. Since the function $g\circ f:(U,\tau_{R'}(X))\to(W,\tau_{R''}(Z))$ is Nsg –closed in $(W,\tau_{R''}(Z))$, $(g\circ f)(D)$ is Nsg –closed in $(W,\tau_{R''}(Z))$. Since the function $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is strongly Nsg –continuous, $g^{-1}[(g\circ f)(D)]$ is nano closed in $(V,\tau_{R'}(Y))$. i.e., since the functions $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is injective, $f(D)$ is nano closed in $(V,\tau_{R'}(Y))$ for every nano closed set D in $(U,\tau_R(X))$. Thus $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano closed.

Definition 3.7: A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is said to be Nsg –open function if the image $f(A)$ is Nsg –open in $(V,\tau_{R'}(Y))$ for each nano open set A in $(U,\tau_R(X))$.

Definition 3.8: A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg –open function if and only if $f(NInt(A))\subseteq NsgInt(f(A))$ for each subset A of $(U,\tau_R(X))$.

Proof: Suppose that $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a Nsg –open function. Let $A\subseteq U$. Then $NInt(A)$ is nano open in $(U,\tau_R(X))$. Since $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg –open, $f(NInt(A))$ is Nsg –open in $(V,\tau_{R'}(Y))$. Also, as $NInt(A)\subseteq A$, it follows that $f(NInt(A))\subseteq f(A)$. Thus $f(NInt(A))$ is Nsg –open contained in $f(A)$. Hence $f(NInt(A))\subseteq NsgInt(f(A))$.

Conversely, let $f(NInt(A))\subseteq NsgInt(f(A))$ for every subset A of $(U,\tau_R(X))$. Let F be a nano open set in $(U,\tau_R(X))$, then $NInt(F)=F$. Now $f(F)\subseteq f(NInt(F))\subseteq NsgInt(f(F))=f(F)$. Hence $f(F)=NsgInt(f(F))$. Thus $f(F)$ is Nsg –open in $(V,\tau_{R'}(Y))$ for every nano open set F in $(U,\tau_R(X))$. Hence $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is a Nsg –open function.

4. NANO SG-HOMEOMORPHISMS

In this section, a new form of homeomorphisms namely, Nsg –homeomorphism is introduced and some of the properties are studied.

Definition 4.1 A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is said to be a Nsg –homeomorphism if

(i) f is one to one and onto

(ii) f is a Nsg –continuous

(iii) f is a Nsg –open

Theorem 4.2: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be an one to one onto mapping. Then f is Nsg –homeomorphism if and only if f is Nsg –closed and Nsg –continuous.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a Nsg–homeomorphism. Then f is Nsg–continuous. Let A be an arbitrary nano closed set in $(U,\tau_R(X))$. Then $U-A$ is nano open. Since f is Nsg–open, $f(U-A)$ is Nsg–open in $(V,\tau_{R'}(Y))$. That is, $V-f(A)$ is Nsg–open in $(V,\tau_{R'}(Y))$. Hence $f(A)$ is Nsg–closed in $(V,\tau_{R'}(Y))$ for every nano closed set A in $(U,\tau_R(X))$. Hence $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg–closed. Conversely, let f be Nsg–closed and Nsg–continuous function. Let G be a nano open set in $(U,\tau_R(X))$. Then $U-G$ is nano closed in $(U,\tau_R(X))$. Since f is Nsg–closed, $f(U-G)$ is Nsg–closed in $(V,\tau_{R'}(Y))$. That is, $f(U-G)=V-f(G)$ is Nsg–closed in $(V,\tau_{R'}(Y))$. Hence $f(G)$ is Nsg–open in $(V,\tau_{R'}(Y))$ for every nano open set G in $(U,\tau_R(X))$. Thus f is Nsg–open and hence $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg–homeomorphism.

Theorem 4.3: A one to one map f of $(U,\tau_R(X))$ onto $(V,\tau_{R'}(Y))$ is a Nsg–homeomorphism if and only if $f(NsgCl(A)=NCl(f(A))$ for every subset A of $(U,\tau_R(X))$.

Proof: If $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg–homeomorphism, then f is Nsg–continuous and Nsg–closed. If $A\subseteq U$, it follows that $f(NsgCl(A)\subseteq NCl(f(A))$ since f is Nsg–continuous. Since $NsgCl(A)$ is nano closed in $(U,\tau_R(X))$ and f is Nsg–closed map, $f(NsgCl(A)$ is Nsg–closed in $(V,\tau_{R'}(Y))$. Also $NsgCl(f(NsgCl(A)))=f(NsgCl(f(A)))$. Since $A\subseteq NsgCl(A)$, $f(A)\subseteq f(NsgCl(A))$ and hence $NCl(f(A))\subseteq NCl(f(NsgCl(A)))=f(NsgCl(A))$. Thus $NCl(f(A))\subseteq f(NsgCl(A))$. Therefore, $f(NsgCl(A)=NCl(f(A))$ if f is Nsg–homeomorphism.

Conversely, if $f(NsgCl(A)=NCl(f(A))$ for every subset A of $(U,\tau_R(X))$, then f is Nsg–continuous. If A is nano closed in $(U,\tau_R(X))$, then A is Nsg–closed in $(U,\tau_R(X))$. Then $NsgCl(A)=A$ which implies $f(NsgCl(A))=f(A)$. Hence, by the given hypothesis, it follows that $NCl(f(A))=f(A)$. Thus $f(A)$ is nano closed in V and hence Nsg–closed in V for every nano closed set A in U. That is, f is Nsg–closed. Thus $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg–homeomorphism.

Example 4.4: Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$ and let $X=\{a,b\}\subseteq U$. The nano open sets are $\tau_R(X)=\{U,\phi,\{a\},\{a,b,d\},\{b,d\}\}$. Let $V=\{x,y,z,w\}$ with $V/R'=\{\{x\},\{w\},\{y,z\}\}$ and let $Y=\{x,z\}\subseteq V$. The nano open sets are $\tau_{R'}(Y)=\{V,\phi,\{x\},\{x,y,z\},\{y,z\}\}$. Define a function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ as $f(a)=y,\ f(b)=x,\ f(c)=w,\ f(d)=z$ Then f is bijective, Nsg–continuous and Nsg–open and so the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg–homeomorphism.

Theorem 4.5: If a function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano homeomorphism, then it is Nsg–homeomorphism but not conversely.

Proof: As the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano homeomorphism, by definition, f is bijective, nano continuous and nano open. And hence $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg–continuous. Since $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano open, the image of every nano open set in $(U,\tau_R(X))$ is nano open in $(V,\tau_{R'}(Y))$ and hence Nsg–open in $(V,\tau_{R'}(Y))$ every nano open set is Nsg–open. Thus the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg–open. Therefore, every nano homeomorphism $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg–homeomorphism.

The converse of the Theorem 4.5 need not be true as seen from the following example.

Example 4.6: In Example 4.4, the map f is a Nsg–homeomorphism. Now $f^{-1}(V)=U$, $f^{-1}(\phi)=\phi$, $f^{-1}(x)=\{b\}$, $f^{-1}(\{y,z\})=\{a,d\}$, $f^{-1}(\{x,y,z\})=\{a,b,d\}$. Hence the inverse images of nano open sets in $(V,\tau_{R'}(Y))$ are not nano open in $(U,\tau_R(X))$ and hence f is not nano continuous. Thus the map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is not nano homeomorphism.

Theorem 4.7: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a Nsg –continuous function. Then the following statement are equivalent.

(i) f is a Nsg –open function
(ii) f is a Nsg –homeomorphism
(iii) f is a Nsg –closed function.

Proof:
(i)⇒(ii): By the given hypothesis, the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is bijective, Nsg –continuous and Nsg –open. Hence the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg –homeomorphism.

(ii)⇒(iii): By the given hypothesis, the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg –homeomorphism and hence Nsg –open. Let A be the nano closed set in $(U,\tau_R(X))$. Then A^C is nano open in $(U,\tau_R(X))$

By assumption, $f(A^C)$ is Nsg –open in $(V,\tau_{R'}(Y))$ .i.e., $f(A^C)=(f(A))^C$ is Nsg –open in $(V,\tau_{R'}(Y))$ and hence $f(A)$ is Nsg –closed in $(V,\tau_{R'}(Y))$ for every nano closed set A in $(U,\tau_R(X))$. Hence the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg –closed function.

(iii)⇒(i): Let F be a nano open set in $(U,\tau_R(X))$. Then F^C is nano closed set in $(U,\tau_R(X))$. By the given hypothesis, $f(F^C)$ is Nsg –closed in $(V,\tau_{R'}(Y))$. Now, $f(F^C)=(f(F))^C$ is Nsg –closed, i.e., $f(F)$ is Nsg –open in $(V,\tau_{R'}(Y))$ for every nano open set F in $(U,\tau_R(X))$. Hence $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg -open function.

The composition of two Nsg -homeomorphisms need not always be a Nsg -homeomorphism as seen from the following example.

Example 4.8: Let $(U,\tau_R(X))$, $(V,\tau_{R'}(Y))$ and $(W,\tau_{R''}(Z))$ be three nano topological spaces and let $U=V=W=\{a,b,c,d\}$, then the nano open sets are $\tau_R(X)=\{U,\phi,\{a\},\{a,b,d\},\{b,d\}\}$, $\tau_{R'}(Y)=\{V,\phi,\{b\},\{a,b,c\},\{a,c\}\}$ and $\tau_{R''}(Z)=\{W,\phi,\{c\},\{a,b,c\},\{a,b\}\}$. Define two functions $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ and $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ as $f(a)=c, f(b)=b, f(c)=a, f(d)=d$ and $g(a)=a, g(b)=b, g(c)=c, g(d)=d$. Here the functions f and g are Nsg –continuous and bijective. Also the image of every nano open set in $(U,\tau_R(X))$ is Nsg –open in $(V,\tau_{R'}(Y))$. i.e., $f(\{a,b,d\})=\{a,b,c\}$, $f(\{b,d\})=\{a,c\}$, $f(\{a\})=\{b\}$. Thus the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nsg –open and thus Nsg –homeomorphism. The map $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ is also Nsg –continuous, bijective and Nsg –open. Hence g is also Nsg –homeomorphism. But their composition $g\circ f:(U,\tau_{R'}(X))\to(W,\tau_{R''}(Z))$ is not a Nsg –homeomorphism because for the nano open set $F=\{a,b\}$ in $(W,\tau_{R''}(Z))$, $(g\circ f)^{-1}(F)=f^{-1}(g^{-1}(\{a,b\}))=f^{-1}(\{a,b\})=\{a,d\}$ is not Nsg –open in $(U,\tau_R(X))$. Hence the composition $g\circ f:(U,\tau_{R'}(X))\to(W,\tau_{R''}(Z))$ is not Nsg –continuous and thus not a Nsg -homeomorphism. Thus the composition of two Nsg –homeomorphisms need not be a Nsg -homeomorphism.

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