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MAXIMAL WEAKLY OPEN SETS IN FUZZY TOPOLOGICAL SPACES

SANJIVAPPA K DEMBARE*

Assistant professor, Department of Mathematics,
Government first Grade College, Navanagar, Bagalkot-587103, Karnataka, India.

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ABSTRACT

In this paper,maximal weakly open sets are introduced and characterized in fuzzy topological spaces. A non-empty fuzzy open set A of a fuzzy topological space (X, τ) is said to be a fuzzy maximal weakly open set if any fuzzy open set which is contained in A is either 1 or A.

Keywords: *Fuzzy set,Fuzzy open set,fuzzy closed set,Fuzzy minimal weakly open set.*

Mathematics subject classification (2000): *54A40 and 03E72.*

1. INTRODUCTION

In the year 2001 and 2003, F.Nakaoka and N.oda [1] [2] [3]introduced and studied minimal open (resp.minimal closed) sets which are sub classes of open (resp.closed) sets. The complements of minimal open sets and maximal open sets are called maximal closed sets and minimal closed sets respectively. In the year 2000 M. Sheik john [4] introduced and studied weakly closed sets and weakly open sets in topological spaces. In the year 2014 R.S.Wali and Vivekananda Dembre [5] [6] introduced and studied minimal weakly open sets and maximal weakly closed sets and maximal weakly open sets and minimal weakly closed sets in topological spaces. In the year 1965, L.A.Zadeh [7] introduced the concept of fuzzy subset as a generalization of that of an ordinary subset. The introduction of fuzzy subsets paved the way for rapid research work in many areas of mathematical science. In the year 1968, C.L.Chang [8] introduced the concept of fuzzy topological spaces as an application of fuzzy sets to topological spaces. Subsequently several researchers contributed to the development of the theory and applications of fuzzy topology. The theory of fuzzy topological spaces can be regarded as a generalization theory of topological spaces. An ordinary subset A or a set X can be characterized by a function called characteristic function $\mu_A : X \longrightarrow [0,1]$ of A, defined by

$$\mu_A(x) = \begin{cases} 1, & if \quad x \in A. \\ 0, & if \quad x \notin A. \end{cases}$$

Thus an element $x \in X$ isin A if μ_A (x) = 1 and is not in A if μ_A (x) = 0. In general if X is a set and A is a subset of X then A has the following representation. A = {(x ,μ_A (x)): x∈X}, here μ_A (x) may be regarded as the degree of belongingness of x to A, which is either 0 or 1. Hence A is the class of objects with degree of belongingness either 0 or 1 only. Prof. L.A.Zadeh [6] introduced a class of objects with continuous grades of belongingness ranging between 0 and 1; he called such a class as fuzzy subset. A fuzzy subset A in X is characterized as a membership function $\mu_A: X \rightarrow [0,1]$, which associates with each point in x a real number $\mu_A(x)$ between 0 and 1 which represents the degree or grade membership of belongingness of x to A.

1.1. Definition[1]: A proper non-empty open subset U of a topological space X is said to be minimal open set if any open set which is contained in U is φ or U.

1.2.Definition[2] :A proper non-empty open subset U of a topological space X is said to be maximal open set if any open set which is contained in U is X or U.

1.3.Definition[3]:A proper non-empty closed subset F of a topological space X is said to be minimal closed set if any closed set which is contained in F is φ or F.

*Corresponding Author: Sanjivappa K Dembare**
Assistant professor, Department of Mathematics,
Government first Grade College, Navanagar, Bagalkot-587103, Karnataka, India.

1.4.Definition[3]:A proper non-empty closed subset F of a topological space X is said to be maximal closed set if any closed set which is contained in F is X or F

1.5.Definition[4]:A subset A of (X,τ) iscalled weakly closed set if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.

1.6.Definition[4]: A subset A in(X,τ) is called weakly open set in X if A^c is weakly closed set in X.

1.7 Definition[5]: A proper non-empty weakly open subset U of X is said to be minimal weakly open set if any weakly open set which is contained in U is $\emptyset$ or U.

1.8 Definition[6]: A proper non-empty weakly closed subset U of X is said to be maximal weakly open set if any weakly open set which is contained in U is X or U.

1.9 Definition[7]: A fuzzy subset A in a set X is a function $A : X \to [0, 1]$. A fuzzy subset in X is empty iff its membership function is identically 0 on X andis denoted by 0 or μ_ϕ.The set X can be considered as a fuzzy subset of X whosemembership function is identically 1 on X and is denoted by $\mu_x$or I_x. In fact everysubset of X is a fuzzy subset of Xbut not conversely. Hence the concept of a fuzzysubset is a generalization of the concept of a subset.

1.10 Defnition[7]: If A and B are any two fuzzy subsets of a set X, thenA is said to be included in B or A is contained in B iff $A(x) \leq B(x)$ for all x in X. Equivalently, $A \leq B$ iff $A(x) \leq B(x)$ for all x in X.

1.11Defnition[7]: Two fuzzy subsets A and B are said to be equal if $A(x) = B(x)$ for every x in X. Equivalently $A = B$ if $A(x) = B(x)$ for every x in X.

1.12Defnition[7]: The complement of a fuzzy subset A in a set X, denoted by A' or $1 - A$, is the fuzzy subset of X defined by $A'(x) = 1 - A(x)$ for all x in X. Note that $(A')' = A$.

1.13 Defnition[7]: The union of two fuzzy subsets A and B in X, denoted by $A \vee B$, is a fuzzy subset in X defined by $(A \vee B)(x) = \mathrm{Max}\{\mu_A(x), \mu_B(X)\}$ for all x in X.

1.14 Defnition[7]: The intersection of two fuzzy subsets A and B in X, denoted by $A \wedge B$, is a fuzzy subset in X defined by $(A \wedge B)(x) = \mathrm{Min}\{A(x), B(x)\}$ for all x in X.

1.15 Defnition[7]: A fuzzy set on X is ‘Crisp‘ if it takes only the values 0 and 1 on X.

1.16 Defnition[7]: Let X be a set and be a family of fuzzy subsets of X. τ is called a fuzzy topology on X iff τ satisfies the following conditions.

(i) $\mu_\phi; \mu_X \in \tau$: That is 0 and $1 \in \tau$
(ii) If $G_i \in \tau$ for $i \in I$ then $\vee G_i \in \tau i \in I$
(iii) If $G,H \in \tau$ then $G \wedge H \in \tau$

The pair (X,τ) is called a fuzzy topological space (abbreviated as fts). The membersof $\tau$are called fuzzy open sets and a fuzzy set Ain X is said to be closed iff $1 - A$ is an fuzzy open set in X.

1.17 Remark[7]: Every topological space is a fuzzy topological space but not conversely.

1.18 Defnition[7]: Let X be a fts and A be a fuzzy subset in X. Then $\wedge$ {B: B is a closed fuzzy set in X and $B \geq A$} is called the closure of A and is denoted by A or cl(A).

1.19 Defnition[7]: Let A and B be two fuzzy sets in a fuzzy topological space (X,τ) and let $A \geq B$. Then B is called an interior fuzzy set of A if there exists $G \in \tau$ suchthat $A \geq G \geq B$, the least upper bound of all interior fuzzy sets of A is called theinterior of A and is denoted by A^0.

2.0 FUZZY MAXIMAL WEAKLY OPEN SETS IN FUZZY TOPOLOGICAL SPACES

2.1 Definition: A non-empty fuzzy open set A of a fuzzy topological space (X, τ) is said to be a fuzzy maximal weakly open set if any fuzzy open set which is contained in A is either 1 or A.

2.2 Lemma:

(i) If A is any fuzzy maximal weakly open set and α is a fuzzy open set, then $A \vee \alpha = 1$ or $\alpha \leq A$
(ii) If A and B are fuzzy minimal weakly open sets then $A \vee B = 1$ or $A = B$.

2.3 Theorem: If A,B and C are fuzzy maximal weakly open sets such that $A \neq B$ and if $A \wedge B < C$, then $A = C$ or $B = C$.

Proof: Let A,Band C be fuzzy maximal weakly open sets such that $A \neq B$ and $A \wedge B < C$. If $A = C$ then there is nothing to prove. But if $A \neq C$, then we have to prove that $B = C$.

Now, $B \wedge C = B \wedge (C \wedge 1)$
$= B \wedge [C \wedge (A \vee B)]$ by the Lemma 2.2 $A \vee B = 1$.
$= B \wedge [(C \wedge A) \vee (C \wedge B)]$
$= (B \wedge C \wedge A) \vee (B \wedge C \wedge B)$
$= (B \wedge C) \vee (B \wedge C)$, by hypothesis.
$= B \wedge (A \vee C) = B \wedge 1 = B$
$\Rightarrow B \wedge C = B \Rightarrow B \leq C$.

From the definition of fuzzy maximal weakly open sets, it follows that $B = C$.

2.4 Theorem: If A,B,C are fuzzy maximal weakly open sets which are different from each other, then $A \wedge B \nless A \wedge C$.

Proof: LetA, B and C be fuzzy maximal weakly open sets which are different from each other such that $A \wedge B < A \wedge C$ then we see that
$(A \wedge B) \vee (B \wedge C) < (A \wedge C) \vee (B \wedge C)$
$= (A \vee C) \wedge B < (A \vee B) \wedge C$
$= 1 \wedge B < 1 \wedge C$ (by theLemma2.2)
$= B < C$.

This shows that B=C from the definition of the fuzzy maximal weakly open sets, which contradicts the fact that $A \neq B \neq C$. Therefore $A \wedge B \nless A \wedge C$.

2.5 Theorem: Assume that $|\wedge| \geq 2$. If A_i is a fuzzy maximal weakly open set for any element i of $\wedge$ and $A_i \neq Aj$ for any elements i and j of $\wedge$with $i \neq j$, then

(i) $1 - \vee_{i \in \wedge\setminus[j]} A_i < Aj$ for some element j of $\wedge$.
(ii) $\vee_{i \in \wedge\setminus[j]} A_i \neq 0$.

2.6 Remark: If A_i is a fuzzy maximal weakly open set for any element i of $\wedge$ and $A_i \neq Aj$ for any elements i and j of $\wedge$ with $i \neq j$. If $|\wedge| \geq 3$,then $A_i \wedge Aj \neq 0$ for any elements i and j of $\wedge$ with $i \neq j$.

2.7 Theorem: If A_i is a fuzzy maximal weakly open set for any element i of $\wedge$ and $A_i \neq Aj$ for any elements i and j of $\wedge$ with $i \neq j$; If $|\wedge| \geq 2$, then $\wedge_{i \in \wedge\setminus[j]} A_i \nless Aj \nless \wedge_{i \in \wedge\setminus[j]} A_i$ for any element j of $\wedge$.

Proof: Let j be any element of $\wedge$ such that $\wedge_{i \in \wedge\setminus[j]} A_i Aj$ .Then $1 = 1 - \left(\wedge_{i \wedge\setminus[j]} A_i \wedge \wedge A_i <\right) \left(\underset{i \in \wedge\setminus[j]}{Aj}\right)$ by the theorem 2.5 which

implies that Aj,which is contradiction to our assumption.Therefore $\wedge_{i \in \wedge\setminus\{j\}} A_i \nless Aj$ (i)

Again let $A\, j\, (\wedge_{i \in \wedge\setminus[j]} A_i)$, then $Aj\ A_i$ for some element i of $\wedge$, which implies that $Aj < A_i$ by definition

of fuzzy maximal weakly open sets. This contradicts our assumption. Therefore $Aj \nless \wedge_{i \in \wedge\setminus[j]} A_i$ (ii)

From (i) and (ii) $\wedge_{i \in \wedge\setminus[j]} A_i \nless Aj \nless \wedge_{i \in \wedge\setminus[j]} A_i$.

2.8 Remark: If A_i is a fuzzy maximal weakly open set for any element i of $\wedge$ and $A_i \neq Aj$ for any elements i and j of $\wedge$ with $i \neq$ jand if Γ is a proper non-empty subset of $\wedge$ then
$\wedge_{i \in \wedge\setminus[j]} A_i \nless \wedge_{k \in \Gamma} A_k \nless \wedge_{i \in \wedge\setminus[j]} A_i$.

2.9 Theorem: (Decomposition Theorem) Assume that $|\wedge| \geq 2$;. If A_i is a fuzzy maximal weakly open set for any element i of ∧ and $A_i \neq Aj$ for any elements i and j of ∧ with i ≠j, then for any element j of ∧.
$A_i = (\wedge A_i) \vee (1 - (\wedge Ai)$
i∈∧i∈∧\{j}

Proof: Let j be any element of ∧ then

$(\wedge A_i) \vee [1 - (\wedge Ai)] = [(\wedge A_i)] \vee [1 - (\wedge A_i)]$
i∈∧ i∈∧\{j}i∈∧\{j}i∈∧\{j}

$= [(\wedge A_i) \vee 1 - (\wedge A_i)] \wedge [Aj \vee 1 - (\wedge A_i)]$
i∈∧\{j}i∈∧ {j}i∈∧\{j}

$=1 \wedge Aj$ this implies A_i for any element of ∧.

2.10 Remark: If A_i is a fuzzy maximal weakly open set for any element i of ∧ and $A_i \neq Aj$for any elements i and j of ∧ with i ≠j and if Γ is a proper non-empty subset of ∧, then $\wedge A_i < \wedge A_{k.}$ i∈∧≠ k∈Γ

2.11 Remark: Let A and B be any fuzzy subsets of X. If A ∨ B = 1; A∧ B is a fuzzy closed set and A is fuzzy open set then B is fuzzy closed set.

2.12 Remark: Assume that $|\wedge| \geq 2$. If A_i is a fuzzy maximal weakly open set for any element i of ∧ and $A_i \neq Aj$ for any elements i and j of ∧ with i ≠j and if $\wedge A_i = 1$ then $\{A_i \setminus i \in \wedge\}$ is the set of all fuzzy maximal weakly open sets of a fuzzy topology (X, τ).

2.13 Remark: If A_i is a fuzzy open set for any element i of ∧ and $A_i \vee Aj = 1$ for any elements i and j of ∧with i ≠j and if ∧Ai is a fuzzy closed set, then ∧Ai is a fuzzy closed set for any element i of ∧.

2.14 Remark: If Ai is a fuzzy maximal weakly open set for any element i of ∧ and Ai≠ Aj, for any elements i and j of ∧ with i ≠j and if ∧Ai is a fuzzy closed set, then Ai is a fuzzy closed set for any element i of ∧.

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