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THE EFFECT OF HALL CURRENTS, RADIATION ABSORPTION AND CHEMICAL REACTION ON TRANSIENT MIXED CONVECTIVE HEAT AND MASS TRANSFER FLOW IN A VERTICAL WAVY CHANNEL UNDER AN INCLINED MAGNETIC FIELD

K. MADHU SUDHANA*[1], Dr. B. SREENIVASA REDDY[2], Dr. M. SREEDHAR BABU[2]

[1]Dept. of Mathematics,
RGU I II T, R.K. Valley, Vempalli(M), Kadapa (d), (A.P.), India.

[2]Asst.Professor, Dept. of Mathematics,
Yogi Vemana University, Kadapa, (A.P.), India.

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ABSTRACT

The topic deals with the effect of hall currents, radiation absorption and chemical reaction on transient mixed convective heat and mass transfer flow in a vertical wavy channel under an inclined magnetic field. We investigate the effect of chemical reaction Hall currents on unsteady mixed convective heat and mass transfer flow of a viscous, electrically conducting fluid in a vertical channel under the influence of an inclined magnetic fluid with heat sources. The equations governing the flow, heat and mass transfer are solved by employing perturbation technique with aspect ratio δ as perturbation parameter. The velocity, temperature and concentration distributions are investigated for different values of G, M, m, k, N, α and $x+\gamma t$. The rate of heat and mass transfer are numerically evaluated for different variations of the governing parameters.

Key words: *Nusselt numbers, Sherwood number, porous media, radioactive, recirculation pattern*

1. INTRODUCTION

There are many physical processes in which buoyancy forces resulting from combined thermal and species diffusion play an important role in the convective transfer of heat and mass. The engineering applications include the chemical distillatory processes, formation and dispersion of fog, design of heat exchangers, channel type solar energy collectors, and thermo-protection systems. Therefore, the characteristics of natural convection heat and mass transfer are relatively important. Convection flows driven by temperature and concentration differences have been studied extensively in the past.

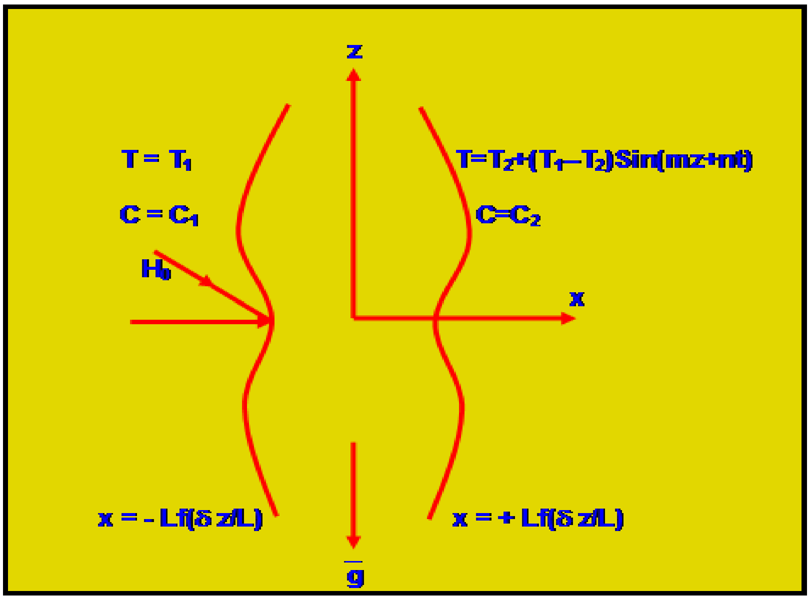
The study of heat and mass transfer from an irregular surface has many applications. It is often encountered in heat transfer devices to enhance heat transfer. For examples, flat-plate solar collectors and flat-plate condensers in refrigerators. The natural convective heat transfer from an isothermal vertical wavy surface was first studied by Yao [64, 65]. Vajravelu and Nayfeh [61] have investigated the influence of the wall waviness on friction and pressure drop of the generated coquette flow. Vajravelu and Sastry [59] have analysed the free convection heat transfer in a viscous, incompressible fluid confined between long vertical wavy walls in the presence of constant heat source. Later Vajravelu and Debnath [60] have extended this study to convective flow in a vertical wavy channel in four different geometrical configurations. This problem has been extended to the case of wavy walls by McMichael and Deutsch [33], Deshikachar *et al.* [19], Rao *et al.* [47, 48] and Sree Ramachandra Murthy [57]. Hyan Goo Kwon *et al.* [24] have analyzed that the Flow and heat/mass transfer in a wavy duct with various corrugation angles in two dimensional flow regimes. Kumar [31] has studied heat transfer with radiation and temperature dependent heat source in MHD free convection flow confined between two vertical wavy walls. Mahdy [32] have studied the mixed convection heat and mass transfer on a vertical wavy plate embedded in a saturated porous media (PST/PSE). Comini *et al.* [13] have analyzed the Convective heat and mass transfer in wavy finned-tube exchangers. Jer-Huan Jang *et al.* [27] have analyzed that the Mixed convection heat and mass transfer along a vertical wavy surface. Cheng [9, 10] has investigated coupled heat and mass transfer by natural convection flow along a wavy conical surface and vertical wavy surface in a porous medium.

Corresponding Author: K. Madhu Sudhana*[1]

Heat generation in a porous media due to the presence of temperature dependent heat sources has number of applications related to the development of energy resources. It is also important in engineering processes pertaining to flows in which a fluid supports an exothermic chemical or nuclear reaction. Proposal of disposing the radioactive waste material b burying in the ground or in deep ocean sediment is another problem where heat generation in porous medium occurs, Foroboschi and Federico [20] have assumed volumetric heat generation of the type

$$\theta = \begin{cases} \theta_0 (T - T_0) & for \quad T \geq T_0 \\ 0 & for \quad T < T_0 \end{cases}$$

David Moleam [16] has studied the effect of temperature dependent heat source Q = 1/ a + bT such as occurring in the electrical heating on the steady state transfer within a porous medium. Chandrasekahr [6], Palm *et al.* [41] reviewed the extensive work and mentioned about several authors who have contributed to the force convection with heat generating source. Mixed convection flows have been studied extensively for various enclosure shapes and thermal boundary conditions. Due to the super position of the buoyancy effects on the main flow there is a secondary flow in the form of a vortex recirculation pattern.



Configuration of the Problem

2. FORMULATION AND SOLUTION OF THE PROBLEM

We consider the unsteady flow of an incompressible, viscous ,electrically conducting fluid confined in a vertical channel bounded by two wavy walls under the influence of an inclined magnetic field of intensity Ho lying in the plane (x-z).The magnetic field is inclined at an angle α_1 to the axial direction and hence its components are $(0, H_0 Sin(\alpha_1), H_0 Cos(\alpha_1))$.In view of the traveling thermal wave imposed on the wall $x = +Lf(mz)$ the velocity field has components(u,0,w)The magnetic field in the presence of fluid flow induces the current($(J_x, 0, J_z)$.We choose a rectangular Cartesian co-ordinate system O(x,y,z) with z-axis in the vertical direction and the walls at $x = \pm Lf(mz)$.

When the strength of the magnetic field is very large we include the Hall current so that the generalized Ohm's law is modified to

$$\overline{J} + \omega_e \tau_e \overline{J} x \overline{H} = \sigma(\overline{E} + \mu_e \overline{q} x \overline{H}) \tag{2.1}$$

where $\overline{q}$ is the velocity vector. $\overline{H}$ is the magnetic field intensity vector. $\overline{E}$ is the electric field, J is the current density vector, ω_e is the cyclotron frequency, τ_e is the electron collision time,σ is the fluid conductivity and μ_e is the magnetic permeability. Neglecting the electron pressure gradient, ion-slip and thermo-electric effects and assuming the electric field E=0, equation (2.1) reduces

$$j_x - m H_0 J_z \operatorname{Sin}(\alpha_1) = -\sigma\mu_e H_0 w \operatorname{Sin}(\alpha_1) \tag{2.2}$$

$$J_z + m H_0 J_x \operatorname{Sin}(\alpha_1) = \sigma\mu_e H_0 u \operatorname{Sin}(\alpha_1) \tag{2.3}$$

where m= $\omega_e \tau_e$ is the Hall parameter.

On solving equations (2.2) & (2.3) we obtain

$$j_x = \frac{\sigma\mu_e H_0 \operatorname{Sin}(\alpha_1)}{1+m^2 H_0^2 \operatorname{Sin}^2(\alpha_1)}(mH_0 u \operatorname{Sin}(\alpha_1) - w) \tag{2.4}$$

$$j_z = \frac{\sigma\mu_e H_0 \operatorname{Sin}(\alpha_1)}{1+m^2 H_0^2 \operatorname{Sin}^2(\alpha_1)}(u + mH_0 w \operatorname{Sin}(\alpha_1)) \tag{2.5}$$

where u, w are the velocity components along x and z directions respectively,

The Momentum equations are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) + \mu_e(-H_0 J_z \operatorname{Sin}(\alpha_1)) \tag{2.6}$$

$$\frac{\partial w}{\partial t} u\frac{\partial W}{\partial x} + w\frac{\partial W}{\partial z} = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial z^2}\right) + \mu_e(H_0 J_x \operatorname{Sin}(\alpha_1)) \tag{2.7}$$

Substituting J_x and J_z from equations (2.4) & (2.5) in equations (2.6) & (2.7) we obtain

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{\sigma\mu_e H_{00}^2 \operatorname{Sin}^2(\alpha_1)}{1+m^2 H_0^2 \operatorname{Sin}^2(\alpha_1)}(u + mH_0 w \operatorname{Sin}(\alpha_1)) - \rho g \tag{2.8}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial W}{\partial x} + w\frac{\partial W}{\partial z} = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial z^2}\right) - \frac{\sigma\mu_e H_0^2 \operatorname{Sin}^2(\alpha_1)}{1+m^2 H_0^2 \operatorname{Sin}^2(\alpha_1)}(w - mH_0 u \operatorname{Sin}(\alpha_1)) \tag{2.9}$$

The energy equation is

$$\rho C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial z}\right) = k_f\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) + Q + Q_1'(C - C_0) \tag{2.10}$$

The diffusion equation is

$$\left(\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + w\frac{\partial C}{\partial z}\right) = D_1\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial z^2}\right) - k_1(C - C_0) \tag{2.11}$$

The equation of state is

$$\rho - \rho_0 = -\beta(T - T_o) - \beta^\bullet(C - C_0)) \tag{2.12}$$

Where T, C are the temperature and concentration in the fluid. k_f is the thermal conductivity, Cp is the specific heat constant pressure, k is the permeability of the porous medium, β is the coefficient of thermal expansion, $\beta^\bullet$ is the volumetric coefficient of expansion with mass fraction coefficient, D_1 is the molecular diffusivity, Q is the strength of the heat source, k_1 is the chemical reaction coefficient, Q_1' is the radiation absorption coefficient.,

The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$q = \frac{1}{L}\int_{-Lf}^{Lf} w\,dz \tag{2.13}$$

The boundary conditions are

u= 0, w=0 $T=T_1$, $C=C_1$ on $x = -Lf(mz)$ (2.14)

w=0, w=0, $T=T_2 + ((T_1-T_2)\operatorname{Sin}(mz+nt)$, $C=C_2$ on $x = -Lf(mz)$ (2.15)

Eliminating the pressure from equations (2.8) & (2.9) and introducing the Stokes Stream function ψ as

$$u = -\frac{\partial\psi}{\partial z},\; w = \frac{\partial\psi}{\partial x} \tag{2.17}$$

The equations (2.8)-(2.11) in terms of ψ are

$$\frac{\partial(\nabla^2\psi)}{\partial t}-\frac{\partial\psi}{\partial z}\frac{\partial(\nabla^2\psi)}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial(\nabla^2\psi)}{\partial z}=\mu\nabla^4\psi+\beta g\frac{\partial(T-T_o)}{\partial x}+\beta^{\bullet}g\frac{\partial(C-C_o)}{\partial x}-\left(\frac{\sigma\mu_e^2H_0^2\,\mathrm{Sin}^2(\alpha_1)}{1+m^2H_0^2\,\mathrm{Sin}^2(\alpha_1)}\right)\nabla^2\psi \quad (2.18)$$

$$\rho C_p\left(\frac{\partial T}{\partial t}+\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial T}{\partial x}\right)=k_f\left(\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial z^2}\right)+Q+Q_1'(C-C_o) \quad (2.19)$$

$$\left(\frac{\partial C}{\partial t}+\frac{\partial\psi}{\partial x}\frac{\partial c}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial C}{\partial x}\right)=D_1\left(\frac{\partial^2C}{\partial x^2}+\frac{\partial^2C}{\partial z^2}\right)-k_1(C-C_0) \quad (2.20)$$

On introducing the following non-dimensional variables

$$(x',z')=(x/L,mz),\psi'=\frac{\psi}{qL},\ \theta=\frac{T-T_2}{T_1-T_2},C'=\frac{C-C_2}{C_1-C_2}$$

The equation of momentum, energy and diffusion in the non-dimensional form are

$$\nabla^4\psi-M_1^2\nabla^2\psi+\frac{G}{R}\left(\frac{\partial\theta}{\partial z}+N\frac{\partial C}{\partial z}\right)=\delta R(\delta\frac{\partial}{\partial t}(\nabla^2\psi)+\left(\frac{\partial\psi}{\partial z}\frac{\partial(\nabla^2\psi)}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial(\nabla^2\psi)}{\partial z}\right) \quad (2.21)$$

$$\delta P\left(\delta\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial\theta}{\partial x}\right)=\left(\frac{\partial^2\theta}{\partial x^2}+\delta^2\frac{\partial^2\theta}{\partial z^2}\right)+\alpha+Q_1C \quad (2.22)$$

$$\delta Sc\left(\delta\frac{\partial C}{\partial t}+\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial C}{\partial x}\right)=\left(\frac{\partial^2C}{\partial x^2}+\delta^2\frac{\partial^2C}{\partial z^2}\right)-KC \quad (2.23)$$

$$\nabla^2=\frac{\partial}{\partial x^2}+\delta^2\frac{\partial}{\partial z^2}$$

where $G=\frac{\beta g\,\Delta T_eL^3}{\nu^2}$ (Grashof Number), $\delta=mL$ (Aspect ratio)

$M^2=\frac{\sigma\mu_e^2H_o^2L^2}{\nu^2}$ (Hartman Number), $M_1^2=\frac{M^2Sin^2(\alpha_1)}{1+m^2}$

$R=\frac{qL}{\nu}$ (Reynolds Number), $P=\frac{\mu C_p}{k_f}$ (Prandtl Number)

$\alpha=\frac{QL^2}{k_fC_p(T_1-T_2)}$ (Heat Source Parameter), $Sc=\frac{\nu}{D_1}$ (Schmidt Number)

$N=\frac{\beta^{\bullet}(C_1-C_2)}{\beta(T_1-T_2)}$ (Buoyancy ratio), $k=\frac{k_1L^2}{D_1}$ (Chemical reaction parameter)

$Q_1=\frac{Q_1'L^2(C_1-C_2)}{k_f(T_1-T_2)}$ (Radiation absorption parameter)

The corresponding boundary conditions are

$$\psi(1)-\psi(-1)=1,$$

$$\frac{\partial\psi}{\partial z}=0,\frac{\partial\psi}{\partial x}=0,\theta=1,C=1\quad at\ x=-f(z)$$

$$\frac{\partial\psi}{\partial z}=0,\frac{\partial\psi}{\partial x}=0,\theta=\mathrm{Sin}(z+\gamma t),\ C=0\ \ at\ x=+f(z) \quad (2.24)$$

3. ANALYSIS OF THE FLOW

On introducing the transformation

$$\eta = \frac{x}{f(z)} \tag{3.1}$$

The equations (2.21)-(2.23) reduce to

$$F^4\psi - (M_1^2 f^2)F^2\psi + \left(\frac{Gf^3}{R}\right)\left(\frac{\partial\theta}{\partial z} + N\frac{\partial C}{\partial z}\right) = (\delta Rf)(\delta\frac{\partial}{\partial t}(F^2\psi) + \left(\frac{\partial\psi}{\partial z}\frac{\partial(F^2\psi)}{\partial\eta} - \frac{\partial\psi}{\partial\eta}\frac{\partial(F^2\psi)}{\partial z}\right) \tag{3.2}$$

$$(\delta Pf)\left(\delta\frac{\partial\theta}{\partial t} + \frac{\partial\psi}{\partial\eta}\frac{\partial\theta}{\partial z} - \frac{\partial\psi}{\partial z}\frac{\partial\theta}{\partial\eta}\right) = \left(\frac{\partial^2\theta}{\partial\eta^2} + \delta^2 f^2\frac{\partial^2\theta}{\partial z^2}\right) + (\alpha f^2) + (Q_1 f^2)C \tag{3.3}$$

$$(\delta Scf)\left(\delta f^2\frac{\partial C}{\partial t} + \frac{\partial\psi}{\partial\eta}\frac{\partial C}{\partial z} - \frac{\partial\psi}{\partial z}\frac{\partial C}{\partial\eta}\right) = \left(\frac{\partial^2 C}{\partial\eta^2} + \delta^2 f^2\frac{\partial^2 C}{\partial z^2}\right) - (kf^2)C \le \tag{3.4}$$

Assuming the aspect ratio δ to be small we take the asymptotic solutions as

$$\psi(x,z,t) = \psi_0(x,z,t) + \delta\psi_1(x,z.,t) + \delta^2\psi_2(x,z,t) + ...$$
$$\theta(x,z,t) = \theta_o(x,z,t) + \delta\theta_1(x,z,t) + \delta^2\theta_2(x,z,t) + ...$$
$$C(x,z,t) = C_o(x,z,t) + \delta C_1(x,z,t) + \delta^2 C_2(x,z,t) + ... \tag{3.5}$$

Substituting (3.5) in equations (3.2)-(3.4) and equating the like powers of δ the equations and the respective boundary conditions to the zero th order are

$$\frac{\partial^2\theta_0}{\partial\eta^2} = -(\alpha f^2) = -(Q_1 f^2)C_o \tag{3.6}$$

$$\frac{\partial^2 C_0}{\partial\eta^2} - (kf^2)C_0 = 0 \tag{3.7}$$

$$\frac{\partial^4\psi_0}{\partial\eta^4} - (M_1^2 f^2)\frac{\partial^2\psi_0}{\partial\eta^2} = -\left(\frac{Gf^3}{R}\right)\left(\frac{\partial\theta_0}{\partial z} + N\frac{\partial C_0}{\partial z}\right) \tag{3.8}$$

With

$$\left.\begin{array}{l} \psi_0(+1) - \psi_0(-1) = 1 \\ \frac{\partial\psi_0}{\partial\eta} = 0,\ \frac{\partial\psi_0}{\partial z} = 0,\ \theta_0 = 1,\ C_0 = 1\ at\ \ \eta = -1 \\ \frac{\partial\psi_0}{\partial\eta} = 0,\ \frac{\partial\psi_0}{\partial z} = 0,\ \theta_0 = \mathrm{Sin}(z+\gamma t),\ C_0 = 0\ \ at\ \ \eta = +1 \end{array}\right\} \tag{3.9}$$

and to the first order are

$$\frac{\partial^2\theta_1}{\partial\eta^2} = (PRf)\left(\frac{\partial\psi_0}{\partial\eta}\frac{\partial\theta_0}{\partial z} - \frac{\partial\psi_0}{\partial z}\frac{\partial\theta_0}{\partial\eta}\right) \tag{3.10}$$

$$\frac{\partial^2 C_1}{\partial\eta^2} - (kf^2)C_1 = (ScRf)\left(\frac{\partial\psi_0}{\partial\eta}\frac{\partial C_0}{\partial z} - \frac{\partial\psi_0}{\partial z}\frac{\partial C_0}{\partial\eta}\right) \tag{3.11}$$

$$\frac{\partial^4\psi_1}{\partial\eta^4} - (M_1^2 f^2)\frac{\partial^2\psi_1}{\partial\eta^2} = -\left(\frac{Gf^3}{R}\right)\left(\frac{\partial\theta_1}{\partial z} + N\frac{\partial C_1}{\partial z}\right) + +(Rf)\left(\frac{\partial\psi_0}{\partial\eta}\frac{\partial^3\psi_0}{\partial z^3} - \frac{\partial\psi_0}{\partial \overline{z}}\frac{\partial^3\psi_0}{\partial\eta\partial z^2}\right) \tag{3.12}$$

With

$$\left.\begin{array}{l}\psi_1(+1)-\psi_1(-1)=0\\ \dfrac{\partial\psi_1}{\partial\eta}=0,\ \dfrac{\partial\psi_1}{\partial\overline{z}}=0,\quad \theta_1=0,\ C_1=0\ \ at\ \ \eta=-1\\ \dfrac{\partial\psi_1}{\partial\eta}=0,\ \dfrac{\partial\psi_1}{\partial\overline{z}}=0,\quad \theta_1=0,\ C_1=0\ \ at\ \ \eta=+1\end{array}\right\} \tag{3.13}$$

4. SOLUTIONS OF THE PROBLEM

Solving the equations (3.6) - (3.8) and (3.10) – (3.12) subject to the boundary conditions (3.9) & (3.13) we obtain

$$\theta_{0_0}=0.5\alpha(x^2-1)+0.5\,\mathrm{Sin}(z+\gamma t)(1+x)+0.5(1-x)$$

$$C_0=0.5\left(\frac{Ch(\beta_1 x)}{Ch(\beta_1)}-\frac{Sh(\beta_1 x)}{Sh(\beta_1)}\right)+a_3\left(\frac{Ch(\beta_1 x)}{Ch(\beta_1)}-1\right)$$

$$\psi_0=a_9\,\mathrm{Cosh}(M_1x)+a_{10}\,\mathrm{Sinh}(M_1x)+a_{11}x+a_{12}+\varphi_1(x)$$

$$\varphi_1(x)=-a_6x+a_7x^2-a_8x^3$$

Similarly the solutions to the first order are

$$\begin{aligned}\theta_1=&\ a_{36}(x^2-1)+a_{37}(x^3-x)+a_{38}(x^4-1)+a_{39}(x^5-x)+a_{40}(x^6-1)\\ &+(a_{41}+xa_{43})(Ch(M\ x)-Ch(M_1)+\ a_{42}(Sh(M_1x)-xSh(M_1))\\ &+a_{44}(xSh(M_1x)-Sh(M_1))\end{aligned}$$

$$\begin{aligned}C_1=&\ a_{47}\left(1-\frac{Ch(\beta_1x)}{Ch(\beta_1)}\right)+a_{48}\left(x-\frac{Sh(\beta_1x)}{Sh(\beta_1)}\right)+a_{49}\left(x^2-\frac{Ch(\beta_1x)}{Ch(\beta_1)}\right)\\ &+a_{50}\left(x^3-\frac{Sh(\beta_1x)}{Sh(\beta_1)}\right)+a_{51}\left(x^4-\frac{Ch(\beta_1x)}{Ch(\beta_1)}\right)+a_{52}\left(Ch(M_1x)-Ch(M_1)\frac{Ch(\beta_1x)}{Ch(\beta_1)}\right)\\ &+a_{53}\left(Sh(M_1x)-Sh(M_1)\frac{Sh(\beta_1x)}{Sh(\beta_1)}\right)+a_{54}\left(xCh(M_1x)-Ch(M_1)\frac{Sh(\beta_1x)}{Sh(\beta_1)}\right)\\ &+a_{55}\left(xSh(M_1x)-Sh(M_1)\frac{Ch(\beta_1x)}{Ch(\beta_1)}\right)+b_3\left(Sh(\beta_2x)-Sh(\beta_2)\frac{Sh(\beta_1x)}{Sh(\beta_1)}\right)\\ &+b_4\left(Sh(\beta_3x)-Sh(\beta_3)\frac{Sh(\beta_1x)}{Sh(\beta_1)}\right)+b_5\left(Ch(\beta_2x)-Ch(\beta_2)\frac{Ch(\beta_1x)}{Ch(\beta_1)}\right)\\ &+b_6\left(Ch(\beta_3x)-Ch(\beta_2)\frac{Ch(\beta_1x)}{Ch(\beta_1)}\right)+b_7\left(xSh(\beta_1x)-Sh(\beta_1)\frac{Ch(\beta_1x)}{Ch(\beta_1)}\right)\\ &+b_8\left(x^2Sh(\beta_1x)-Sh(\beta_1)\frac{Ch(\beta_1x)}{Ch(\beta_1)}\right)+b_9\left(x^3Sh(\beta_1x)-Sh(\beta_1)\frac{Ch(\beta_1x)}{Ch(\beta_1)}\right)\\ &+b_{11}\left(xCh(\beta_1x)-Ch(\beta_1)\frac{Sh(\beta_1x)}{Sh(\beta_1)}\right)+b_{12}(x^2Ch(\beta_1x)-Ch(\beta_1))\\ &+b_{13}\left(x^3Ch(\beta_1x)-Ch(\beta_1)\frac{Sh(\beta_1x)}{Sh(\beta_1)}\right)\end{aligned}$$

$$\psi_1=d_2\,\mathrm{Cosh}(M_1x)+d_3\,\mathrm{Sinh}(M_1x)+d_4x+d_5+\varphi_4(x)$$

$$\varphi_4(x) = b_{65}x + b_{66}x^2 + b_{67}x^3 + b_{68}x^4 + b_{69}x^5 + b_{70}x^6 + b_{71}x^7 + (b_{72}x + +b_{74}x^2 + b_{77}x^3)\,\mathrm{Cosh}(M_1x)$$
$$+(b_{73}x + b_{75}x^2 + b_{76}x^3)\,\mathrm{Sinh}(M_1x) + b_{78}\,\mathrm{Cosh}(\beta_1x) + b_{79}\,\mathrm{Sinh}(\beta_{1x}x)$$

5. NUSSELT NUMBER and SHERWOOD NUMBER

The rate of heat transfer (Nusselt Number) on the walls has been calculated using the formula

$$Nu = \frac{1}{(\theta_m - \theta_w)}\left(\frac{\partial\theta}{\partial x}\right)_{x=\pm1}$$

where $\theta_m = 0.5\int_{-1}^{1}\theta\,dx$

$$(Nu)_{x=+1} = \frac{1}{\theta_m - \mathrm{Sin}(z+\gamma t)}(b_{24} + \delta b_{22})$$

$$(Nu)_{x=-1} = \frac{1}{(\theta_m - 1)}(b_{25} + \delta b_{23})$$

$$\theta_m = b_{26} + \delta\, b_{27}$$

The rate of mass transfer (Sherwood Number) on the walls has been calculated using the formula

$$Sh = \frac{1}{(C_m - C_w)}\left(\frac{\partial C}{\partial x}\right)_{x=\pm1}$$

where $C_m = 0.5\int_{-1}^{1}C\,dx$

$$(Sh)_{x=+1} = \frac{1}{C_m}(b_{18} + \delta\, b_{16})$$

$$(Sh)_{x=-1} = \frac{1}{(C_m - 1)}(b_{19} + \delta\, b_{17})$$

$$C_m = b_{20} + \delta\, b_{21}$$

where $a_1, a_2, \ldots, a_{90}, b_1, b_2, \ldots, b_{79}$ are constants given in the appendix

6. DISCUSSION OF THE NUMERICAL RESULTS

We investigate the effect of Hall currents on the unsteady convective heat and mass transfer flow in a vertical wavy channel in the presence of heat generating sources under the influence of an inclined magnetic field. The equations governing the flow, heat and mass transfer are solved by employing a perturbation technique with the aspect ratio δ as a perturbation parameter. The unsteadiness in the flow is due to the traveling thermal waves imposed on the walls.

Figs.2-12 represent the axial velocity (w)for different variations of G, R, M, D^{-1}, m, β, λ. α, x+γt The actual axial flow is in the vertically downward direction and hence w>0 represents the reversal flow. Fig.2 represents 'w' with 'G' and Reynolds number 'R'. It is found that w exhibits a reversal flow with G<0 and the region of reversal flow enlarges with increase in G. The maximum |w| occurs at η=0. |w| experiences an enhancement with increase in |G|. An increase in 'R' enhances |w| in the entire region. Fig-3 represents w with Hartman number M and Hall parameters m. It is found that at M = 5, w exhibits a reversal flow in the region -0.8 ≤ η ≤ 0.4 and as M increases, the region of reversal flow reduces in its size. |w| reduces with M≤5 and for higher M = 7, |w| depreciates in the region -0.8 ≤ η ≤ 0 .2 and enhances in the remaining region and for still higher M = 9, |w| enhances in the regions adjacent to η = ±1 and reduces in the central region. The variation of w in m shows that w exhibits a reversal flow for m≥2.5 and region of reversal flow reduces with increase in m. |w| enhances with increase in m≤2.5 and reduces with m>2.5. Lesser the permeability of the porous medium larger the axial velocity in the flow region (fig.4).Fig-6 represents the variation of w with buoyancy ratio N. It is found that when the molecular buoyancy force dominates over the thermal buoyancy force |w| enhances in the flow region when buoyancy forces act in the same direction and for the forces acting in opposite directions, |w| depreciates in the flow region. An increase in α≤4 enhances |w| and reduces with higher α≥6 (fig.9). Fig-5 represents w with Schmidt number Sc. It is found that lesser the molecular diffusivity larger |w| and for further lowering of the diffusivity smaller |w| and for still lowering of diffusivity larger |w|. Fig-10 represents the effect of wall waviness on w. It is found that the higher the dilation of the channel walls lesser |w| in the flow region and for further higher dilation larger |w|. Fig-7 represents w with chemical reaction parameter k. Higher the chemical reaction parameter k larger |w|. An increase in Q_1 results in an enhancement in |w| (fig.8). Fig-11 represents the variation of w with inclination λ of the magnetic field. It is found that |w| depreciates with increase in $\lambda \le \frac{\pi}{2}$ and enhances with higher $\lambda = \pi$ and again depreciates with still higher $\lambda = 2\pi$. Fig-12 represents variation of w with phase $x+\gamma t$ of boundary temperature. An increase in $x + \gamma t \le \frac{\pi}{2}$ enhances |w| and depreciates with higher $x+\gamma t \ge \pi$.

The average Nusselt number (Nu) is exhibited in tables. 1-6 for different values of G, R, M, D^{-1},m, Sc, β, α, N, k, Q_1 and $x+\gamma t$. It is found that average Nusselt number enhances with increases in |G|. The variation of Nu with Hartmann number M shows that higher the Lorentz force smaller |Nu| and for further higher Lorentz force ($M \ge 10$) larger |Nu| at both the walls. An increase in $m<1.5$ reduces |Nu| and enhances with higher $m \ge 2.5$ at $\eta = \pm 1$. With reference to D^{-1} we find that the rate of heat transfer increases with increase in D^{-1} at both the walls. With respect to Schmidt number Sc we find that lesser the molecular diffusivity larger |Nu| at both the walls. The variation of θ with β shows that higher the dilation of the channel walls larger |Nu| for G>0 and for higher $|\beta| \ge 0.7$ larger |Nu| in the heating case and smaller |Nu| in the cooling case at $\eta = 1$ and at $\eta = -1$, larger |Nu| for all G (tables. 1&4). The variation of θ with α shows that an increase in α reduces |Nu| at $\eta = 1$ and enhances at $\eta = -1$. When the molecular buoyancy force dominates over the thermal buoyancy force, the rate of heat transfer reduces at $\eta = \pm 1$, when buoyancy forces act in same direction and for the forces acting in opposite directions |Nu| enhances at $\eta = 1$ and reduces at $\eta = -1$. An increase in the chemical reaction parameter $k \le 1.5$ enhances |Nu| at $\eta = -1$ and reduces at $\eta = -1$ and for higher $k \ge 2.5$ it reduces at $\eta = -1$ and enhances at $\eta = +1$. An increase in the radiation absorption parameter Q_1 enhances |Nu| at $\eta = 1$ and at $\eta = -1$, |Nu| enhances with $Q_1 \le 1.5$ and reduces with $Q_1 \ge 2.5$ (tables. 2&5). From tables 3&6 we find that the rate of heat transfer reduces at $\eta = 1$ and enhances at $\eta = -1$ with increase in $x + \gamma t \le \frac{\pi}{2}$ and for higher $x+\gamma t \ge \pi$, |Nu| enhances at $\eta = 1$ and reduces at $\eta = -1$.

The rate of mass transfer (Sherwood number) (Sh) at $\eta = \pm 1$ is shown in tables.7-12, for different variations. It is found that the rate of mass transfer enhances with increase in |G| at $\eta = \pm 1$.With respect to M, we find that the rate of mass transfer at $\eta = 1$ reduces with $M \le 5$ and enhances with $M \ge 10$ and at $\eta = -1$, it enhances with M for all G. An increase in $m \le 1.5$ reduces |Sh| at $\eta = \pm 1$ and enhances with higher $m \ge 2.5$. With reference to D^{-1} we find that the rate of mass transfer increases at both the wall with increase in D^{-1} .The variation of Sh with Sc shows that lesser the molecular diffusivity larger |Sh| at $\eta = 1$ and at $\eta = -1$ larger |Sh| and for further lowering of the diffusivity smaller |Sh|. Higher the of dilation of the channel walls larger |Sh| at $\eta = 1$. At $\eta = -1$, for $|\beta| \le 0.5$, larger |Sh| for G>0 and smaller |Sh| for G<0 and for still higher $|\beta| \ge 0.7$, larger |Sh| for all G (tables.7&10). The variation of Sh with α shows that |Sh| depreciates with α at $\eta = 1$. A $\eta = -1$, |Sh| depreciates with $\alpha \le 4$ and enhances with $\alpha \ge 6$ in heating case and in cooling case |Sh| depreciates with α.The rate of mass transfer enhances with increase in N>0 and reduces with N<0. The variation of |Sh| with chemical reaction parameter k shows that |Sh| at $\eta = 1$ reduces with $k \le 1.5$ and enhances with higher $k \ge 2.5$ while at $\eta = -1$, |Sh| depreciates for all G (tables. 8&11). From tables 8&12, the rate of mass transfer depreciates with increase in $x + \gamma t \le \frac{\pi}{2}$ and enhances with higher at $x+\gamma t \ge \pi$ at both the walls.

CONCLUSION

The impact of chemical reaction on boundary layer flow, heat and mass transfer analysis over wavy channel by taking inclined magnetic field, heat source/sink and heat generation /absorption into the account. The governing partial differential equations with represents the flow, temperature and concentration equations are transformed into the ordinary differential equations using non-dimensional variables. These equations together with boundary conditions are solved using perturbation method. The important findings of the problem are listed below.

- It is observed that the primary velocity exhibits reversal flow in the regions y = -1 and y = 1. As the values of M increases the fluid velocity decreases in the region left side of y=0, whereas the primary velocity of the fluid elevates in the right side of y = 0.
- It is found that the higher the dilation of the channel walls lesser |w| in the flow region and for further higher dilation larger |w|.
- The primary velocity |w| enhances with the improving values of chemical reaction parameter (k).
- As the values of heat source parameter increases the |w| also increases in the fluid region.
- The temperature of the fluid enhances with the higher values of M.

- With respect to M, we find that the rate of mass transfer at η = 1 reduces with M≤5 and enhances with M≥10 and at η = -1, it enhances with M for all G.
- The variation of Nu with Hartmann number M shows that higher the Lorentz force smaller |Nu| and for further higher Lorentz force (M≥10) larger |Nu| at both the walls.
- An increase in $x+\gamma t \le \frac{\pi}{2}$ leads to an enhancement in the actual temperature and for further higher x+γt = π depreciates in the flow region and for still higher x+γt = 2π we notice an enhancement in the actual temperature in the entire region

Table – 1: Nusselt number (Nu) at η = -1

G	I	II	III	IV	V	VI	VII	VIII
10^3	0.84807	8.49226	18.46640	0.49555	0.31766	0.24111	9.414443	34.77412
$3x10^3$	0.64259	8.46882	12.87020	0.24784	0.04344	-0.04589	9.390341	34.48839
-10^3	1.05343	8.51569	24.06260	0.74313	0.59173	0.52794	9.43850	35.05983
$-3x10^3$	1.25868	8.53909	29.65880	0.99056	0.86563	0.81461	9.46255	35.34553
M	2	5	10	2	2	2	2	2
m	0.5	0.5	0.5	1.5	2.5	3.5	0.5	0.5
D^{-1}	10^2	10^2	10^2	10^2	10^2	10^2	$2x10^2$	$3x10^2$

Table – 2: Nusselt number (Nu) at η = -1

G	I	II	III	IV	V	VI	VII
10^3	-9.23091	-12.11372	0.84807	45.48327	0.74990	12.65173	16.55357
$3x10^3$	-6.88343	-9.08440	0.64259	46.84024	-2.46809	-5.57878	-8.68947
-10^3	-11.57849	-15.14316	1.05343	44.12621	3.96778	6.88212	9.79648
$-3x10^3$	-13.92616	-18.17271	1.25868	42.76904	7.18555	13.11242	19.03929
Sc	0.24	06.	1.3	2.01	1.3	1.3	1.3
Q_1	0.5	0.5	0.5	0.5	1.5	2.5	3.5

Table – 3: Nusselt number (Nu) at η = -1

G	I	II	III	IV	V	VI	VII	VIII
10^3	0.84807	2.36200	2.69940	4.84361	7.66118	0.59982	-6.01953	3.02482
$3x10^3$	0.64259	1.56700	2.30140	3.97976	6.52332	0.45450	-4.56108	2.29195
-10^3	1.05343	2.15700	2.69730	2.70732	2.79882	0.74507	-7.47719	3.757229
$-3x10^3$	1.25868	1.95200	2.49530	- 3.42912	-4.06378	0.94900	-8.93404	4.48939
K	0.5	1.5	2.5	0.5	0.5	05.	0.5	0.5
α	2	2	2	4	6	2	2	2
x+γt	π/4	π/4	π/4	π/4	π/4	π/2	π	2π

Table – 4: Nusselt number (Nu) at η = -1

G	I	II	III	IV	V	VI
10^3	0.84807	1.61045	3.14310	-0.73323	-5.94848	-4.00712
$3x10^3$	0.64259	1.41270	2.97655	-0.65632	-5.07739	-2.92070
-10^3	1.05343	1.80814	3.0963	0.18982	-6.81964	-5.09359
$-3x10^3$	1.25868	2.00577	3.47613	1.11282	-7.69088	-6.18011
R	35	70	140	35	35	35
N	1	1	1	2	-0.5	-0.8

Table – 5: Nusselt number (Nu) at η = +1

G	I	II	III	IV	V	VI
10^3	0.84807	1.53567	0.15299	0.12513	-0.41718	1.12052
$3x10^3$	0.64259	1.38720	-0.14992	-0.10000	-0.33156	0.92891
-10^3	1.05343	1.68406	0.45572	0.4009	-0.50292	1.81203
$-3x10^3$	1.25868	1.83237	0.75826	0.70487	-0.68877	2.50344
λ	π/4	π/2	π	2π	π/4	π/4
β	0.5	0.5	0.5	0.5	0.3	0.7

Table – 6: Nusselt number (Nu) at η = -1

G	I	II	III	IV	V	VI	VII	VIII
10^3	2.21259	2.04264	2.44740	2.38630	2.47162	2.50776	-2.58156	-18.00035
$3x10^3$	3.94029	4.85867	16.95600	3.99265	4.01571	4.02485	1.54687	-8.65330
-10^3	0.48481	5.94396	6.93870	0.77986	0.92743	0.99054	-10.83836	-36.69445
$-3x10^3$	-1.24305	9.84530	10.43010	-1.82668	-2.61688	-0.52679	-10.83836	-36.69445
M	2	5	10	2	2	2	2	2
m	0.5	0.5	0.5	1.5	2.5	3.5	0.5	0.5
D^{-1}	10^2	10^2	10^2	10^2	10^2	10^2	$2x10^2$	$3x10^2$

Table – 7: Nusselt number (Nu) at η = +1

G	I	II	III	IV	V	VI	VII
10^3	9.33545	11.51723	2.21259	-28.13209	2.04689	1.88119	1.71550
$3x10^3$	9.30579	11.15669	3.94029	-27.12052	5.71409	7.48788	6.26169
-10^3	9.36505	11.87769	0.48481	-29.14372	-1.62038	-3.72557	-5.83076
$-3x10^3$	9.39457	12.23807	-1.24305	-30.15543	-5.28774	-9.33242	-13.37711
Sc	0.24	06.	1.3	2.01	1.3	1.3	1.3
Q_1	0.5	0.5	0.5	0.5	1.5	2.5	3.5

Table – 8: Nusselt number (Nu) at η = +

G	I	II	III	IV	V	VI	VII	VIII
10^3	2.21259	3.38200	4.93080	0.56032	-0.33580	2.20259	2.61259	1.91259
$3x10^3$	3.94029	4.83300	6.67840	-0.87561	-0.46892	3.34029	4.04029	3.24029
-10^3	0.48481	0.93000	1.18330	1.19614	-0.20285	0.40481	0.68481	0.42481
$-3x10^3$	-1.24305	-2.47900	-2.43570	1.83186	-0.67006	-1.19310	-1.64305	-1.20305
K	0.5	1.5	2.5	0.5	0.5	05.	0.5	0.5
α	2	2	2	4	6	2	2	2
x+γt	π/4	π/4	π/4	π/4	π/4	π/2	π	2π

Table – 9: Nusselt number (Nu) at η = +1

G	I	II	III	IV	V	VI
10^3	2.21259	1.60420	0.38096	3.68095	6.49449	5.01648
$3x10^3$	3.94029	3.32768	2.08300	6.61623	6.41093	4.57067
-10^3	0.48481	-0.11932	-1.32110	0.74564	6.57800	5.46224
$-3x10^3$	-1.24305	-1.84288	-3.02319	-2.18969	6.66145	5.90797
R	35	70	140	35	35	35
N	1	1	1	2	-0.5	-0.8

Table – 10: Nusselt number (Nu) at η = +1

G	I	II	III	IV	V	VI
10^3	2.21259	1.86088	2.54886	2.56927	5.48769	-9.75756
$3x10^3$	3.94029	3.81750	4.03475	4.13010	6.35225	-6.68952
-10^3	0.48481	-0.09579	1.06284	1.22831	4.62305	-12.82568
$-3x10^3$	-1.24305	-1.05252	-2.42331	-1.57276	3.75833	-15.89386
λ	π/4	π/2	π	2π	π/4	π/4
β	0.5	0.5	0.5	0.5	0.3	0.7

Table – 11: Sherwood number (Sh) at η = -1

G	I	II	III	IV	V	VI	VII	VIII
10^3	-0.60283	01.48065	2.27096	-0.60956	-0.70510	-0.72088	0.89160	3.14650
$3x10^3$	0.91369	4.19855	7.03995	0.71001	0.90117	0.55272	4.53455	12.26601
-10^3	-2.01728	-3.08903	-4.72418	-1.95171	-2.01691	-1.90149	-3.19892	-5.73169
$-3x10^3$	-3.33961	-6.51917	-7.96411	-3.14641	-3.24416	-2.998964	-6.84567	-14.37800
M	2	5	10	2	2	2	2	2
m	0.5	0.5	0.5	1.5	2.5	3.5	0.5	0.5
D^{-1}	10^2	10^2	10^2	10^2	10^2	10^2	$2x10^2$	$3x10^2$

Table – 12: Sherwood number (Sh) at η = -1

G	I	II	III	IV	V	VI	VII
10^3	-0.05300	-0.07553	-0.60283	-1.53381	-0.61283	-0.62283	-0.66283
$3x10^3$	0.71485	1.05916	2.91369	02.02094	0.92369	0.94369	0.96368
-10^3	-0.80454	-1.20237	-2.01728	-3.02698	-2.02728	-2.21728	-2.61728
$-3x10^3$	-1.54027	-2.32142	-3.33961	-4.46216	-3.36961	-3.43961	-3.63961
Sc	0.24	06.	1.3	2.01	1.3	1.3	1.3
Q_1	0.5	0.5	0.5	0.5	1.5	2.5	3.5

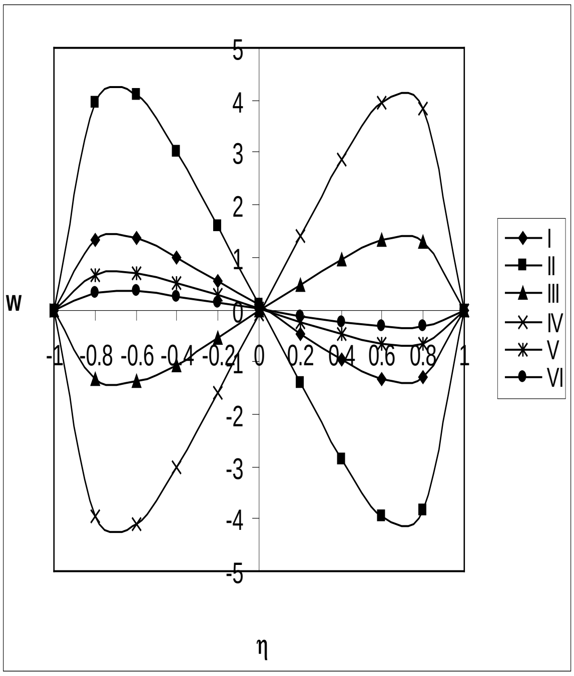


Fig. 1: Variation of w with G, R

	I	II	III	IV	V	VI
G	10^3	$2x10^3$	-10^3	$-2x10^3$	10^3	10^3
R	35	35	35	35	70	140

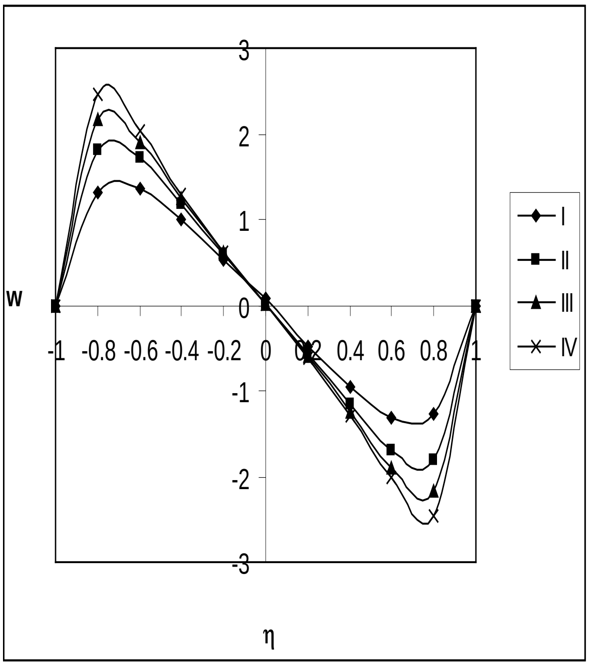


Fig. 2: Variation of w with M

	I	II	III	IV
M	2	5	7	9

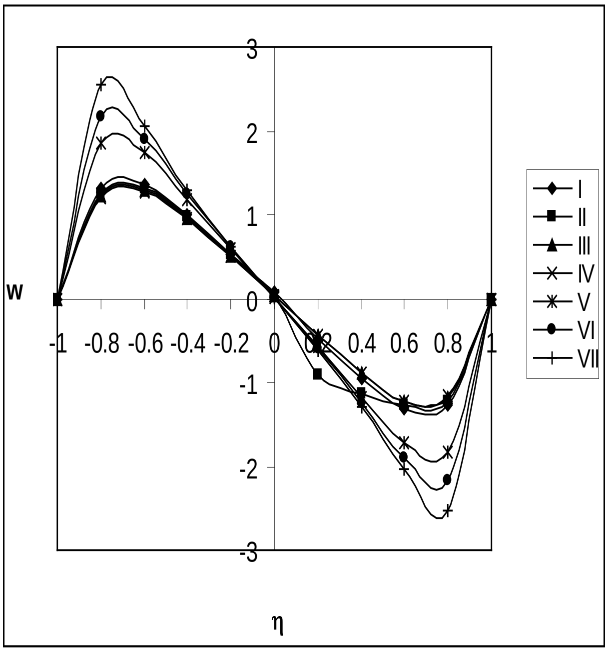


Fig. 3: Variation of w with m, D^{-1}

	I	II	III	IV	V	VI	VII
M	0.5	1.5	2.5	3.5	0.5	0.5	0.5
D^{-1}	10^2	10^2	10^2	10^2	$2x10^2$	$3x10^2$	$5x10^2$

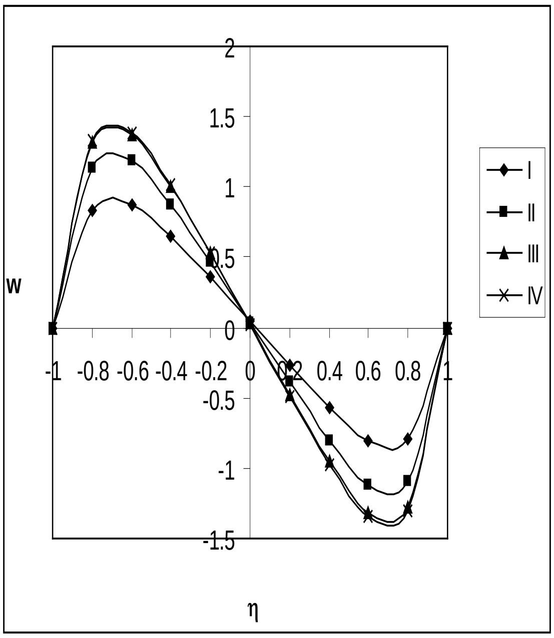


Fig. 4: Variation of w with Sc

	I	II	III	IV
Sc	0.24	0.6	1.3	2.01

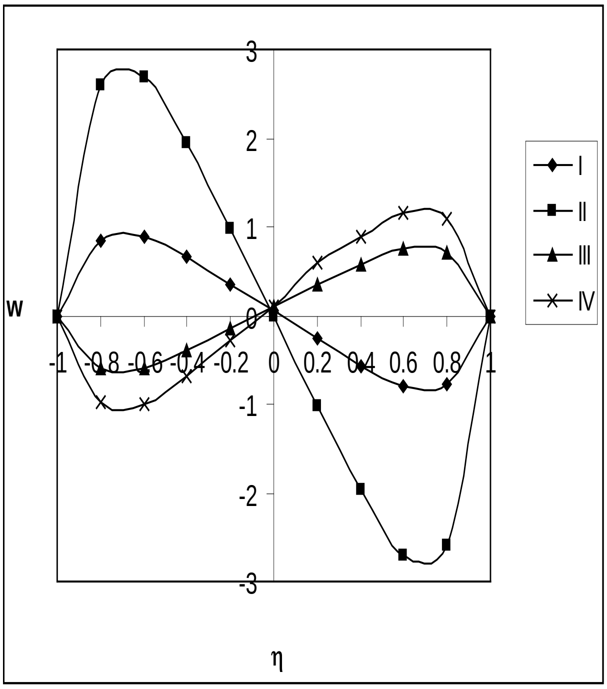


Fig. 5: Variation of w with N

	I	II	III	IV
N	1	2	-0.5	-0.8

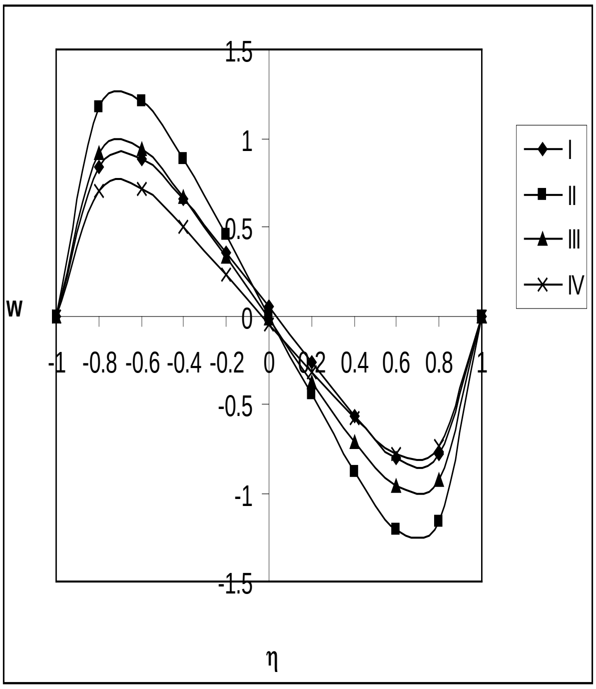


Fig. 6: Variation of w with K

	I	II	III	IV
K	0.5	1.5	2.5	3.5

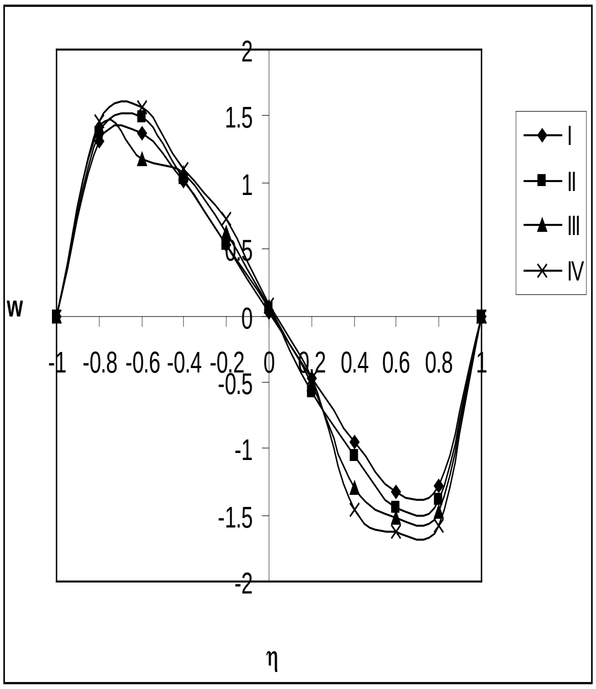


Fig. 7: Variation of w with Q_1

	I	II	III	IV
Q_1	0.5	0.5	0.5	0.5

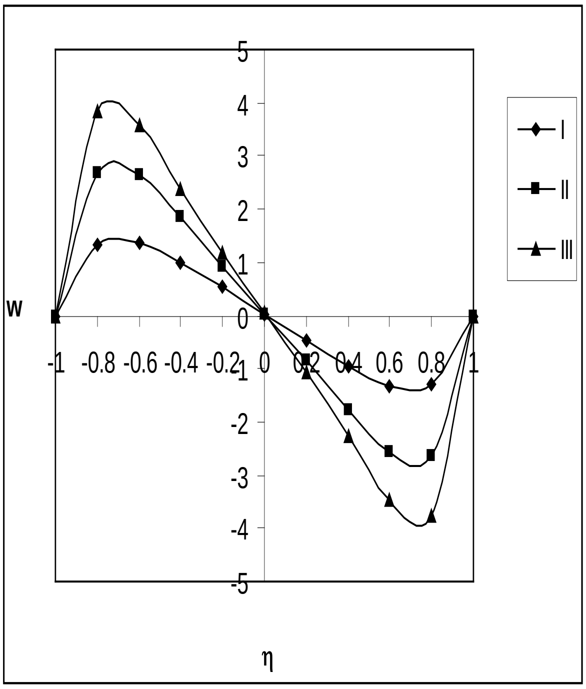


Fig. 8: Variation of w with K

	I	II	III
α	2	4	6

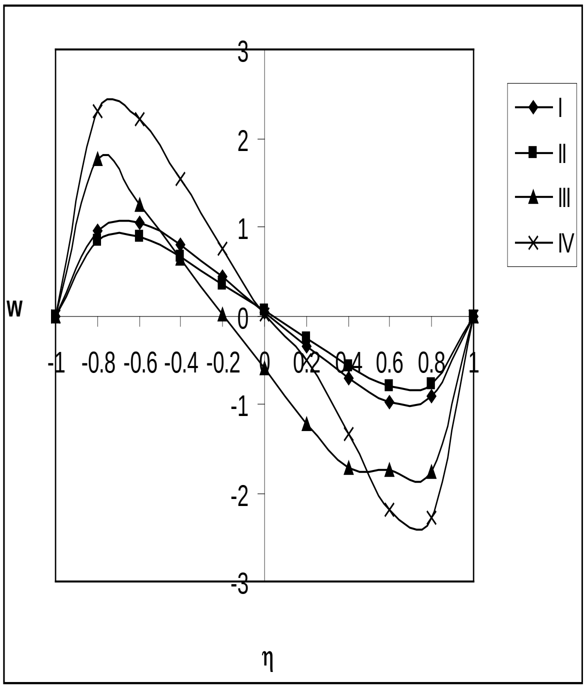


Fig. 9: Variation of w with β

	I	II	III	IV
β	0.3	0.5	0.7	0.9

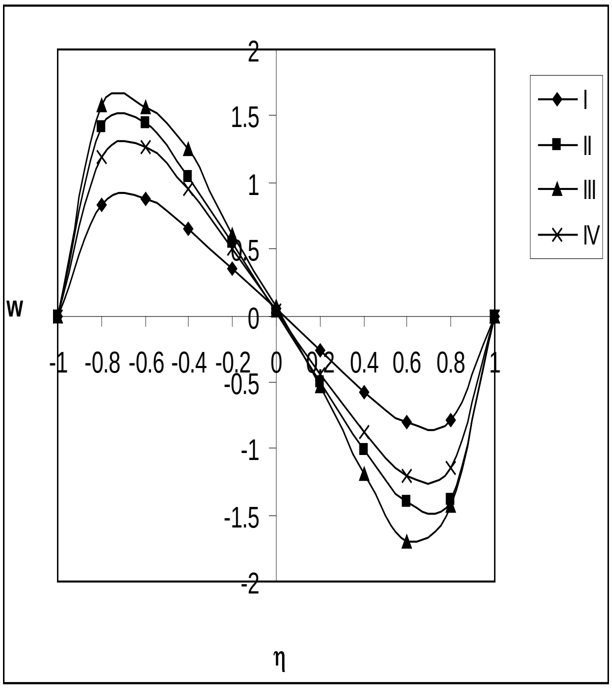


Fig. 10: Variation of w with λ

	I	II	III	IV
λ	π/4	π/2	π	2π

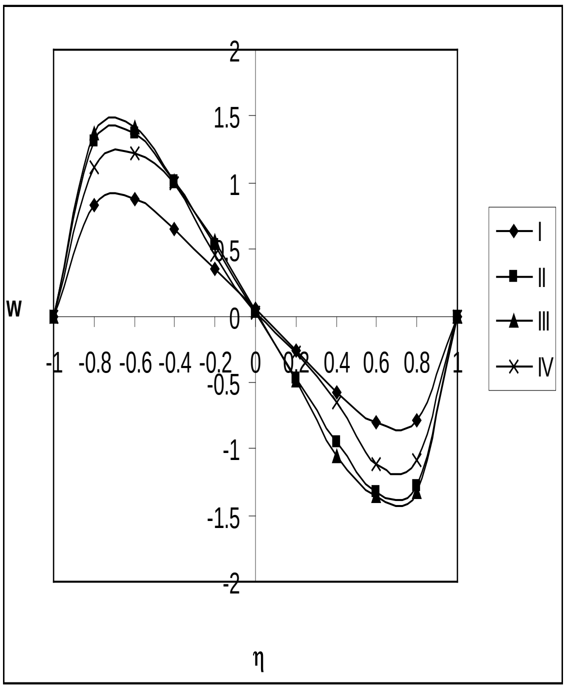


Fig. 11: Variation of w with x+γt

	I	II	III	IV
x+γt	π/4	π/2	π	2π

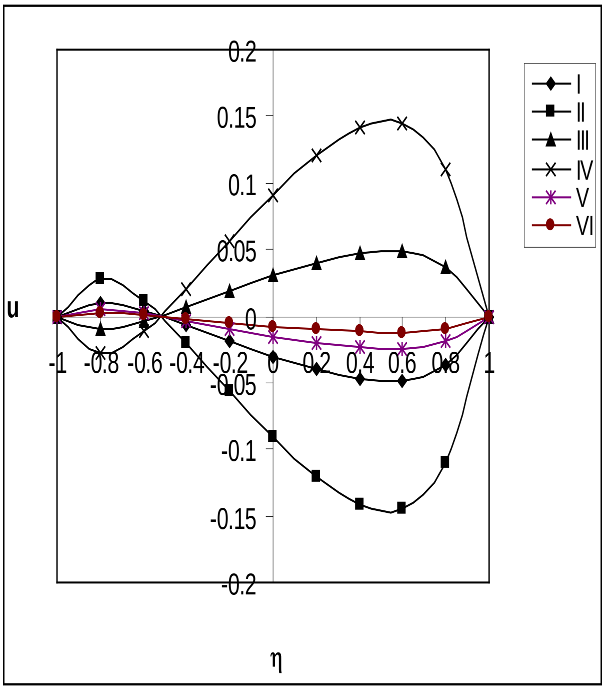


Fig. 12: Variation of u with G, R

	I	II	III	IV	V	VI
G	10^3	$2x10^3$	-10^3	$-2x10^3$	10^3	10^3
R	35	35	35	35	70	140

APPENDIX

$D_1 = z + \gamma t \quad M_1^2 = \frac{M^2 Sin^2\alpha_1}{1+M^2 Sin^2\alpha_1}$

$\beta_1^2 = KSc\,,\ a_1 = \frac{Sin\ D_1 - 1}{2},\quad a_2 = \frac{\alpha}{2} - \left(\frac{1+SinD_1}{2}\right), a_3 = \frac{ScS_0\alpha}{N\beta_1^2}, a_4 = \frac{G\ CosD_1}{R}, a_5 = \frac{G}{R}CosD_1$

$a_6 = \frac{a_5}{M_1^4}, a_7 = \frac{a_4}{2M_1^2}\ a_8 = \frac{a_5}{6M_1^2}, a_9 = -\frac{a_{14}}{M_1\,ShM_1}, a_{10} = \frac{(a_{13}-a_{15})}{M_1\,ChM_T ShM_1}, a_{11} = a_{13} - M\, a_{10}\, Ch\, M_1$

$a_{15} = a_6 + a_8, a_{13} = a_6 + 3a_8, a_{14} = 2a_7, a_{21} = M_1, a_{10},, a_{22} = \frac{-2a_7}{ShM_1}, a_{23} = 2a_{27}, a_{24} = -3a_8$

$a_{26} = \frac{SinD_1 - 1}{2}, a_{17} = d_{11} - a_6^1, a_{18} = a_7^1, a_{19} = -a_8^1, a_{20} = a_6^1 - a_7^1,\ a_{27} = \frac{a_{20}CosD_1 + a_{23}a_{26}}{2}$

$a_{28} = \frac{a_{20}CosD_1 - a_{25}\alpha - a_{26}a_{23}}{2}, a_{29} = \frac{(a_{17}+a_{18})CosD_1}{2}, a_{30} = \frac{(a_{18}+a_{19})CosD_1}{2}, a_{31} = \frac{a_{19}CosD_1}{2}$

$a_{32} = \frac{a_9' CosD_1}{2} + a_{26}a_{21}, a_{33} = \frac{a_{16}CosD_1}{2} - \alpha a_{26}a_{16}, a_{34} = \frac{a_9' CosD}{2} - \alpha a_{21}, a_{35} = \frac{a_{16}CosD_1}{2} - \alpha a_{22}$

$a_{36} = \left(\frac{a_{27}}{2}\right)\frac{ScS_0}{N}, a_{37} = \left(\frac{a_{28}}{6}\right)\frac{ScS_0}{N}, a_{38} = \left(\frac{a_{29}}{12}\right)\frac{ScS_0}{N}, a_{39} = \left(\frac{a_{30}}{20}\right)\frac{ScS_0}{N}, a_{40} = \left(\frac{a_{31}}{30}\right), \frac{ScS_0}{N},$

$a_{41} = \frac{(M_1 a_{32} - 2a_{35})ScN_1^0}{M_1^2 N}, a_{42} = \frac{(M_1 a_{33} - 2a_{34})ScS_0}{M_1^2 N}, a_{43} = \frac{a_{34}ScS_0}{M_1^2 N}$

$a_{44} = \frac{a_{35}}{M_1^2}, a_{45} = -(a_{37} + a_{39} + a_{42}\, Sh\, M_1 + a_{43}\, Ch\, M_1), a_{46} = -(a_{36} + a_{38} + a_{40} + a_{41}\, Ch\, M_1 + a_{44}\, Sh\, M_1)$

$a_{47} = \frac{-a_{27}}{\beta_1^2}, a_{48} = \frac{\beta_1^2 a_{24} - 2a_{29}}{\beta_1^4}, a_{49} = \frac{(\beta_1^3 a_{29} - 3a_{30})}{\beta_1^5}, a_{50} = \frac{(\beta_4 a_{30} - 4a_{31})}{\beta_1^4}\ a_{51} = \frac{a_{31}}{\beta_1^2}, a_{52} = \frac{a_{32}}{M_1^2 - \beta_1^2},$

$a_{53} = \frac{-a_{323}}{M_1^2 - \beta_1^2} + \frac{2M_1 a_{34}}{(M_1^2 - \beta_1^2)^2}, a_{54} = \frac{-a_{34}}{M_1^2 - \beta_1^2}\ a_{55} = \frac{-a_{35}}{M_1^2 - \beta_1^2}\ b_1 = \frac{\beta_1}{2Ch\beta_1} + \frac{a_3\beta_1}{Ch\beta_1}\ b_2 = \frac{\beta_1}{2Sh\beta_1},$

$b_3 = -Sc\left(\frac{a_1' b_1 - b_2 b_{13}}{2}\right) b_4 = -Sc\left(\frac{a_9' b_1 + b_2 b_{16}}{2}\right), b_5 = -Sc\left(\frac{b_1 a_{16} - b_2 a_9'}{2}\right), b_6 = -Sc\left(\frac{b_1 a_{16} + b_2 a_9'}{2}\right),$

$b_7 = -Sc(a_{17}b_1), b_8 = -Sca_{18}b_1, b_9 = +Sca_{17}b_1$

$b_{10} = -Scb_1a_{20}, b_{11} = +Sca_{17}b_1, b_{12} = Sca_{18}b_2, b_{13} = Sca_{19}b_2, b_{14} = Scb_2a_{20},$

$b_{15} = -(a_{48} + a_{50} + a_{53}\, Sh\, M_1 + a_{54}\, Ch\, M_1 + b_3\ Sh\ \beta_2 + b_4\ Sh\ \beta_5 + b_8\, Sh\, \beta_1 + b_{10}\, Sh\, \beta_1 + b_{11}\, Ch\, \beta_1 + b_{13}\, Ch\, \beta_1)/Sh\, \beta_1$

$b_{16}/b_{17} = \mp a_{47}\, \beta_1\, Th\, \beta_1 + a_{48}\,(1 - \beta_1\, cth\, \beta_1) \pm a_{41}\,(2 - \beta_1\, Th\, \beta_1) + a_{50}\,(3 - \beta_1\, cth\, \beta_1)$
$\pm a_{51}\,(4 - \beta_1\, Th\, \beta_1) \pm a_{52}\,(M_1\, Sh\, M_1 - \beta_1\, Ch\, M_1\, Th\, \beta_1)$
$+ a_{51}\,(M_1\, Ch\, M_1 - \beta_1\, Sh\, M_1\, cth\, \beta_1) + a_{54}\,(Ch\, M_1 + M_1\, Sh\, M_1$
$- \beta_1\, Ch\, M_1\, Th\, \beta_1) \pm a_{55}\,(Sh\, M_1 + M_1\, Ch\, M_1 - \beta_1\, Sh\, M_1\, Th\, \beta_1)$
$+ b_3\,(\beta_2\, Ch\, \beta_2 - \beta_2\, Sh\, \beta_2\, cth\, \beta_1) + b_4\,(\beta_3\, Ch\, \beta_3 - \beta_1\, Sh\, \beta_3\, cth\, \beta_1)$
$\pm b_5\,(\beta_2\, Sh\, \beta_2 - \beta_1\, Ch\, \beta_2\, Th\, \beta_1) \pm b_6\,(\beta_3\, Sh\, \beta_3 - \beta_1\, Ch\, \beta_3\, Th\, \beta_1)$
$\pm b_7\,(Sh\, \beta_1 + \beta_1\, Ch\, \beta_1 - \beta_1\, Sh\, \beta_1\, Th\, \beta_1) + b_8\, Sh\, \beta_1 \pm b_9\,(3\, Sh\, \beta_1$
$+ \beta_1\, Ch\, \beta_1 - \beta_1\, Sh\, \beta_1\, Th\, \beta_1) + b_{11}\,(Ch\, \beta_1 + \beta_1\, Sh\, \beta_1 - \beta_1\, Ch\, \beta_1\, cth\, \beta_1)$
$\pm b_{12}\,(2Ch\, \beta_1) + b_{13}\,(3Ch\, \beta_1 + \beta_1\, Sh\, \beta_1 - \beta_1\, Ch\, \beta_1\, cth\, \beta_1)$

$b_{18}/b_{19} = \pm b_1\, Sh\, \beta_1 + b_1\, Ch\, \beta_1$

$b_{20} = \frac{Th\beta_1}{\beta_1} + \frac{a_3 Th\beta_1 - 2a_3}{\beta_1}$

$$b_{21} = a_{47}(2 - 2Th\beta_1) + 2a_{49}\left(\frac{1}{3} - \frac{Th\beta_1}{\beta_1}\right) + 2a_{51}\left(\frac{1}{5} - \frac{Th\beta_1}{\beta_1}\right) + 2a_{52}\left(\frac{ShM_1}{M_1} - \frac{ChM_1Th\beta_1}{M_1}\right)$$
$$+2a_{55}\left(\frac{ChM_1}{M_1} - \frac{ShM_1}{M_1^2} - \frac{ChM_1cth\beta_1}{\beta_1}\right) + 2b_5\left(\frac{Sh\beta_2}{\beta_2} - \frac{Ch\beta_2Th\beta_1}{\beta_1}\right) + 2b_6\left(\frac{Sh\beta_3}{\beta_3} - \frac{Ch\beta_3Th\beta_1}{\beta_1}\right)$$
$$+2b_9\left(\frac{Ch\beta_1}{\beta} - \frac{3Sh\beta_1}{\beta_1^2} + \frac{3Sh\beta_1}{\beta_1^3}\right) + 4b_{12}\left(\frac{Sh\beta_1}{\beta_1^3} - \frac{Ch\beta_1}{\beta_1^2}\right)$$

$b_{22}/b_{23} = \pm 2a_{36} + 3a_{37} \pm 4a_{38} + 5a_{39} \pm 6a_{40} \pm M_1a_{41}\, Sh\, M_1 + M_1a_{41}\, Ch\, M_1 + a_{43}\,(M_1\, Sh\, M_1 + Ch\, M_1)$
$\pm a_{44}\,(M_1\, Ch\, M_1 + Sh\, M_1) + a_{45}$

$$b_{24} / b_{25} = \pm\alpha + \frac{\sin D_1 - 1}{2}, b_{26} = \frac{-2\alpha}{3} + \frac{(\sin D_1 - 1)}{2}$$

$$b_{27} = \frac{-4}{3}a_{36} - \frac{8}{5}a_{38} - \frac{12}{7}a_{40} + 2a_{41}\left(\frac{ShM_1}{M_1} - ChM_1\right) + 2a_{44}\left(\frac{ChM_1}{M} - \frac{ShM_1}{M_1^2}\right)$$

$$b_{28} = 0.5(a_{21}a'_{21} + a_{22}a'_2)\beta_1, b_{29} = 0.5(a_{21}a'_{22} + a_{22}a'_{21})\beta_1, b_{30} = 0.5(a_{21}a'_{21} - a_{22}a'_{21})\beta_1 + a_{25}(a'_{23} + a'_{25}),$$

$$b_{31} = a_{21}(a'_{23} + a'_{25}) + a_{25}a'_{22}M_1\; b_{32} = a_{22}a'_{23} + a_{25}a'_{21}M_1, b_{33} = 2a'_{24}a_{21}, b_{34} = a_{24}a'_{22}M_1, b_{35} = a'_{21}a_{24}\beta_1,$$

$$b_{36} = a_{23}a'_{25} + 2a_{25}a'_{24},\; b_{37} = 2a'_{24}a_{23} + a_{24}a'_{23} + a_{24}a'_{25}, b_{38} = 2a_{24}a'_{24}, b_{39} = 0.5(a_{21}a'_{21} + a_{22}a'_{22})\beta_1)$$

$$b_{40} = 0.5(a'_{21}a_{22} + a''_{22}a_{21})\beta_1^2,\quad b_{41} = \frac{a_{21}a'_{21}\beta_1^2}{2} + 2(a'_{21}a_{24} + a_{24}a'_{25}) - a_{22}a'_{22}\beta_1^2,\; b_{42} = a'_{25}a_{21}\beta_1^2$$

$$b_{43} = a'_{25}a_{22}\beta_1^2 + 2a_{24}a_{22}, b_{44} = a'_{23}a_{21}\beta_1^2, b_{45} = a'_{23}a_{22}\beta_1^2, b_{46} = a'_{24}a_{21}\beta_1^2, b_{47} = a'_{24}a_{22}\beta_1^2\; b_{48} = 2a_{24}a'_{23},$$

$$b_{49} = 2a_{24}a'_{24}, b_{50} = \frac{-G}{R}(a'_{45} + Na'_{48}) + R(b_{48} - b_{36}), b_{51} = \frac{-G}{R}(a'_{36} + Na'_{49}) + R(b_{49} - b_{37}),$$

$$b_{52} = \frac{-G}{R}(a'_{37} + Na'_{50}) + R(b_{38})\; b_{53} = \frac{-G}{R}(a'_{46} + Na'_{47}) + R(b_{41} - b_{30}), b_{54} = \frac{-G}{R}(a'_{38} + Na'_{51}),$$

$$b_{55} = \frac{-G}{R}a'_{39}, b_{56} = \frac{-G}{R}a'_{40}\; b_{57} = \frac{-G}{R}(a'_{41} + Na'_{52}) + R(b_{42} - b_{31}),$$

$$b_{58} = \frac{-G}{R}(a'_{42} + Na'_{53}) + R(b_{43} - b_{32})\; b_{59} = \frac{-G}{R}(a'_{43} + Na'_{54}) + R(b_{44} - b_{33}),$$

$$b_{60} = \frac{-G}{R}(a'_{44} + Na'_{55}) + R(b_{45} - b_{34}), b_{61} = R(b_{46} - b_{35})\; b_{62} = R(b_{47} - b_{35}), b_{63} = \frac{-G}{R}Nb'_{14}$$

$$b_{64} = \frac{-G}{R}Nb'_{15}, b_{65} = \frac{-b_{50}}{M_1^4} - \frac{b_{53}}{M_1^2}, b_{66} = \frac{-b_{50}}{2M_1^2} - \frac{b_{51}}{M_1^4}\; b_{67} = \frac{-b_{51}}{3M_1^2} - \frac{b_{52}}{M_1^4}, b_{68} = \frac{-b_{52}}{4M_1^2} - \frac{b_{54}}{M_1^4},$$

$$b_{69} = \frac{-b_{54}}{5M_1^2} - \frac{b_{55}}{M_1^4}, b_{70} = \frac{-b_{55}}{6M_1^2} - \frac{b_{56}}{M_1^4}, b_{71} = \frac{-b_{56}}{7M_1^2}\; b_{72} = \frac{b_{58}}{M_1^3} - \left(\frac{2}{M_1} + \frac{1}{2M_1}\right)b_{59},$$

$$b_{73} = \frac{b_{57}}{2M_1^3} - \left(\frac{2}{M_1} + \frac{1}{2M_1}\right)b_{60}, b_{74} = \frac{b_{60}}{2} - \left(\frac{2}{M_1} + \frac{1}{2M_1}\right)b_{61}\; b_{75} = \frac{b_{59}}{2} - \left(\frac{2}{M_1} + \frac{1}{2M_1}\right)b_{61},$$

$$b_{73} = \frac{b_{61}}{3}, b_{74} = \frac{b_{62}}{3}$$

$$d_6 = \frac{\phi'_4(+1) + \phi'_4(-1)}{2}, d_7 = \frac{\phi'_4(+1) + \phi'_4(-1)}{2}, d_8 = \frac{\phi_4(+1) - \phi_4(-1)}{2},$$

$$d_2 = \frac{-d_7}{M_1ShM_1}\; d_3 = \frac{(d_8 - d_6)}{(M_1ChM_1 - ShM_1)}$$

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