

## A STRONGER FORM OF SEMI STAR OPEN SETS

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### ABSTRACT

In this paper, we introduce a new class of sets, namely Semi star regular open sets, using regular open sets and the generalized closure operator due to W.Dunham we analyze the characterizations of Semi star regular open sets. Further, we study some fundamental properties of Semi star regular open sets and study their interrelationships with their known star generalized closed sets. We also define Semi star regular interior.

**Keywords:** Semi regular-open set, generalized closure, semi star regular open set.

**AMS Classification:** 54A05.

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### 1. INTRODUCTION

In 1963, Norman Levine [10] introduced the concept of semi-open set in topological spaces which is a weaker form of open sets. Velicko [7] introduced the notion of Regular open sets in topological spaces. Levine [10] generalized the concept of closed sets to generalized closed sets in 1970. Using generalized closed sets, Dunham [1] introduced the concept of closure operator  $Cl^*$  and the generalized closure of a set  $A$  is denoted by  $Cl^*(A)$ . Recently S.Pious Missier and A.Robert [14] introduced a nearly open sets, namely Semi $^*\alpha$ -open sets in a topological space. In this paper we introduced a new class of nearly open sets, Semi $^*$ regular open sets in topological spaces and studied their properties.

### 2. PRELIMINARIES

Throughout this paper  $(X, \tau)$  will always denote a topological space on which no separation axioms are assumed, unless explicitly stated. If  $A$  is a subset of a topological space  $(X, \tau)$ ,  $Cl(A)$  and  $Int(A)$  denote the closure and the interior of  $A$  respectively.

**Definition 2.1:** A subset  $A$  of a topological space  $(X, \tau)$  is generalized closed (briefly g-closed) [9] if  $Cl(A) \subseteq U$  whenever  $U$  is an open set in  $X$  containing  $A$ .

**Definition 2.2:** If  $A$  is a subset of a topological space  $(X, \tau)$ , the generalized closure [4] of  $A$  is defined as the intersection of all g-closed sets in  $X$  containing  $A$  and is denoted by  $Cl^*(A)$ .

**Definition 2.3:** A subset  $A$  of a topological space  $(X, \tau)$  is semi-open [10] (respectively semi $^*$ -open [11]) if there is an open set  $U$  in  $X$  such that  $U \subseteq A \subseteq Cl(U)$  ( respectively  $U \subseteq A \subseteq Cl^*(U)$  ) or equivalently if  $A \subseteq Cl(Int(A))$  (respectively  $A \subseteq Cl^*(Int(A))$ ).

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**Definition 2.4:** A subset  $A$  of a topological space  $(X, \tau)$  is regular open [6] [respectively  $\alpha$ -open [3], preopen [8]] if  $A = \text{Int}(\text{Cl}(A))$  [respectively  $A \subseteq \text{Int}(\text{Cl}(\text{Int}(A)))$ ,  $A \subseteq \text{Int}(\text{Cl}(A))$ ].

**Definition 2.5:** A subset  $A$  of a topological space  $(X, \tau)$  is

- (i) a regular generalized closed [6] (briefly rg-closed) if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open in  $X$
- (ii) a generalized pre regular closed (briefly gpr-closed) if  $\text{pcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open in  $X$
- (iii) a regular weakly generalized closed (briefly rwg-closed) if  $\text{cl}(\text{int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open in  $X$ .

**Definition 2.6:** A subset  $A$  is regular semi open [5] (respectively semi  $\alpha$ -open [12], semi preopen [13]) if there is a regular open set (respectively  $\alpha$ -open, preopen)  $U$  in  $X$  such that  $U \subseteq A \subseteq \text{Cl}(U)$  or equivalently if  $A = \text{SInt}(\text{Scl}(A))$  [4]. (respectively  $A \subseteq \text{Cl}(\alpha\text{-int}(A))$ ,  $A \subseteq \text{Cl}(p\text{-int}(A))$ ).

**Definition 2.7:** A topological space  $X$  is  $\mathbf{T}_{1/2}$  if every g-closed set in  $X$  is closed. A space  $X$  is locally indiscrete if every open set in  $X$  is closed.

A space  $X$  is extremally disconnected if the closure of every open set in  $X$  is open.

### 3. SEMI\* REGULAR OPEN SET

**Definition 3.1:** A subset  $A$  of a Topological space  $(X, \tau)$  is called a Semi\*regular open set (briefly s\*r-open) if there exists a regular open set  $U$  in  $X$  such that  $U \subseteq A \subseteq \text{Cl}^*(U)$ .

The Class of all Semi\*regular open sets in  $(X, \tau)$  is denoted by  $\text{S}^*\text{RO}(X, \tau)$  or simply  $\text{S}^*\text{RO}(X)$ .

**Example 3.2:** Let  $X = \{a, b, c, d\}$   $\tau = \{X, \emptyset, \{a\}, \{d\}, \{a, d\}, \{a, c, d\}\}$ . In the space  $(X, \tau)$ , the subsets  $\{a, b\}$ ,  $\{b, d\}$   $\{a, b, d\}$  are Semi star regular open sets.

**Theorem 3.3:** For a subset  $A$  of a topological space  $(X, \tau)$  the following are equivalent.

- (i)  $A$  is Semi\* regular open
- (ii)  $A = \text{Cl}^*(r\text{-int}(A))$
- (iii)  $\text{Cl}^*(A) = \text{Cl}^*(r\text{-int}(A))$
- (iv)  $\text{Cl}^*(A) = \text{Cl}^*(A \cap \text{int}(\text{cl}(A)))$

**Proof:**

(i)  $\Rightarrow$  (ii): If  $A$  is Semi\* regular open, then there is a regular open set  $U$  such that  $U \subseteq A \subseteq \text{Cl}^*(U)$ .

Now  $U \subseteq A \Rightarrow U = r\text{-int}(U) \subseteq r\text{-int}(A) \Rightarrow A \subseteq \text{Cl}^*(U) \subseteq \text{Cl}^*(r\text{-int}(A))$  and  $A \supseteq \text{Cl}^*(r\text{-int}(A))$ . Hence  $A = \text{Cl}^*(r\text{-int}(A))$ .

(ii)  $\Rightarrow$  (iii): By assumption,  $A = \text{Cl}^*(r\text{-int}(A))$ . Since  $\text{Cl}^*$  is a Kuratowski Operator, we have  $\text{Cl}^*(A) = \text{Cl}^*(\text{Cl}^*(r\text{-int}(A))) = \text{Cl}^*(r\text{-int}(A))$ .

(iii)  $\Rightarrow$  (i): Take  $U = r\text{-int}(A)$ .

Then  $U$  is a regular open set in  $X$ . (ie)  $U \subseteq A \subseteq \text{Cl}^*(A) = \text{Cl}^*(r\text{-int}(A)) = \text{Cl}^*(U)$

(iii)  $\Rightarrow$  (iv): follows from the fact that for any subset  $A$ ,  $r\text{-int}(A) = A \cap (\text{int}(\text{Cl}(A)))$

**Theorem 3.4:** Arbitrary Union of Semi\*regular open sets in  $X$  is also Semi\*regular open set in  $X$ .

**Proof:** Let  $\{A_i\}$  be a collection of semi\*regular open sets in  $X$ . Since each  $A_i$  is Semi\*regular open set, there is a regular-open set  $U_i$  in  $X$  such that  $U_i \subseteq A_i \subseteq \text{Cl}^*(U_i)$ .

Then  $\cup U_i \subseteq \cup A_i \subseteq \cup \text{Cl}^*(U_i) \subseteq \text{Cl}^*(\cup U_i)$ . Since  $\cup U_i$  is regular-open, by Definition 3.1,  $\cup A_i$  is Semi\*regular open set.

**Remark 3.5:** The intersection of two Semi\*regular open sets need not be Semi\*regular open as seen from the following example.

**Example 3.6:** Let  $X = \{a, b, c, d\}$  with the topology  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ . In the space  $(X, \tau)$ , the subsets  $\{a, d\}$  and  $\{b, d\}$  are semi\*regular open sets but their intersection  $\{d\}$  is not a Semi\*regular open set.

**Remark 3.7:**  $\text{S}^*\text{RO}(X, \tau)$  is a topology if it is closed under finite intersection.

**Theorem 3.8:** If  $A$  is Semi\*regular open in  $X$ , then  $A$  can be expressed as  $A=U\cup B$  where (i)  $U$  is regular open in  $X$  (ii)  $B$  is nowhere dense in  $X$  (iii)  $U\cap B=\emptyset$ .

**Proof:** Since  $A$  is Semi\*regular open set in  $X$ , there is a regular open set  $U$  such that  $U\subseteq A\subseteq Cl^*(U)$ . Then  $A=U\cup B$  where  $B=A\setminus U$ . Then  $B\subseteq Cl^*(U)-U$  and therefore  $B$  is nowhere dense in  $X$ .

**Theorem 3.9:** Let  $A$  be a Semi\*regular open set and  $B\subseteq X$  such that  $A\subseteq B\subseteq Cl^*(A)$ . Then  $B$  is Semi\*regular open set.

**Proof:** Since  $A$  is Semi\*regular open set, there is a regular open set  $U$  in  $X$  such that  $U\subseteq A\subseteq Cl^*(U)$ . Then  $U\subseteq A\subseteq B\subseteq Cl^*(A)\subseteq Cl^*(U)$ . Hence  $B$  is Semi\*regular open.

**Theorem 3.10:**

- (i) Every Semi\*regular open set is Semi\* $\alpha$ -open.
- (ii) Every Semi\*regular open set is Semi\*pre-open.
- (iii) Every Semi\*regular open set is Semi\*open.
- (iv) Every Semi\* regular open set is semi open.
- (v) Every Semi\*regular open set is Semi  $\alpha$ -open.
- (vi) Every Semi\*regular open set is Semi pre-open.
- (vii) Every Semi\*regular open set is regular generalized open set.
- (viii) Every Semi\*regular open set is generalized pre regular open set.
- (ix) Every Semi\*regular open is regular weakly generalized open set.

**Proof:** Let  $A$  be a Semi\* regular open set. Then there is a regular open set  $U$  such that  $U\subseteq A\subseteq Cl^*(U)$ . Since every regular open set is  $\alpha$  open,  $A$  is semi\*  $\alpha$ -open. This proves (i).

Since every semi\* $\alpha$ -open set is semi\*preopen, this proves (ii).

Suppose  $A$  is a Semi\*regular open set. Then  $A\subseteq Cl^*(r-int(A))$  and since every regular open set is open,  $A\subseteq Cl^*(int(A))$ .

Thus  $A$  is Semi\*open. This proves (iii). (iv) follows from (iii) since every semi\*open set is semi open and (v) follows from (iv) since every semi open set is semi  $\alpha$ -open.

Since every  $\alpha$ -open set is pre-open, this proves (vi). Since  $A$  is Semi\*regular open set, there is a regular open set  $U$  in  $X$  such that  $U\subseteq A\subseteq Cl^*(U)$ . If we take  $A\subseteq U$ , then  $A$  is a regular open set and every regular open set is regular generalized open set. This proves (vii). In a similar way, we can prove (viii) and (ix).

**Remark 3.11:** The Converse of each of the statements in Theorem 3.10 is not true as shown in the following examples.

**Example 3.12:** Let  $X=\{a, b, c\}$  with the topology  $\tau = \{X, \Phi, \{a\}, \{a, b\}, \{a, c\}\}$ . The subset  $\{a, b\}$  is Semi\*  $\alpha$ -open but it is not Semi\*regular open.

**Example 3.13:** In the topological space  $(X, \tau)$  where  $X=\{a, b, c, d\}$   $\tau = \{X, \Phi, \{a\}, \{d\}, \{a, d\}, \{a, c, d\}\}$ , the subset  $\{a, b\}$  is semi\*pre-open but it is not Semi\*regular open.

**Example 3.14:** Let  $X=\{a, b, c, d\}$  with the topology  $\tau = \{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$ . The subset  $\{a, b, c\}$  is semi\* open but it is not Semi\*regular open.

**Example 3.15:** Let  $X= \{a, b, c, d\}$  with the topology  $\tau = \{X, \Phi, \{b\}, \{c\}, \{b, c\}, \{a, c, d\}\}$ . The subsets  $\{a, c\}$  and  $\{b, c\}$  are semi open but they are not semi\*regular open set.

**Example 3.16:** Consider the topological space  $(X, \tau)$  where  $X= \{a, b, c, d\}$   $\tau = \{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}\}$ . The subset  $\{a, b, d\}$  is semi  $\alpha$ -open but it is not semi\*regular open.

**Example 3.17:** Let  $X=\{a, b, c, d\}$  with the topology  $\tau = \{X, \Phi, \{a\}, \{c\}, \{a, c\}\}$ . The subset  $\{b, c, d\}$  is semi pre open but not semi\*regular open.

**Example 3.18:** Consider  $X=\{a, b, c, d\}$  with the topology  $\tau = \{X, \Phi, \{a\}, \{c, d\}, \{a, c, d\}\}$ . The subset  $\{a, d\}$  is regular generalized open set but not semi\*regular open.

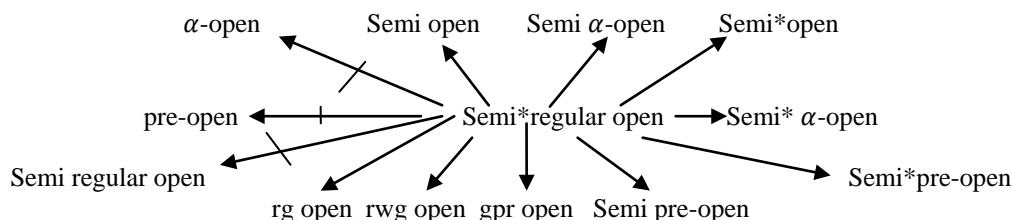
**Example 3.19:** Let  $X=\{a, b, c, d\}$  with the topology  $\tau = \{X, \Phi, \{a\}, \{d\}, \{a, b\}, \{a, b, d\}\}$ . The subset  $\{b, d\}$  is generalized pre open set but not semi\*regular open.

**Example 3.20:** Let  $X=\{a, b, c, d\}$  with the topology  $\tau = \{X, \Phi, \{a\}, \{b, c, d\}\}$ . The subset  $\{a, c, d\}$  is a regular weakly generalized open set but not semi\*regular open.

**Example 3.21:** Let  $X= \{a, b, c, d, e\}$  be a topological space with the topology  $\tau = \{X, \Phi, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}, \{a, b, c, d\}\}$ . The subset  $\{a, e\}$  is semi\*regular open but not  $\alpha$ -open.

**Example 3.22:** In the topological space  $(X, \tau)$  where  $X = \{a, b, c, d\}$  and  $\tau = \{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}\}$ , the subset  $\{a\}$  is semi\*regular open but not pre open.

From the above discussions we have the following diagram.



**Example 3.23:** Let  $(X, \tau)$  be a topological space where  $X=\{a, b, c, d\}$   $\tau =\{X, \phi, \{a\}, \{b, c, d\}\}$ . Here  $\tau = \alpha$ -open = regular open = semi open = semi  $\alpha$ -open= semi regular open=semi\*open = semi\*  $\alpha$ -open= semi\*regular open= $\{X, \phi, \{a\}, \{b, c, d\}\}$  and all the subsets of  $X$  are preopen, semi preopen and semi\*preopen.

**Results 3.24:** In a topological space  $(X, \tau)$ ,

- (i) Every semi regular open set is the union of a regular open set and the largest nowhere dense subset of  $X$ . Converse is also true.
- (ii) Every semi\*regular open set is the union of regular open set and nowhere dense subset of  $X$ . But the converse is not true.

**Example 3.25:** Consider the topological space  $X=\{a, b, c, d\}$  with the topology  $\tau =\{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ . Semi regular open set= $\{\{a, c, d\}, \{b, c, d\}\}$ . Here  $\{a\}, \{b\}$  are regular open sets and  $\{c, d\}$  is the largest nowhere dense subset of  $X$ .

**Example 3.26:** Consider the topological space  $X=\{a, b, c, d, e\}$  with the topology  $\tau =\{X, \Phi, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}, \{a, b, c, d\}\}$ . Here Regular open set =  $\{a\}$  and  $\{e\}$  is nowhere dense subset of  $X$ .  $\{a, e\}$  is the semi regular open set. Therefore the converse is true.

**Example 3.27:** Consider the topological space  $X=\{a, b, c, d\}$  with the topology  $\tau =\{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ .  $\{a, d\}$  is the semi\*regular open set. Here  $\{a\}$  is a regular open set and  $\{d\}$  is a nowhere dense.

**Example 3.28:** In the above topological space  $\{c, d\}$  is nowhere dense set but  $\{a, c, d\}$  is not a semi\*regular open set. Therefore the converse is not true.

**Remark 3.29:** In a extremally disconnected space and in a locally indiscrete space, regular open set = semi regular open set = semi\*regular open set.

In a  $T_{1/2}$  space, semi regular open set = semi\* regular open set.

**Example 3.30:** Consider  $X=\{a, b, c, d\}$   $\tau =\{X, \Phi, \{b\}, \{c\}, \{b, c\}, \{a, c, d\}\}$ . Then  $(X, \tau)$  is a extremally disconnected space. Regular open set=semi regular open set=semi\*regular open set =  $\{\{b\}, \{a, c, d\}\}$ .

**Example 3.31:** Consider  $X=\{a, b, c, d\}$  with the topology  $\tau =\{X, \Phi, \{a\}, \{b, c, d\}\}$ . Then  $(X, \tau)$  is a locally indiscrete space Regular open set=semi regular open set=semi\*regular open set =  $\{\{a\}, \{b, c, d\}\}$ .

**Example 3.32** Consider  $X=\{a, b, c\}$  with the topology  $\tau =\{X, \Phi, \{a\}, \{b\}, \{a, b\}\}$ . Then  $(X, \tau)$  is a  $T_{1/2}$  space, semi regular open set = semi\*regular open set =  $\{\{a, c\}, \{b, c\}\}$ .

**Theorem 3.33:** Let  $A$  be semi\*regular open set and  $B \subseteq X$  such that  $r-int(A) \subseteq B \subseteq Cl^*(A)$ . Then  $B$  is semi\*regular open.

**Proof:** Since  $A$  is semi\*regular open, by Theorem 3.3, we have  $Cl^*(A) = Cl^*(r-int(A))$ . Since  $r-int(A) \subseteq B$ ,  $r-int(A) \subseteq r-int(B)$  and hence  $Cl^*(r-int(A)) \subseteq Cl^*(r-int(B))$ . Therefore by assumption, we have  $B \subseteq Cl^*(A) = Cl^*(r-int(A)) \subseteq Cl^*(r-int(B))$ . Hence  $B \subseteq Cl^*(r-int(B))$ . Again by using theorem 3.3,  $B$  is semi\*regular open.

#### 4. SEMI\*REGULAR CLOSED SET

**Definition 4.1:** A Subset  $A$  of a topological space  $(X, \tau)$  is called a Semi\*regular closed sets if the complement  $A^c$  of  $A$  is Semi\*regular open.

**Theorem 4.2:** If  $\{A_\alpha\}$  is a collection of Semi\*regular closed sets, then  $\bigcap A$  is also a Semi\*regular closed set.

**Proof:**  $A_\alpha$  is Semi\*regular closed sets for all. Then  $X - A$  is Semi\*regular open for all  $\alpha$  (ie)  $\bigcup (X - A_\alpha)$  is semi\*regular open. Then  $X - \bigcap A$  is semi\*regular open. Therefore  $\bigcap A$  is semi\*regular closed.

**Remark 4.3:** Union of two semi\*regular closed sets need not be semi\* regular closed as seen from the following example.

**Example 4.4:** Consider the space  $(X, \tau)$  with  $X = \{a, b, c, d\}$   $\tau = \{X, \Phi, \{a\}, \{d\}, \{a, d\}, \{a, c, d\}\}$ . The subsets  $\{a, c\}$  and  $\{c, d\}$  are semi\*regular closed sets but  $\{a, c, d\}$  is not semi\*regular closed set.

**Theorem 4.5:**

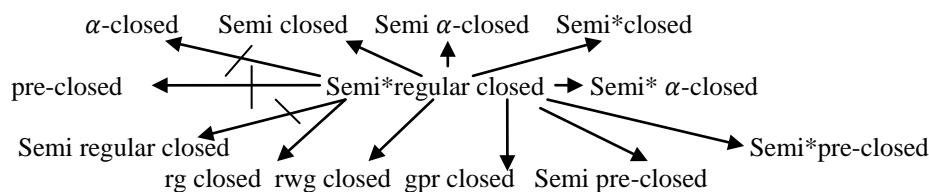
- (i) Every Semi\*regular closed set is Semi\* $\alpha$ - closed.
- (ii) Every Semi\*regular closed set is Semi\*pre- closed.
- (iii) Every Semi\*regular closed set is Semi\* closed.
- (iv) Every Semi\* regular closed set is Semi closed.
- (v) Every Semi\*regular closed set is Semi  $\alpha$ - closed.
- (vi) Every Semi\*regular closed set is Semi pre- closed.
- (vii) Every Semi\*regular closed set is regular generalized closed set.
- (viii) Every Semi\*regular closed set is generalized pre regular closed set.
- (ix) Every Semi\*regular closed set is regular weakly generalized closed set.

**Proof:** Let  $A$  be a Semi\* regular closed set. Then  $A^c$  is Semi\*regular open set. Then there is a regular open set  $U$  such that  $U \subseteq A^c \subseteq Cl^*(U)$ . Since every regular open set is  $\alpha$  open,  $A^c$  is semi\*  $\alpha$ -open. Therefore  $A$  is Semi\*  $\alpha$ - closed. This proves (i). In a similar way, remaining proof of this theorem follows from theorem 3.10.

**Remark 4.6:** In some Topologies, semi\*regular closed set does not imply  $\alpha$ -closed, pre closed and semi regular closed.

**Example 4.7:** Consider the topological space  $X = \{a, b, c, d\}$  with the topology  $\tau = \{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ . The subsets  $\{a, c\}$  and  $\{b, c\}$  are semi\*regular closed but not  $\alpha$ -closed, pre closed and semi regular closed.

From the above discussions we have the following diagram.



**Theorem 4.8:** A subset  $A$  of a topological space  $(X, \tau)$  is semi\*regular closed set if and only if there is a regular closed set  $B$  in  $(X, \tau)$  such that  $Int^*(B) \subseteq A \subseteq B$  where  $Int^*(B)$  is the generalized interior of  $B$ .

**Proof:** Suppose  $A$  is semi\*regular closed set. Then  $A^c$  is semi\*regular open. Then there exists a regular open set  $U$  in  $X$  such that  $U \subseteq A^c \subseteq Cl^*(U)$ . (ie)  $U^c \supseteq A \supseteq (Cl^*(U))^c$

Since  $(Cl^*(U))^c = Int^*(U^c)$ ,  $U^c \supseteq A \supseteq Int^*(U^c)$  where  $U^c$  is regular closed set in  $X$ .

Conversely suppose there is a regular closed set  $B$  in  $(X, \tau)$  such that  $Int^*(B) \subseteq A \subseteq B$

(ie)  $(Int^*(B))^c \supseteq A^c \supseteq B^c$ . Since  $[Int^*(B)]^c = Cl^*(B^c)$  and  $B^c$  is a regular open set, by the definition 3.1,  $A^c$  is semi\*regular open hence  $A$  is Semi\*regular closed set.

#### 5. SEMI\*r-INTERIOR OF A SET

**Definition 5.1:** The semi\*regular-interior of  $A$  is defined as the union of all semi\*regular open sets contained in  $A$  and is denoted by  $S^*rInt(A)$ .

**Definition 5.2:** Let  $A$  be a subset of a topological space  $(X, \tau)$ . A point  $x$  in  $X$  is called a semi\*-r-interior point of  $A$  if there is a semi\*regular open subset of  $A$  that contains  $x$ .

**Theorem 5.3:** If  $A$  is any subset of a topological space  $(X, \tau)$ , then

- (i)  $S^*rInt(A)$  is the largest semi\* regular open set contained in  $A$ .
- (ii)  $A$  is semi\*regular open set if and only if  $S^*rInt(A)=A$
- (iii)  $S^*rInt(A)$  is the set of all semi\*-r-interior points of  $A$ .
- (iv)  $A$  is semi\*regular open if and only if every point of  $A$  is a semi\*-r-interior point of  $A$ .

**Proof:**

- (i) Being the union of all semi\*regular open subsets of  $A$ , by theorem 3.4,  $S^*rInt(A)$  is semi\*regular open and contains every semi\*regular open subset of  $A$ . This proves (i)
- (ii)  $A$  is semi\*regular open implies  $s^*rInt(A) = A$ .  
On the other hand, Suppose  $s^*rInt(A) = A$ . By (i),  $s^*rInt(A)$  is semi\* regular open and hence  $A$  is semi\* regular open.
- (iii) By definition 5.2,  $x \in s^*rInt(A)$  if and only if  $x$  belongs to some semi\*regular open subset  $U$  of  $A$ . (ie) if and only if  $x$  is a semi\*-r-interior point of  $A$ .
- (iv) follows from (ii) and (iii)

**Theorem 5.4:** In any topological space  $(X, \tau)$ , the following hold

- (i)  $s^*rInt(\Phi)=\Phi$
- (ii)  $s^*rInt(X)=X$
- (iii) If  $A$  is the subset of  $X$ ,  $s^*rInt(A) \subseteq A$
- (iv) If  $A$  and  $B$  are subsets of  $X$  and  $A \subseteq B$  then  $s^*rInt(A) \subseteq s^*rInt(B)$
- (v)  $s^*rInt(s^*rInt(A))=s^*rInt(A)$
- (vi)  $s^*rInt(A) \subseteq s^*int(A) \subseteq s^* \alpha Int(A) \subseteq s^*pInt(A) \subseteq A$
- (vii)  $srInt(A) \subseteq s \alpha Int(A) \subseteq spInt(A) \subseteq A$
- (viii)  $s^*rInt(A) \cup s^*rInt(B) \subseteq s^*rInt(A \cup B)$
- (ix)  $s^*rInt(A \cap B) \subseteq s^*rInt(A) \cap s^*rInt(B)$
- (x)  $rInt(s^*rInt(A))=rInt(A)$  only if  $A$  is semi\*regular open.

**Proof:** (i), (ii), (iii) and (iv) follow from definition 5.2. By theorem 5.3(i),  $S^*rInt(A)$  is semi\*regular open and by theorem 5.3(ii),  $S^*rInt(S^*rInt(A))=S^*rInt(A)$ . Thus (v) is proved and (vi) follows from theorem 3.8, the fact that every semi\*regular open set is semi\*open, every semi\*open set is semi\*  $\alpha$ -open and every semi\*  $\alpha$ -open set is semi\*pre open. (vii) follows from the result that every semi regular open set is semi  $\alpha$ -open set and every semi  $\alpha$ -open set is semi pre open set. Since  $A \subseteq A \cup B$ , from (iv) we have  $S^*rInt(A) \subseteq S^*rInt(A \cup B)$ . Similarly  $S^*rInt(B) \subseteq S^*rInt(A \cup B)$ . This proves (viii). In a similar way we can prove (ix). Since  $A$  is semi\*regular open, by theorem 5.3. (ii),  $S^*rInt(A) = A$ . This proves (x).

**Remark 5.5:** The inclusions in (vi), (vii) (viii) and (ix) of Theorem 5.4 may be strict and equality may also hold.

**Example 5.6:** Consider the topological space with  $X = \{a, b, c, d, e\}$  and  $\tau = \{X, \Phi, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}, \{a, b, c, d\}\}$ .

Let  $A = \{a, b, d, e\}$ . Then  $s^*rInt(A) = \{a, e\}$ ,  $s^*Int(A) = \{a, e\}$ ,  $s^* \alpha Int(A) = \{a, e\}$ ,  $s^*Int(A) = \{a, b, d, e\}$ .

Here  $s^*rInt(A) = s^*Int(A) = s^* \alpha Int(A) \subseteq s^*pInt(A)$

Let  $X = \{a, b, c, d\}$  with the topology  $\tau = \{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ . Let  $A = \{a, b, c\}$ . Then  $s^*rInt(A) = \Phi$ ,  $s^*Int(A) = \{a, b, c\}$ ,  $s^* \alpha Int(A) = \{a, b, c\}$ ,  $s^*Int(A) = \{a, b, c\}$

Here  $s^*rInt(A) \subseteq s^*Int(A) = s^* \alpha Int(A) = s^*Int(A) = A$

**Example 5.7:** Let  $(X, \tau)$  be a topological space where  $X = \{a, b, c, d\}$  and  $\tau = \{X, \Phi, \{a\}, \{d\}, \{a, d\}, \{a, c, d\}\}$ . Let  $A = \{a, b, d\}$ . Then  $srInt(A) = \Phi$ ,  $s \alpha Int(A) = spInt(A) = \{a, b, d\}$

Here  $srInt(A) \subseteq s \alpha Int(A) = spInt(A) = A$

Let  $A = \{b, c, d\}$  Here  $srInt(A) = s \alpha Int(A) = spInt(A) = A$

**Example 5.8:** Consider the topological space in example 5.6.

Let  $A = \{a, e\}$   $B = \{b, c, d\}$   $A \cup B = \{a, b, c, d, e\}$   $A \cap B = \Phi$   
 $s^*rInt(A) = \{a, e\}$   $s^*rInt(B) = \Phi$   $s^*rInt(A \cup B) = X$ ,  $s^*rInt(A \cap B) = \Phi$

Here  $s^*rInt(A) \cup s^*rInt(B) \subseteq s^*rInt(A \cup B)$  and  $s^*rInt(A \cap B) \subseteq s^*rInt(A) \cap s^*rInt(B)$

**Example 5.9:** Consider the topological space in example 3.27.

Let  $A = \{a, d\}$   $B = \{b, d\}$   $A \cup B = \{a, b, d\}$

Then  $s^*rInt(A) = \{a, d\}$ ,  $s^*rInt(B) = \{b, d\}$ ,  $s^*rInt(A \cup B) = \{a, b, d\}$ .

Here  $s^*rInt(A \cup B) = s^*rInt(A) \cup s^*rInt(B)$

Let  $A = \{a, d\}$ ,  $B = \{a, c, d\}$ ,  $A \cap B = \{a, d\}$

Then  $s^*rInt(A) = \{a, d\}$ ,  $s^*rInt(B) = \{a, d\}$ ,  $s^*rInt(A \cap B) = \{a, d\}$

Here  $s^*rInt(A \cap B) = s^*rInt(A) \cap s^*rInt(B)$

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