International Journal of Mathematical Archive-7(7), 2016, 93-101
IJMA Available online through www.ijma.info ISSN 2229 – 5046

ON QUATERNION-k-NORMAL MATRICES

K. GUNASEKARAN, R. KAVITHA*

**Ramanujan Research Centre, PG and Research Department of Mathematics,
Government Arts College (Autonomous), Kumbakonam-612 002, Tamil Nadu, India.**

(Received On: 08-07-16; Revised & Accepted On: 28-07-16)

ABSTRACT

The concept of quaternion- k-normal (q-k-normal) matrices is introduced. Some basic theorems of q-k-normal and q-k-unitary matrices are discussed.

AMS Classifications: *15A09, 15A57, 15A24, 15A33, 15A15.*

Key words: *Quaternion matrix, q-k-normal matrix and q-k-unitary matrix.*

1. INTRODUCTION

Let H be the set of all quaternion numbers. Let $H_{n\times n}$ be the set of all quaternion square matrix over H called by quaternion matrix [6], for a matrix $A\in H_{n\times n}$. $\overline{A}, A^T, A^*, A^{-1}$ and $A^\dagger$ denotes conjugate, transpose, Conjugate transpose, inverse and Moore-Penrose inverse of A respectively. Let k be a fixed product of disjoint transpositions in $S_n=\{1,2,3...n\}$ and let K be the permutation matrix associated with k. The concept of q-k-normal matrices is introduced as generalization of q-k-real and q-k-hermitian and q-k-normal matrices [3, 5]. The q-k-unitary is also discussed in this paper. Clearly K satisfies $K^2=I$, $K=K^T=K^*=KI$.

2. DEFINITIONS AND SOME THEOREMS

Definition 2.1: A matrix $A\in H_{n\times n}$ is said to be quaternion-k-normal denoted by q-k-normal if $AKA^*K=KA^*KA$ where K is a permutation matrix associated with $k(x)$ in S_n.

Example 2.2: A = $\begin{bmatrix} 6+2i & 3 \\ 2 & 6+4i \end{bmatrix}$ is an q-k-normal matrix

Theorem 1: Let $A,B\in H_{n\times n}$. If A and B are q-k-normal with $AKB^*K=KB^*KA$ and $BKA^*K=KA^*KB$ then $A+B$ is q-k-normal.

Proof:

$$\begin{aligned}(A+B)\left[K(A+B)^*K\right] &= (A+B)K\left(A^*+B^*\right)K \\ &= (A+B)\left(KA^*K+KB^*K\right) \\ &= AKA^*K+AKB^*K+BKA^*K+BKB^*K \\ &= \left(KA^*K\right)A+\left(KB^*K\right)A+\left(KA^*K\right)B+\left(KB^*K\right)B\end{aligned}$$

Corresponding Author: R. Kavitha*
Ramanujan Research Centre, PG and Research Department of Mathematics,
Government Arts College (Autonomous), Kumbakonam-612 002, Tamil Nadu, India.

$$=\left(KA^{*}K+KB^{*}K\right)A+\left(KA^{*}K+KB^{*}K\right)B$$
$$=\left(KA^{*}K+KB^{*}K\right)(A+B)$$
$$=\left[K\left(A+B\right)^{*}K\right](A+B)$$
$$(A+B)\left[K\left(A+B\right)^{*}K\right]=\left[K\left(A+B\right)^{*}K\right](A+B)$$

Therefore $A+B$ is q-*k*-normal.

Theorem 2: If A is q-*k*-normal then A^{-1} is q-*k*-normal.

Proof:

$$A\left(KA^{*}K\right)=\left(KA^{*}K\right)A$$
$$\left(A^{-1}\right)K\left(A^{-1}\right)^{*}K=A^{-1}K\left(A^{*}\right)^{-1}K$$
$$=A^{-1}K^{-1}\left(A^{*}\right)^{-1}K^{-1}\left[\because K=K^{-1}\right]$$
$$=\left(AKA^{*}K\right)^{-1}$$
$$=K^{-1}\left(A^{*}\right)^{-1}K^{-1}A^{-1}$$
$$=K\left(A^{-1}\right)^{*}K\left(A^{-1}\right)$$
$$\left(A^{-1}\right)K\left(A^{-1}\right)^{*}K=K\left(A^{-1}\right)^{*}K\left(A^{-1}\right)$$

Therefore A^{-1} is q-*k*-normal.

Theorem 3: If A is q-*k*-normal then A^{T} is q-*k*-normal.

Proof:

$$A\left(KA^{*}K\right)=\left(KA^{*}K\right)A$$
$$A^{T}K\left(A^{T}\right)^{*}K=A^{T}K\left(A^{*}\right)^{T}K$$
$$=A^{T}K^{T}\left(A^{*}\right)^{T}K^{T}\left[\because K=K^{T}\right]$$
$$=\left(AKA^{*}K\right)^{T}$$
$$=K^{T}\left(A^{*}\right)^{T}K^{T}\left(A^{T}\right)$$
$$=K^{T}\left(A^{T}\right)^{*}K^{T}\left(A^{T}\right)$$
$$\left(A^{T}\right)K\left(A^{T}\right)^{*}K=K\left(A^{T}\right)^{*}K\left(A^{T}\right)$$

Therefore A^{T} is q-*k*-normal.

Theorem 4: If A is q-*k*-normal then A^{*} is q-*k*-normal.

Proof:

$$A\left(KA^{*}K\right)=\left(KA^{*}K\right)A$$
$$\left(A^{*}\right)K\left(A^{*}\right)^{*}K=\left(A^{*}\right)K^{*}\left(A^{*}\right)^{*}K^{*}\quad\left[\because K=K^{*}\right]$$
$$=\left(AKA^{*}K\right)^{*}$$
$$=K^{*}\left(A^{*}\right)^{*}K^{*}A^{*}$$

$$= K\left(A^*\right)^* K\left(A^*\right)$$

$$\left(A^*\right)K\left(A^*\right)^* K = K\left(A^*\right)^* K\left(A^*\right)$$

Therefore A^* is q-*k*-normal.

Theorem 5: If A is q-*k*-normal then A^2 is q-*k*-normal.

Proof:

$$A\left(KA^*K\right) = \left(KA^*K\right)A$$

$$A^2\left[K\left(A^2\right)^* K\right] = A^2K\left(A^*\right)^2 K$$

$$= AA\left(KA^*K\right)\left(KA^*K\right)$$

$$= A\left(KA^*K\right)A\left(KA^*K\right)$$

$$= \left(KA^*K\right)A\left(KA^*K\right)A$$

$$= \left(KA^*K\right)\left(KA^*K\right)AA$$

$$= \left[K\left(A^*\right)^2 K\right]\left(A^2\right)$$

$$= K\left(A^2\right)^* K\left(A^2\right)$$

$$\left(A^2\right)K\left(A^2\right)^* K = K\left(A^2\right)^* K\left(A^2\right)$$

Therefore A^2 is q- *k* -normal.

Theorem 6: If A is q-*k*-normal then A^t is q-*k*-normal.

Proof:

$$AKA^*K = KA^*KA$$

$$A^tK\left(A^t\right)^* K = \left(AAA....A\ t\ times\right)K\left(A^*A^*A^*...A^*\ t\ times\right)K$$

$$= AAA...A\left(KA^*K\right)\left(KA^*K\right)....\left(KA^*K\right)$$

Therefore A^t is q-*k*-normal.

Theorem 7: If A is q-*k*-normal then $A^\dagger$ is q-*k*-normal.

Proof:

$$A\left(KA^*K\right) = \left(KA^*K\right)A$$

$$\left(A^\dagger\right)K\left(A^\dagger\right)^* K = A^\dagger K^\dagger\left(A^*\right)^\dagger K^\dagger \quad \left[\because K = K^\dagger\right]$$

$$= \left(AKA^*K\right)^\dagger$$

$$= K^\dagger\left(A^*\right)^\dagger K^\dagger A^\dagger$$

$$= K\left(A^*\right)^\dagger KA^\dagger$$

$$= K\left(A^\dagger\right)^* K\left(A^\dagger\right)$$

$$\left(A^\dagger\right)K\left(A^\dagger\right)^* K = K\left(A^\dagger\right)^* K\left(A^\dagger\right)$$

Therefore $A^\dagger$ is q-*k*-normal.

Theorem 8: If A is q-k-normal then αA is q-k-normal.

Proof:

$$A\left(KA^{*}K\right)=\left(KA^{*}K\right)A$$

$$\begin{aligned}(\alpha A)K(\alpha A)^{*}K &= \alpha AK\overline{\alpha}A^{*}K\\ &=\alpha\overline{\alpha}\left(AKA^{*}K\right)\\ &=\alpha\overline{\alpha}\left(KA^{*}KA\right)\\ &=K\alpha\overline{\alpha}A^{*}KA\\ &=K\left(\overline{\alpha}A^{*}\right)K(\alpha A)\\ &=K(\alpha A)^{*}K(\alpha A)\end{aligned}$$

$$(\alpha A)K(\alpha A)^{*}K=K(\alpha A)^{*}K(\alpha A)$$

Therefore αA is q-k-normal.

Theorem 9: Let A and B are q-k-normal in $H_{n\times n}$ then AB is q-k-normal if $A\left(KB^{*}K\right)=\left(KB^{*}K\right)A$ and $B\left(KA^{*}K\right)=\left(KA^{*}K\right)B.$

Proof:

$$A\left(KA^{*}K\right)=\left(KA^{*}K\right)A$$

$$\begin{aligned}(AB)\left[K(AB)^{*}K\right] &= (AB)KB^{*}A^{*}K\\ &=(AB)\left(KB^{*}K\right)\left(KA^{*}K\right)\\ &=A\left(KB^{*}KB\right)KA^{*}K\\ &=\left(KB^{*}K\right)A\left(KA^{*}K\right)B\\ &=\left(KB^{*}K\right)\left(KA^{*}K\right)AB\\ &=\left(KB^{*}A^{*}K\right)(AB)\\ &=\left[K(AB)^{*}K\right](AB)\end{aligned}$$

$$(AB)\left[K(AB)^{*}K\right]==\left[K(AB)^{*}K\right](AB)$$

Therefore AB is q-k-normal.

Definition 2.3: A matrix $A\in H_{n\times n}$ is said to be quaternion-k-unitary (q-k-unitary) if $AKA^{*}K=KA^{*}KA=I.$

Example 2.4: A= $\begin{bmatrix}\frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}}\end{bmatrix}$ is an q-k-unitary matrix.

Definition 2.5: Let $A,B\in H_{n\times n}$. The matrix B is said to be quaternion-k-unitarily equivalent (q-k-unitarily equivalent) to A if there exists an q-k-unitary matrix U such that $B=KU^{*}KAU.$

Example 2.6: Let $A=\begin{bmatrix}1+i & 2i\\ 3+2i & 3\end{bmatrix}$ and $B=\begin{bmatrix}2+2i & 2+3i\\ -2+2i & -3+2i\end{bmatrix}$

Then if we take $U=\begin{bmatrix}\frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}}\end{bmatrix}$ it can be verified that $UKU^*K=KU^*KU=I$ and $B=KU^*KAU$

Hence B is q-𝓀-unitarily equivalent to A.

3. EQUIVALENT CONDITIONS ON Q-𝒦-NORMAL MATRICES

Theorem 3.1: Let $A\in H_{n\times n}$. If A is q-𝓀-unitarily equivalent to a diagonal matrix, then A is q-𝓀- normal.

Proof: Let $A\in H_{n\times n}$. If A is q-𝓀-unitarily equivalent to a diagonal matrix D, then there exists an q-𝓀-unitary matrix P such that $KP^*KAP=D$ Which implies that

$$\begin{aligned}&PKP^*KAPKP^*K=PDKP^*K\\ &A=PDKP^*K \quad \text{as } PKP^*K=I\\ &A^*=\left(PDKP^*K\right)^*\\ &\quad=K^*PK^*D^*P^*=KPKD^*P^*\end{aligned}$$

Now $$\begin{aligned}AKA^*K&=\left(PDKP^*K\right)K\left(KPKD^*P^*\right)K\\ &=PDKP^*K\left(KK\right)PKD^*P^*K\\ &=PDKP^*K\left(K^2\right)PKD^*P^*K \quad \left[\because K^2=I\right]\\ &=PD\left(KP^*KP\right)KD^*P^*K \qquad \left[\because KP^*KP=I\right]\\ &=PDIKD^*P^*K\\ &=PDKD^*P^*K\\ &=PD\left(KD^*K\right)\left(KP^*K\right)\end{aligned}$$

$$\begin{aligned}KA^*KA&=K\left(KPKD^*P^*\right)K\left(PDKP^*K\right)\\ &=K^2PKD^*\left(KK\right)P^*KPDKP^*K\\ &=PKD^*K\left(KP^*KP\right)DKP^*K\\ &=P\left(KD^*K\right)DKP^*K\\ &=P(KD^*K)D(KP^*K)\end{aligned}$$

Therefore D and KD^*K are each diagonal $D(KD^*K)=\left(KD^*K\right)D$ and hence $AKA^*K=KA^*KA$ so that A is q-𝓀-normal.

Theorem 3.2: If A is q-𝓀-hermitian then $A^{-1}KA^*K$ is q-𝓀-unitary.

Proof: $$\begin{aligned}A^{-1}\left(KA^*K\right)K\left(A^{-1}KA^*K\right)^*K&=A^{-1}AK\left(A^{-1}A\right)^*K\\ &=IK\ I^*K\\ &=I.\end{aligned}$$

Theorem 3.3: If A is q-*k*-normal then $A^{-1}KA^{*}K$ is q-*k*-unitary.

Proof: $\left(A^{-1}KA^{*}K\right)K\left(A^{-1}KA^{*}K\right)^{*}K = A^{-1}KA^{*}\left(KK\right)K^{*}\left(A^{*}\right)^{*}K^{*}\left(A^{-1}\right)^{*}K$

$$\begin{aligned}
&= A^{-1}\left(KA^{*}KA\right)K^{*}\left(A^{-1}\right)^{*}K \\
&= A^{-1}AKA^{*}KK\left(A^{-1}\right)^{*}K \\
&= KA^{*}\left(A^{-1}\right)^{*}K \\
&= K\left(A^{-1}A\right)^{*}K \\
&= K\left(I^{*}\right)K \\
&= I.
\end{aligned}$$

Remark 3.4: From, theorem 3.2 and 3.3 if A is either q-*k*-hermitian or q-*k*-normal then $A^{-1}KA^{*}K$ is q-*k*-unitary.

Theorem 3.5: Let $H, N \in H_{n\times n}$ be invertible. If $B = HNH$, where H is q-*k*-hermitian and N is q-*k*-normal then $B^{-1}KB^{*}K$ is similar to an q-*k*- unitary matrix.

Proof: Let $H, N \in H_{n\times n}$ then be invertible. If $B = HNH$ then

$$\begin{aligned}
B^{-1}KB^{*}K &= \left(HNH\right)^{-1}K\left(HNH\right)^{*}K \\
&= H^{-1}N^{-1}H^{-1}KH^{*}N^{*}H^{*}K \\
&= H^{-1}N^{-1}H^{-1}KH^{*}\left(KK\right)N^{*}H^{*}K \\
&= H^{-1}N^{-1}H^{-1}\left(KH^{*}K\right)KN^{*}H^{*}K \qquad [\because \text{H is q-}k\text{-hermitian}] \\
&= H^{-1}N^{-1}\left(H^{-1}H\right)KN^{*}H^{*}K \qquad \left[H = KH^{*}K\right] \\
&= H^{-1}N^{-1}KN^{*}H^{*}K \\
&= H^{-1}N^{-1}KN^{*}KKH^{*}K \\
&= H^{-1}N^{-1}\left(KN^{*}K\right)\left(KH^{*}K\right)
\end{aligned}$$

Since N is q-*k*- normal from remark 3.4 $N^{-1}KN^{*}K$ is q-*k*- unitary and hence the result follows.

Theorem 3.6: If A is q-*k*-normal and $AB = 0$ then $KA^{*}KB = 0$.

Proof:

$$\begin{aligned}
A\left(KA^{*}K\right) &= \left(KA^{*}K\right)A \\
A\left(KA^{*}K\right)B &= \left(KA^{*}K\right)AB \\
&= \left(KA^{*}K\right)0 \qquad [\because AB = 0] \\
&= 0 \\
A\left(KA^{*}K\right)B &= 0
\end{aligned}$$

Therefore $KA^{*}KB = 0 \quad [\because A \neq 0]$.

Theorem 3.7: If X is an q-*k*-eigenvector of an q-*k*-normal matrix A corresponding to an q-*k*-eigenvalue λ, then X is also an q-*k*-eigenvector of $KA^{*}K$ corresponding to the q-*k*-eigenvalue $\overline{\lambda}$.

Proof: Let $A \in H_{n\times n}$ be q-*k*-normal. Since X is an q-*k*-eigenvector of A corresponding to an q-*k*-eigenvalue λ, $AX = \lambda X$. Since A is q-*k*-normal, it can be easily seen that $A-\lambda I$ and $K\left(A-\lambda I\right)^* K$ commute and hence $A-\lambda I$ is q-*k*-normal. Now

$$AX = \lambda X \Rightarrow \left(A-\lambda I\right)X = 0.$$

Since $A-\lambda I$ is q- *k* - normal by the above theorem 3.6

$$\left[K\left(A-\lambda I\right)^* K\right]X = 0 \Rightarrow \left[K\left(A^* - \overline{\lambda} I\right)K\right]X = 0$$

$$\left[KA^*K - K\overline{\lambda}IK\right]X = 0$$

$$\left(KA^*K\right)X = K\overline{\lambda}KX$$

$$\left(KA^*K\right)X = \overline{\lambda}KKX$$

$$\left(KA^*K\right)X = \overline{\lambda}X$$

Which leads to the result.

Theorem 3.8: If $A \in H_{n\times n}$ is q-*k*-unitary and λ is an q-*k*-eigenvalue of A, then $|\lambda| = 1.$

Proof: Since $A \in H_{n\times n}$ is q-*k*-unitary, A is q- *k* - normal. Since λ is an q-*k*-eigenvalue of A, there exists an q-*k*-eigenvector $V \neq 0$ such that $AV = \lambda V$ which implies $KA^*KV = \overline{\lambda}V$ as A is q-*k*-normal.
Now $V = IV = KA^*KAV$ which leads to

$$V - \left[KA^*KA\right]V = 0$$

$$V\left[1 - KA^*KA\right] = 0$$

$$V\left[1 - AKA^*K\right] = 0$$

$$V\left[1 - \lambda\overline{\lambda}\right] = 0$$

Since $V \neq 0,\ 1 - \lambda\overline{\lambda} = 0$

$$\Rightarrow |\lambda| = 1.$$

Theorem 3.9: Let $A \in H_{n\times n}$. Assume that $A = XP$, where X is q-*k*-unitary and P is non –singular and q-*k*-hermition such that if P^2 commutes with X, then P also commutes with X. Then the following conditions are equivalent.

(i) A is q- *k*-normal
(ii) $XP = PX$
(iii) $AX = VX$
(iv) $AP = PA$

Proof: Let $A = XP$. Since X is q-*k*-unitary $XKX^*K = KX^*KX = I$ and since P is q-*k*-hermitian $KP^*K = P$.
$(i) \Leftrightarrow (ii)$ If A is q-*k*-normal then $AKA^*K = KA^*KA$

Since $A = XP$. $\left(XP\right)K\left(XP\right)^* K = K\left(XP\right)^* K\left(XP\right)$

$$XPKP^*X^*K = KP^*X^*KXP$$

$$XPKP^*\left(KK\right)X^*K = KP^*KKX^*KXP$$

$$XP\left(KP^*K\right)KX^*K = P\left(KX^*KX\right)P$$

$$XPPKX^*K = P^2 \quad \left[\because KX^*KX = I\right]$$
$$XP^2KX^*K = P^2$$

Post multiply by X ,

We have $\quad XP^2\left(KX^*KX\right) = P^2X$
$$XP^2 = P^2X$$

and hence $XP = PX$ by our assumption.

Conversely, if $XP = PX$ then $\left(KP^*K\right)\left(KX^*K\right) = \left(KX^*K\right)\left(KP^*K\right)$
$$\Rightarrow KP^*X^*K = KX^*P^*K$$
$$\Rightarrow P^*X^* = X^*P^*$$

Now $AKA^*K = \left(XP\right)K\left(XP\right)^*K$
$$= XPKP^*X^*K$$
$$= XPKX^*P^*K$$
$$= XP\left(KX^*K\right)\left(KP^*K\right)$$
$$= X\left(KP^*K\right)\left(KX^*K\right)\left(KP^*K\right) \qquad \left[\because P = KP^*K\right]$$
$$= \left(KP^*K\right)\left(KX^*K\right)X\left(KP^*K\right)$$
$$= \left(KP^*X^*K\right)XP$$
$$= K\left(XP\right)^*K\left(XP\right)$$

$(i) \Leftrightarrow (iv)$: If A is q-*k*-normal, then $AP = \left(XP\right)P$
$$= PXP$$
$$= PA,$$

Conversely, if $AP = PA,$ then $\left(XP\right)P = P\left(XP\right)$ post multiply by $P^{-1},$

We have $XP = PX$ and so that A is q-*k*-normal.

Theorem 3.10: Let $A \in H_{n\times n}$. Assume that $A = XP$, where X is q-*k*-unitary and A is non singular and q-*k*-hermition such that if P^2 commutes with X then A also commutes with X . Then the following conditions are equivalent.

(i) A is q-*k*-normal.
(ii) Any q-*k*-eigenvector of X is an q-*k*-eigenvector of A (as long as X has distinct q-*k*-eigenvalues).
(iii) Any q-*k*-eigenvector of A is an q-*k*-eigenvector of X (as long as A has distinct q-*k*-eigenvalues).
(iv) Any q-*k*-eigenvector of X is an q-*k*-eigenvector of A (as long as X has distinct q-*k*-eigenvalues).
(v) Any q-*k*-eigenvector of A is an q- *k*-eigenvector of X (as long as A has distinct q-*k*-eigenvalues).
(vi) Any q- *k*-eigenvector of P is an q-*k*-eigenvector of A (as long as A has distinct q-*k*-eigenvalues).
(vii) Any q- *k*-eigenvector of A is an q-*k*-eigenvector of P (as long as A has distinct q-*k*-eigenvalues)..

Proof:
$(i) \Leftrightarrow (ii)$: Let X have distinct q-*k*-eigenvalues. If we prove $XP = PX \Leftrightarrow$ any q-*k*-eigenvector of X is an q-*k*-eigenvector of $P,$ then $(i) \Leftrightarrow (ii)$ follows by theorem 3.7. Assume that any q-*k*-eigenvector of X is an

q-𝓀-eigenvector of P. If Y is an q-𝓀-eigenvector of X, then X is also an q-𝓀-eigenvector of P. Therefore there exists q-𝓀-eigenvalues λ and μ such that $XY = \lambda Y$ and $PY = \mu Y$. Now $XY = \lambda Y$ implies $PXY = PXY = \lambda\mu Y$. Similarly $PY = \mu Y$ implies $XPY = \lambda\mu Y$. Therefore $PXY = XPY \Rightarrow (PX - XP)Y = 0$ which implies as $PX = XP$ as $Y \neq 0$.

Conversely, assume that $XP = PX$. If Y is an q-𝓀-eigenvector of X, then there exists an q-𝓀-eigenvalue λ such that $XY = \lambda Y$. Let μ be an e q-𝓀-eigenvalue of X such that $XY = \mu Y$. $\therefore \lambda \neq \mu$. Now $XP = PX$ implies $\left(XP - PX\right)Y = 0$ which shows that $XPY = \lambda PY$. Similarly $XY = \mu Y$ implies $XPY = \mu PY$.

Therefore $\lambda PY = \mu PY \Rightarrow (\lambda - \mu)PY = 0 \Rightarrow PY = 0$ as $\lambda - \mu \neq 0. \therefore PY = 0Y$ and hence Y is an q-𝓀-eigenvector of P corresponding to the q-𝓀-eigenvalue 0. In general, if μ is any q-𝓀-eigenvalue of X, then we can prove that Y is also an q-𝓀-eigenvector of P. Therefore any q-𝓀-eigenvector of X is also an q-𝓀-eigenvector of P.

Similar proof holds for other equivalent conditions.

REFERENCES

1. Grone.R, Johnson.CR, E.M.SQ, H.Wolkowicz: normal matrices; Lin.Alg.Appl., 87 (1987) 213-225.
2. Hill.R.D, Water. S.R: on 𝓀-real and 𝓀 -hermitian matrices; Lin.Alg.Appl., Vol 169 (1992) 17-29.
3. Isarel–Adi Ben and Greville Thomas N.E: Generalised inverses: Theory and application; A wiley inerscience publications Newyork (1974).
4. Krishnamoorthy.S and vijayakumar.R: on S-normal matrices; Journal of Analysis and Computation, Vol 2 (2009).
5. Krishnamoorthy.S and vijayakumar.R: Some Equivalent conditions on S-normal matrices; International Journal of Contemporary mathematical sciences, vol 4 (2009) 29-32.
6. Zhang.F: Quaternions and matrices of quaternions; Lin.Alg.Appl., 251 (1997), 21-57.

Source of support: Nil, Conflict of interest: None Declared