

ON REGULAR MILDLY GENERALIZED (RMG)-CLOSED SETS IN TOPOLOGICAL SPACES

\*R. S. WALI, <sup>1</sup>NIRANI LAXMI

\*Department of Mathematics,  
Bhandari & Rathi College, Guledagudd-587 203, Karnataka, India.

<sup>1</sup>Department of Mathematics,  
Rani Channamma University, Belagavi-591 156, Karnataka, India.

(Received On: 27-06-16; Revised & Accepted On: 20-07-16)

---

ABSTRACT

The aim of this paper is to introduce and study the class of Regular Mildly Generalized closed (briefly RMG-closed) sets, which is properly placed between the classes of pre-closed sets due to A.S. Mashhour, M.E. Abd. El-Monsef and S.N. El-Deeb in 1982 and mildly generalized closed sets due to J.K. Park and J.H. Park in 2004. The relations with other notations directly or indirectly connected with generalized closed sets are investigated. Also we investigate some properties of new class of sets.

**Mathematics Subject Classification 2000:** 54A05.

**Keywords:** Pre-closed sets, RMG-closed sets, mildly-g-closed sets, w-closed sets, rwg-closed sets,  $g\alpha$ -closed, rg-open sets.

---

1. INTRODUCTION

In 1970, N. Levine [11] introduced the concept of generalized closed sets in the topological space by comparing the closure of subset with its open supersets. The investigation on generalization of closed set has lead to significant contribution to the theory of separation axiom, covering properties and generalization of continuity. Kong *et. al.* [4] shown some of the properties of generalized closed set have been found to be useful in computer science and digital topology. A.S. Mashhour *et. al* [5], M.Sheik John[17], J.K. Park *et. al* [16], Benchalli and Wali[2] introduced pre-closed sets, weakly closed sets, mildly-g-closed sets and rw-closed sets in topological spaces respectively. In this paper we define new generalization of closed set called Regular Mildly Generalized closed (briefly RMG-closed) set which lies between pre-closed set and mildly-g-closed sets. We also study their some properties.

Throughout this paper  $X$  and  $Y$  denote the topological spaces  $(X, \tau)$  and  $(Y, \sigma)$  respectively and on which no separation axioms are assumed unless otherwise explicitly stated. For any subset  $A$  of a space  $(X, \tau)$ , the closure of  $A$ , interior of  $A$ , semi-interior of  $A$ , semi-closure of  $A$ ,  $\alpha$ -closure of  $A$ ,  $\alpha$ -interior of  $A$  and the complement of  $A$  are denoted by  $cl(A)$  or  $\tau-cl(A)$ ,  $int(A)$ ,  $sint(A)$ ,  $scl(A)$ ,  $\alpha-int(A)$ ,  $\alpha-cl(A)$  and  $A^c$  or  $X - A$  respectively.  $(X, \tau)$  will be replaced by  $X$  if there is no chance of confusion.

Let us recall the following definitions as pre requisites.

2. PRELIMINARIES

**Definition 2.1:** A subset  $A$  of  $X$  is called regular open (briefly r-open) [18] set if  $A = int(cl(A))$  and regular closed (briefly r-closed) [18] set if  $A = cl(int(A))$ .

**Definition 2.2:** A subset  $A$  of  $X$  is called pre-open set [5] if  $A \subseteq int(cl(A))$  and pre-closed [5] set if  $cl(int(A)) \subseteq A$ .

**Definition 2.3:** A subset  $A$  of  $X$  is called semi-open set [8] if  $A \subseteq cl(int(A))$  and semi-closed [8] set if  $int(cl(A)) \subseteq A$ .

---

**Corresponding Author:** \*R. S. Wali, <sup>1</sup>Nirani Laxmi, \*Department of Mathematics,  
Bhandari & Rathi College, Guledagudd-587 203, Karnataka, India.

**Definition 2.4:** A subset  $A$  of  $X$  is called  $\alpha$ -open [11] if  $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$  and  $\alpha$ -closed [11] if  $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$ .

**Definition 2.5:** A subset  $A$  of  $X$  is called  $\beta$ -open [1] if  $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$  and  $\beta$ -closed [1] if  $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$ .

**Definition 2.6:** A subset  $A$  of  $X$  is called  $\delta$ -closed [19] if  $A = \text{cl}_\delta(A)$ , where  $\text{cl}_\delta(A) = \{x \in X: \text{int}(\text{cl}(U)) \cap A \neq \emptyset, U \in \mathcal{A}\}$ .

**Definition 2.7:** Let  $X$  be a topological space. The finite union of regular open sets in  $X$  is said to be  $\pi$ -open [3]. The complement of a  $\pi$ -open set is said to be  $\pi$ -closed [3].

**Definition 2.8:** A subset of a topological space  $(X, \tau)$  is called

1. Generalized closed (briefly g-closed) [9] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
2. Generalized  $\alpha$ -closed (briefly  $g\alpha$ -closed) [6] if  $\alpha\text{-cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\alpha$ -open in  $X$ .
3. Weakly generalized closed (briefly wg-closed) [10] if  $\text{cl}(\text{int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
4. Strongly generalized closed (briefly  $g^*$ -closed) [15] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $X$ .
5. Weakly closed (briefly w-closed) [17] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open in  $X$ .
6. Mildly generalized closed (briefly mildly  $g$ -closed) [16] if  $\text{cl}(\text{int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $X$ .
7. Regular weakly generalized closed (briefly rwg-closed) [10] if  $\text{cl}(\text{int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open in  $X$ .
8. Weakly  $\pi$ -generalized closed (briefly  $w\pi g$ -closed)[13] if  $\text{cl}(\text{int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\pi$ -open in  $X$ .
9. Regular weakly closed (briefly rw-closed) [2] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular semiopen in  $X$ .
10. Generalized pre closed (briefly gp-closed) [7] if  $\text{pcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
11. A subset  $A$  of a space  $(X, \tau)$  is called regular generalized closed (briefly rg-closed) [14] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open set in  $X$ .
12.  $\pi$ -generalized closed (briefly  $\pi g$ -closed) [3] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .

The complements of the above mentioned closed sets are their respective open sets.

**Note:** The definition of Mildly- $g$ -closed sets in  $X$  is used in the investigation of  $wg^*$ -closed set by O. Ravi *et. al* [12].

**Lemma 2.9:**

1. Every  $wg^*$ -closed set (mildly- $g$ -closed set) is  $wg$ -closed set in  $X$  (Theorem 3.4 of [12]).
2. Every  $wg^*$ -closed set (mildly- $g$ -closed set) is  $w\pi g$ -closed set in  $X$  (Theorem 3.6 of [12]).
3. Every  $wg^*$ -closed set (mildly- $g$ -closed set) is  $rwg$ -closed set in  $X$  (Theorem 3.8 of [12]).

### 3. REGULAR MILDLY GENERALIZED CLOSED SETS (BRIEFLY RMG-CLOSED SETS)

**Definition 3.1:** A subset  $A$  of a space  $(X, \tau)$  is called Regular Mildly Generalized closed (briefly RMG-closed) set, if  $\text{cl}(\text{int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular generalized open set in  $X$ . The family of all RMG-closed sets is denoted by  $\text{RMGC}(X)$ .

**Theorem 3.2:** Every pre-closed set is RMG-closed set in  $X$ .

**Proof:** Let  $A$  be any pre-closed set in  $X$ . Suppose  $U$  is rg-open set in  $X$  such that  $A \subseteq U$ . Since  $A$  is pre-closed set in  $X$  i.e.  $\text{cl}(\text{int}(A)) \subseteq A$ . We have  $\text{cl}(\text{int}(A)) \subseteq A \subseteq U$  i.e.  $\text{cl}(\text{int}(A)) \subseteq U$ . Hence  $A$  is RMG-closed set in  $X$ .

The converse of above theorem need not be true as seen from the following example.

**Example 3.3:** Let  $X = \{a, b, c, d\}$  with topology  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ . Let  $A = \{a, b, d\}$  is RMG-closed set but not pre-closed set in  $X$ .

**Theorem 3.4:** Every RMG-closed set is mildly- $g$ -closed set in  $X$ .

**Proof:** Let  $A$  be any RMG-closed set in  $X$ . Suppose  $U$  is  $g$ -open set in  $X$  such that  $A \subseteq U$ . Since every  $g$ -open set is rg-open set in  $X$  i.e.  $U$  is rg-open in  $X$ . As  $A$  is RMG-closed and  $A \subseteq U$  where  $U$  is rg-open. Hence  $\text{cl}(\text{int}(A)) \subseteq U$ . Therefore  $A$  is mildly- $g$ -closed set in  $X$ .

The converse of above theorem need not be true as seen from the following example.

**Example 3.5:** Let  $X = \{a, b, c, d\}$  with topology  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ . Let  $A = \{a, d\}$  and  $B = \{b, d\}$  are mildly- $g$ -closed sets but not RMG-closed sets in  $X$ .

**Corollary 3.6:**

1. Every  $g\alpha$ -closed set is RMG-closed set in X.
2. Every w-closed set is RMG-closed set in X.
3. Every closed set is RMG-closed set in X.
4. Every  $\delta$ -closed set is RMG-closed set in X.
5. Every  $\pi$ -closed set is RMG-closed set in X.
6. Every regular closed set is RMG-closed set in X.

**Proof:**

1. Every  $g\alpha$ -closed set is pre-closed from M.Sheik John [17] and follows from Theorem 3.2.
2. Every w-closed set is pre-closed from M.Sheik John [17] and follows from Theorem 3.2.
3. Every closed set is pre-closed from M.Sheik John [17] and follows from Theorem 3.2.
4. Every  $\delta$ -closed set is pre-closed from N.V. Velicko *et. al* [20], M.Sheik John [17] and follows from Theorem 3.2.
5. Every  $\pi$ -closed set is pre-closed from Dontchev *et. al* [3], M.Sheik John [17] and follows from Theorem 3.2.
6. Every regular closed set is pre-closed from Stone [18], M.Sheik John [17] and follows from Theorem 3.2:

The converse of Corollary 3.6 is not true as shown in below examples.

**Example 3.7:** Let  $X = \{a, b, c, d\}$  with topology  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ .

1. Let  $A = \{a, b, d\}$  is RMG-closed but not  $g\alpha$ -closed set in X.
2. Let  $A = \{a, b, d\}$  is RMG-closed but not w-closed set in X.
3. Let  $A = \{a, b, d\}$  is RMG-closed but not closed set in X.
4. Let  $A = \{a, b, d\}$  is RMG-closed but not  $\delta$ -closed set in X.
5. Let  $A = \{a, b, d\}$  is RMG-closed but not  $\pi$ -closed set in X.
6. Let  $A = \{a, b, d\}$  is RMG-closed but not regular closed set in X.

**Corollary 3.8:**

1. Every RMG-closed set is wg-closed set in X
2. Every RMG-closed set is  $w\pi g$ -closed set in X.
3. Every RMG-closed set is rwg-closed set in X.

**Proof:**

1. From Theorem 3.4 and follows from lemma 2.9 1.
2. From Theorem 3.4 and follows from 2.9 2.
3. From Theorem 3.4 and follows from 2.9 3.

The converse of Remark 3.8 is not true as shown in below examples.

**Example 3.9:** Let  $X = \{a, b, c, d\}$  with topology  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ .

1. Let  $A = \{a, b, d\}$  is wg-closed but not RMG-closed set in X.
2. Let  $A = \{a, c\}$  is  $w\pi g$ -closed but not RMG-closed set in X.
3. Let  $A = \{a, b, c\}$  is rwg-closed but not RMG-closed set in X.

**Remark 3.10:** The following examples show that RMG-closed sets are independent of semi-closed sets, semi pre-closed sets, g-closed sets,  $g^*$ -closed sets, rg-closed sets, rw-closed sets and  $\pi g$ -closed sets.

**Example 3.11:** Let  $X = \{a, b, c, d\}$  with topology  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ . Then

1. Closed sets in  $(X, \tau)$  are  $X, \emptyset, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}$ .
2. RMG- closed sets in  $(X, \tau)$  are  $X, \emptyset, \{c\}, \{d\}, \{a, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$ .
3. Semi-closed sets in  $(X, \tau)$  are  $X, \emptyset, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}$ .
4. Semi pre- closed sets in  $(X, \tau)$  are  $X, \emptyset, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}$ .
5. g-closed set in  $(X, \tau)$  are  $X, \emptyset, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$ .
6.  $g^*$ -closed set in  $(X, \tau)$  are  $X, \emptyset, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$ .
7. rg-closed sets in  $(X, \tau)$  are  $X, \emptyset, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$ .
8. rw-closed sets in  $(X, \tau)$  are  $X, \emptyset, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$ .
9.  $\pi g$ -closed sets in  $(X, \tau)$  are  $X, \emptyset, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$ .

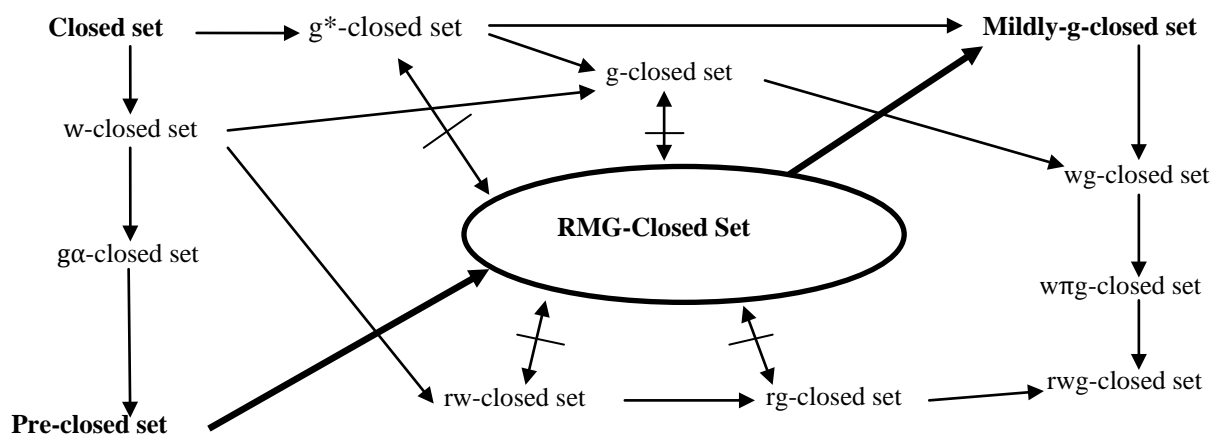
**Remark 3.12:** The following examples shows that RMG-closed sets are dependent on regular closed,  $\pi$ -closed, bw-closed,  $g\alpha$ - closed, wg-closed, rwg-closed sets.

**Examples 3.13:** Let  $X = \{a, b, c, d\}$  with topology  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ . Then

1. Closed sets in  $(X, \tau)$  are  $X, \emptyset, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}$ .
2. RMG-closed sets in  $(X, \tau)$  are  $X, \emptyset, \{c\}, \{d\}, \{a, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$ .
3. Regular closed sets in  $(X, \tau)$  are  $X, \emptyset, \{a, d\}, \{b, c, d\}$ .
4.  $\pi$ -closed sets in  $(X, \tau)$  are  $X, \emptyset, \{d\}, \{a, d\}, \{b, c, d\}$ .
5. w-closed sets in  $(X, \tau)$  are  $X, \emptyset, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}$ .
6.  $g\alpha$ -closed sets in  $(X, \tau)$  are  $X, \emptyset, \{c\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}$ .
7. wg-closed sets in  $(X, \tau)$  are  $X, \emptyset, \{c\}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$ .
8. rwg-closed sets in  $(X, \tau)$  are  $X, \emptyset, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$ .

**Remark 3.14:** From the above discussions and known results we have the following implications in the following diagram, by

$A \longrightarrow B$  mean A implies B but not conversely and  
 $A \longleftrightarrow B$  means A and B are independent of each other.



**Remark 3.15** The intersection of two RMG-closed sets in  $X$  is need not be RMG-closed set in  $X$ .

**Example 3.16:** Let  $X = \{a, b, c, d\}$  with topology  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ . Now  $A = \{a, b, d\}$  and  $B = \{b, c, d\}$  are RMG-closed sets in  $X$ , then  $A \cap B = \{a, b, d\} \cap \{b, c, d\} = \{b, d\}$  which is not RMG-closed set in  $X$ .

**Remark 3.17:** The union of two RMG-closed subsets of  $X$  is need not be RMG-closed set in  $X$ .

**Example 3.18:** Let  $X = \{a, b, c, d\}$  with topology  $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$ . Now  $A = \{b\}$  and  $B = \{c\}$  are RMG-closed sets in  $X$ , then  $A \cup B = \{b\} \cup \{c\} = \{b, c\}$  which is not RMG-closed set in  $X$ .

**Remark 3.19:** Complement of a RMG-closed set need not be RMG-closed set in  $X$ .

**Example 3.20:** Let  $X = \{a, b, c, d\}$  with topology  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ . Then  $A = \{d\}$  is RMG-closed set but  $X - \{d\} = \{a, b, c\}$  is not RMG-closed set in  $X$ .

**Theorem 3.21:** If a subset  $A$  of  $X$  is RMG-closed if and only if  $\text{cl}(\text{int}(A)) - A$  does not contain any non-empty regular generalized closed set in  $X$ .

**Proof:** Let  $F$  be a rg-closed set in  $X$  such that  $F \subseteq (\text{cl}(\text{int}(A)) - A)$ . Since  $X - F$  is rg-open and  $A \subseteq X - F$ . From the definition of 3.1 it follows that  $\text{cl}(\text{int}(A)) \subseteq X - F$ . i.e.  $F \subseteq (X - (\text{cl}(\text{int}(A))))$ . This implies that  $F \subseteq (\text{cl}(\text{int}(A))) \cap (X - (\text{cl}(\text{int}(A)))) = \emptyset$ . Thus  $F = \emptyset$ . Therefore  $\text{cl}(\text{int}(A)) - A$  does not contain any non-empty rg-closed set.

**Corollary 3.22:** If a subset  $A$  of  $X$  is RMG-closed set then  $\text{cl}(\text{int}(A)) - A$  doesnot contain any non-empty regular open set in  $X$ .

**Proof:** Follows from Theorem 3.21 and the fact that every closed set is rg-closed in  $X$ .

**Theorem 3.23:** For an element  $x \in X$ , then  $X - \{x\}$  is RMG-closed set or rg-open set in  $X$ .

**Proof:** Let  $x \in X$ . Suppose  $X - \{x\}$  is not a rg-open, then  $X$  is the only rg-open set containing  $X - \{x\}$  i.e. we have only choice of rg-open set containing  $X - \{x\}$  in  $X$  i.e.  $X - \{x\} \subset X$ . Also we know  $X - \{x\}$  is not RMG-closed. To prove  $X - \{x\}$  is not rg-open. As  $X - \{x\}$  is a subset  $X$  and  $X - \{x\}$  RMG-closed only. Thus we have only choice of rg-open set in  $X$ . Also  $\text{cl}(\text{int}(X - \{x\})) \subset X$ . Therefore by definition  $X - \{x\}$  is RMG-closed, which is a contradiction. Hence  $X - \{x\}$  is rg-open set in  $X$ .

**Theorem 3.24:** If  $A$  is an RMG-closed set in  $X$  such that  $A \subset B \subset \text{cl}(\text{int}(A))$ , then  $B$  is an RMG-closed set in  $X$ .

**Proof:** Let  $A$  be an RMG-closed set in  $X$  such that  $A \subset B \subset \text{cl}(\text{int}(A))$ . Let  $U$  be rg-open set such that  $B \subset U$  then  $A \subset U$ . Since  $A$  is RMG-closed set we have  $\text{cl}(\text{int}(A)) \subset U$ . Now as  $B \subset \text{cl}(\text{int}(A))$  so  $\text{cl}(\text{int}(B)) \subset \text{cl}(\text{int}(\text{cl}(\text{int}(A)))) \subset \text{cl}(\text{int}(A)) \subset U$ . Thus  $\text{cl}(\text{int}(B)) \subset U$  whenever  $B \subset U$  and  $U$  is rg-open in  $X$ . Hence  $B$  is RMG-closed set in  $X$ .

The converse of the above theorem 3.24 need not be true in general.

**Example 3.25:** Let  $X = \{a, b, c, d\}$  with topology  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ . Let  $A = \{d\}$  and  $B = \{c, d\}$ . Now  $A$  and  $B$  both are RMG-closed. But  $A \subset B \not\subset \text{cl}(\text{int}(A))$ .

**Lemma 3.26:** If  $A \subset B \subset X$ , where  $A$  is rg-open relative to  $B$  and  $B$  is open relative to  $X$  then  $A$  is rg-open relative to  $X$  [14].

**Theorem 3.27:** Let  $A \subset Y \subset X$  and  $A$  is RMG-closed set in  $X$ . Then  $A$  is RMG-closed in  $Y$  provided  $Y$  is open set in  $X$ .

**Proof:** Let  $A$  be an RMG-closed set in  $X$ . Now  $Y$  is an open subset in  $X$ . Let  $U$  be any rg-open set in  $Y$  such that  $A \subset U$ . Now  $A \subset U \subset Y \subset X$  by the lemma 3.27,  $U$  is rg-open in  $X$ . Since  $A$  is RMG-closed in  $X$ . So  $\text{cl}(\text{int}(A)) \subset U$ . As  $Y$  is open subspace of  $X$  then  $\text{cl}_Y(\text{int}_Y(A)) \subset \text{cl}(\text{int}(A)) \subset U$  i.e.  $\text{cl}_Y(\text{int}_Y(A)) \subset U$ . Hence  $A$  is RMG-closed set in  $Y$ .

**Theorem 3.28:** If  $A$  is both rg-open and RMG-closed in  $X$  then  $A$  is pre-closed set in  $X$ .

**Proof:** Suppose  $A$  is both rg-open and RMG-closed set in  $X$ . To prove that  $A$  is pre-closed in  $X$ . Suppose  $U$  is a rg-open set in  $X$  such that  $A = U$ . As  $A$  is RMG-closed,  $\text{cl}(\text{int}(A)) \subseteq U$  then  $\text{cl}(\text{int}(A)) \subseteq A$ . Thus is pre-closed set in  $X$ .

**Corollary 3.29:** If  $A$  is regular open and RMG-closed set in  $X$  then it is pre-closed set in  $X$ .

**Theorem 3.30:** If  $A$  is g-open and mildly-g-closed in  $X$  then  $A$  is RMG-closed set in  $X$ .

**Proof:** Let  $A$  is g-open and mildly-g-closed set in  $X$ . Suppose  $U$  is any rg-open set in  $X$  such that  $A \subseteq U$ . As  $A$  is mildly-g-closed and  $A$  is g-open,  $\text{cl}(\text{int}(A)) \subseteq A \subseteq U$ . Hence  $A$  is RMG-closed set in  $X$ .

**Theorem 3.31:** If  $A$  is both open and RMG-closed in  $X$  then  $A$  is closed set in  $X$ .

**Proof:** Let  $A$  is both open and RMG-closed in  $X$ . To prove  $A$  is closed in  $X$ . Now  $A \supseteq \text{cl}(\text{int}(A)) \supseteq \text{cl}(A)$  since  $A$  is open in  $X$  i.e.  $\text{cl}(A) \subseteq A$ . Also  $A \subseteq \text{cl}(A)$ . Hence  $A = \text{cl}(A)$ . Therefore  $A$  is closed set in  $X$ .

**Theorem 3.32:** Let  $A$  be a RMG-closed set in  $X$ . Then  $A$  is regular closed if and only if  $\text{cl}(\text{int}(A)) - A$  is rg-closed in  $X$ .

**Proof:** Let  $A$  is regular closed in  $X$ ,  $\text{cl}(\text{int}(A)) = A$ . Therefore  $\text{cl}(\text{int}(A)) - A = \emptyset$  which is rg-closed in every topological space  $X$ . Conversely, Suppose that  $\text{cl}(\text{int}(A)) - A$  is rg-closed. Since  $A$  is RMG-closed, then by Theorem 3.21,  $\text{cl}(\text{int}(A)) - A = \emptyset$  that is  $\text{cl}(\text{int}(A)) = A$ , Hence  $A$  is regular closed set in  $X$ .

**Theorem 3.33:** Suppose that  $B \subset A \subset X$ ,  $B$  is RMG-closed relative to  $A$  and  $A$  is both regular generalized open and RMG-closed subset of  $X$ , then  $B$  is RMG-closed set relative to  $X$ .

**Proof:** Let  $B \subset G$  and  $G$  be rg-open in  $X$ . Given  $B \subset A \subset X$ . This implies  $B \subset A \cap G$ . Since  $B$  is RMG-closed relative to  $A$ ,  $\text{cl}_A(\text{int}_A(B)) \subset A \cap G$ . Since  $\text{cl}_A(\text{int}_A(B)) = A \cap (\text{cl}(\text{int}(B)))$ , we have  $A \cap \text{cl}(\text{int}(B)) \subseteq A \cap G$ , from which we obtain  $A \subseteq G \cup (\text{cl}(\text{int}(B)))^c$ . Now  $G \cup (\text{cl}(\text{int}(B)))^c$  is rg-open in  $X$ . As  $A$  is RMG-closed,  $\text{cl}(\text{int}(A)) \subseteq G \cup (\text{cl}(\text{int}(B)))^c$ . Since  $\text{cl}(\text{int}(B)) \subseteq \text{cl}(\text{int}(A))$ ,  $\text{cl}(\text{int}(B)) \subseteq G \cup (\text{cl}(\text{int}(B)))^c$ . Therefore  $B$  is RMG-closed set relative to  $X$ .

**Theorem 3.34:** In a topological space  $X$ , if regular generalized open sets of  $X$  are  $\{X, \emptyset\}$ , then every subset of  $X$  is RMG-closed set.

**Proof:** Let  $X$  be any topological space and  $RGO(X) = \{X, \emptyset\}$ . Suppose  $A$  be any arbitrary subset of  $X$ , if  $A = \emptyset$  then  $X$  is the only rg-open set containing  $A$  and so  $cl(int(A)) \subset X$ . Hence by definition 3.1  $A$  is RMG-closed set in  $X$ .

**Remark 3.35:** The converse of the Theorem 3.34 need not be true in general.

**Example 3.36:** Let  $X = \{a, b, c, d\}$  with the topology  $\tau = \{X, \emptyset, \{a, b\}, \{c, d\}\}$ , then every subset of  $X$  is a RMG-closed set in  $X$ , but  $RGO(X) = P(X)$ .

**Definition 3.37:** The intersection of all rg-open subsets of space  $X$  containing  $A$  is called regular generalized kernel of  $A$  and is denoted as  $rgker(A)$ .

**Lemma 3.38:** Let  $X$  be a topological space and  $A$  is an rg-open subset of  $X$  then  $rgker(A) = A$ , but not conversely.

**Proof:** Follows from definition 3.37.

**Lemma 3.39:** For any subset  $A$  of Space  $X$ , then  $A \subset rgker(A)$ .

**Proof:** Follows from definition 3.37.

**Theorem 3.40:** A subset  $A$  is RMG-closed set in  $X$  if and only if  $cl(int(A)) \subset rgker(A)$ .

**Proof:** Let  $A$  be an RMG-closed,  $cl(int(A)) \subset U$ , whenever  $A \subset U$  and  $U$  is rg-open in  $X$ . Let  $x \in cl(int(A))$  and suppose  $x$  does not belong to  $rgker(X)$ , then a rg-open set  $U$  containing  $A$  such that  $x$  is not in  $U$ . Since  $A$  is RMG-closed we have  $x$  is not in  $cl(int(A))$ , which is contradiction. Hence  $x \in rgker(A)$  and so  $cl(int(A)) \subset rgker(A)$ .

Conversely, let  $cl(int(A)) \subset rgker(A)$ . If  $U$  is any rg-open set containing  $A$ , then  $rgker(A) \subset U$  i.e.  $cl(int(A)) \subset rgker(A) \subset U$ . Hence  $A$  is RMG-closed set in  $X$ .

**Theorem 3.41:** If a subset  $A$  of a topological space  $X$  is nowhere dense then it is RMG-closed.

**Proof:** Let a subset  $A$  of  $X$  is nowhere dense then  $cl(int(A)) = \emptyset$ , which is  $A \subset cl(A)$  and also  $A \subset int(cl(A))$ . As  $A$  is nowhere dense, we have  $int(A) = \emptyset$  implies that  $cl(int(A)) = \emptyset$ , therefore  $A$  is RMG-closed in  $X$ .

The converse of above Theorem 3.41 need not be true as shown from the example 3.42.

**Example 3.42:** Let  $X = \{a, b, c, d\}$  with the topology  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$  then subset  $A = \{b, c, d\}$  is RMG-closed set in  $X$  but not nowhere dense i.e.  $int(A) = \{b, c\} \neq \emptyset$ .

#### 4. CONCLUSION

In this article we have focused on regular mildly generalized closed sets in topological space which is lies between pre-closed set and mildly-g-closed set. We got some important conclusions; this new class has very similar properties as mildly-g-closed sets. In future it is useful to extend some more research works in different topological spaces.

#### REFERENCES

1. D. Andrijevic, Semi-preopen sets, Mat. Vesnik, 38(1986), 24-32.
2. S. S. Benchalli and R.S. Wali, On RW-closed sets in topological spaces, Bull.Malaysian.Math. Sci. Soc. (2) 30(2) (2007), 99-110.
3. J. Dontchev and T. Noiri, Quasi-normal spaces and  $\pi$ -g-closed sets, Acta Math. Hungar., 89(3)(2000), 211-219.
4. T.Kong, R. Kopperman and P. Meyer, A topological approach to digital topology, Amer. Math. Monthly, 98 (1991), 901-917.
5. A.S. Mashhour, M.E. Abd. El-Monsef and S.N. El-Deeb, On pre continuous mappings and weak pre-continuous mappings, Proc Math. Phys. Soc. Egypt, 53(1982), 47-53.
6. H. Maki, R. Devi and K. Balachandran, Associated topologies of generalized  $\alpha$ -closed sets and  $\alpha$ -generalized closed sets, Mem. Sci. Kochi Univ.Ser. A. Math., 15(1994), 51-63.
7. H.Maki, J.Umehara and T.Noiri, Every topological space is pre-T1/2, Mem.Fac. Sci. Kochi. Univ. Ser. A. Math., 17(1966), 33-42.
8. N.Levine, Semi-open sets and semi-continuity in topological spaces, Amer.Math. Monthly, 70(1963), 36-41.
9. N.Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19(1970), 89-96.

10. N. Nagaveni, Studies on Generalizations of Homeomorphisms in Topological Spaces, Ph.D.Thesis, Bharathiar University, Coimbatore, 1999.
11. O. Njastad, On some classes of nearly open sets, Pacific J. Math., 15(1965), 961-970.
12. O. Ravi<sup>1\*</sup>, I. Rajasekaran<sup>1</sup> and M. Sathyabama<sup>2</sup>, Weakly  $g^*$ -closed Sets, International Journal of Current Research in Science and Technology, Volume 1, Issue 5 (2015), 45-52.
13. O. Ravi, S. Ganesan and S. Chandrasekar, On weakly  $\pi g$ -closed sets in topological spaces, Italian Journal of Pure and Applied Mathematics (To Appear).
14. N. Palaniappan and K.C.Rao, Regular generalised closed sets, kyungpook math, J., 33(1993), 211-219.
15. A.Pushpalatha, Studies on generalizations of mappings in topological spaces, Ph.D., Thesis, Bharathiar University, coimbatore(2000).
16. J.K. Park and J.H. Park, Mildly generalized closed sets, almost normal and mildly normal spaces, Chaos, Solitions and Fractals, 20(2004), 1103- 1111.
17. M.Sheik John, On  $w$ -closed sets in topology, Acta Ciencia Indica, 4(2000),389-392.
18. M. Stone, Application of the theory of Boolean rings to general topology, Trans. Amer. Math.Soc., 41(1937), 374-481.
19. N.V. Velicko, H-closed Topological Spaces, Tran. Amer. Math. Soc., 78 (1968), 103-118.

**Source of support: Nil, Conflict of interest: None Declared**

***[Copy right © 2016. This is an Open Access article distributed under the terms of the International Journal of Mathematical Archive (IJMA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]***