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REAL AND CLONE DOMINATION NUMBER OF SEMI COMPLEMENTARY SPLITTING GRAPH

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ABSTRACT

A subset D of R is said to be a Real dominating set of S′(G), if every vertex v in R – D is R – adjacent to a vertex of D. The minimum cardinality of vertices in such a set is called the R – domination number of S′(G) and is denoted by $\gamma_R(S'(G))$. A subset D of R is said to be a Clone dominating set of S′(G), if every vertex v′ ϵ C is C – adjacent to a vertex of D. The minimum cardinality of vertices in such a set is called the Clone domination number of S′(G) and is denoted by $\gamma_C(S'(G))$.

KeyWords: *Semi Complementary Splitting Graph -Real adjacent-Clone adjacent –Real domination –Clone domination.*

1. INTRODUCTION

By a graph, we mean a finite simple and undirected graph. The vertex set and edge set of a graph G denoted are by V(G) and E(G) respectively. The **minimum degree** of G is defined as δ(G) = min{deg (v) | v ∈ G}.

The number of **end** vertices in G are denoted by ĕ. For a graph G, the splitting graph S′ of G is obtained by adding a new vertex v′ corresponding to each vertex v of G such that N(v) = N(v′) and it is denoted by S′(G). The new vertex v′ is called the clone vertex of the real vertex v. In S′(G), the vertices are partitioned into two sets R and C. The real vertices are in the set R and the clone vertices are in the set C .

2. PRELIMINARIES

In this paper we discuss **"Real and Clone Domination Number of Semi Complementary Splitting Graph"** for some Standard graphs and some special graphs. We give a brief summary of definitions and other information which are useful for our present investigation.

Definition: 2.1 Semi - Complementary Splitting Graph SCS′(G) is the graph having both real and clone vertices of S′(G), but only the real vertices of S′(G) which are non – adjacent with both the real and clone vertices of S′(G) that are adjacent with both the vertices in SCS′(G).

Example: 2.2 The Semi – Complementary Splitting graph of C_5 - SCS′ (C_5) as shown in Figure: 2.1

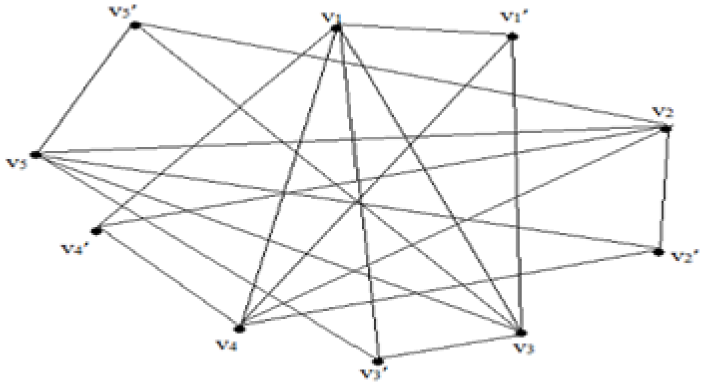

Fig : 2.1

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Definition: 2.3 A dominating set $S \subseteq V(G)$ is a **connected dominating set**, if the induced sub graph S has no isolated vertices. The connected domination number, $\boldsymbol{\gamma_c(G)}$ of G is the minimum cardinality of a connected dominating set of G.
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Definition: 2.4 Two vertices v_i and v_j in R of $S'(G)$ are **Real – adjacent**, if v_i and v_j belongs to the same edge of $S'(G)$ or an edge E_1 containing v_i and an edge E_2 containing v_j are adjacent. We write R – adjacent for Real – adjacent for the sake of convenience.

Definition: 2.5 Two vertices $v_i \in R$ and $v_i' \in C$ are **Clone – adjacent** if v_i and v_i' belongs to the same edge of $S'(G)$. We write C – adjacent for Clone – adjacent for the sake of convenience.

Definition: 2.6 A subset D of R is said to be a **Real dominating set of $S'(G)$**, if every vertex v in R – D is R – adjacent to a vertex of D. The minimum cardinality of vertices in such a set is called the R –domination number of $S'(G)$ and is denoted by $\boldsymbol{\gamma_R(S'(G))}$. Also, we can define Real domination number of $SCS'(G)$ and is denoted by $\boldsymbol{\gamma_R(SCS'(G))}$. We write R - dominating for Real dominating for the sake of convenience.

Definition: 2.7 A subset D of R is said to be a **Clone dominating set** of $S'(G)$, if every vertex $v' \epsilon C$ is C – adjacent to a vertex of D. The minimum cardinality of vertices in such a set is called the Clone domination number of $S'(G)$ and is denoted by $\boldsymbol{\gamma_C(S'(G))}$. Also, we can define Clone domination number of SC $S'(G)$ and is denoted by $\boldsymbol{\gamma_C(SCS'(G))}$. We write C - dominating for Clone dominating for the sake of convenience.

Example: 2.8 Consider the graph G as shown in the figure 2.2.

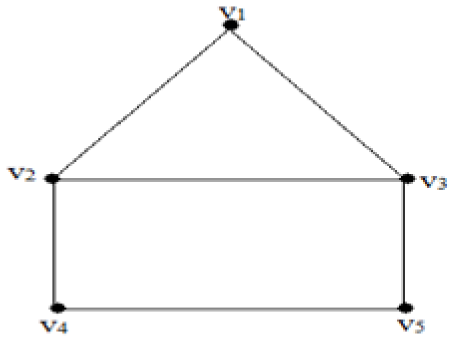


Fig :2.2

We have the following Splitting Graph $S'(G)$ as shown in the figure 2.3.

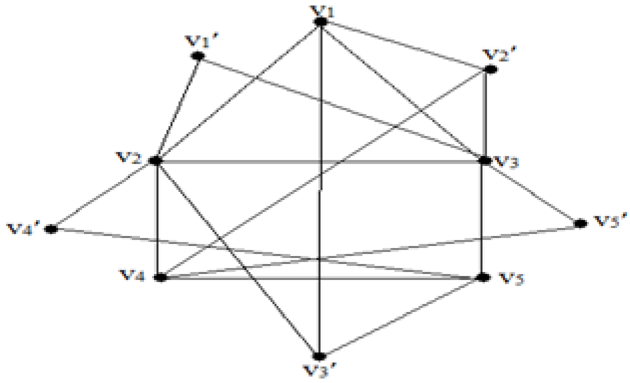


Fig : 2.3

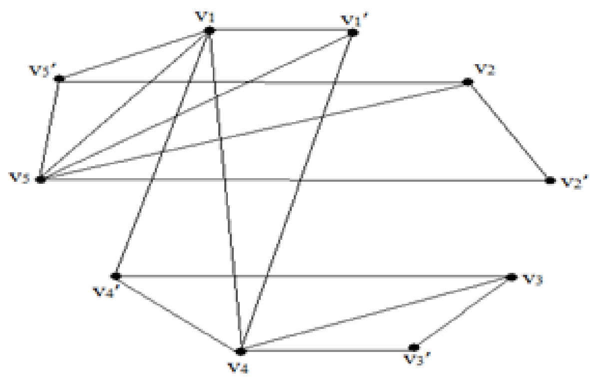


Fig : 2. 4

For the graph G as shown in the figure 2.2, we have the Semi- Complementary Splitting Graph SCS′(G) as shown in the fig: 2.4. For SCS′(G), we have the R - dominating set $\{v_1\}$ and therefore $\gamma_R(SCS'(G)) = 1$; C - dominating set $\{v_1, v_3, v_5\}$ and therefore $\gamma_C(SCS'(G)) = 3$.

3. REAL AND CLONE DOMINATION IN SCS′(G) –STANDARD GRAPHS

Initially, we give the $\gamma_R(SCS'(G))$ and $\gamma_C(SCS'(G))$ for some standard graphs, which are straight forward in the following theorem:

Theorem: 3.1

a) For any path P_n with $n \geq 4$ vertices, $\gamma_R(SCS'(P_n)) = 1$ and $\gamma_C(SCS'(P_n)) = 2$.
b) For any cycle C_n with $n \geq 4$ vertices, $\gamma_R(SCS'(C_n)) = 1$ and $\gamma_C(SCS'(C_n)) = 2$.

Example: 3.2 For P_4, we have the following Semi – Complementary Splitting Graph as shown in figure: 3.1.

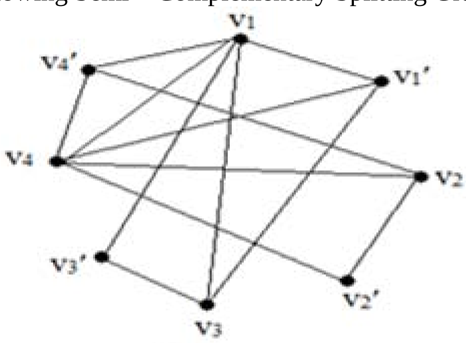


Fig : 3.1

For $SCS'(P_4)$, we have the R-dominating set $\{v_1\}$and therefore $\gamma_R(SCS'(P_4)) = 1$; C-dominating set $\{v_1, v_4\}$and therefore $\gamma_C(SCS'(P_4)) = 2$.

For C_5, we have the Semi – Complementary Splitting Graph as shown in figure: 2.1. For $SCS'(C_5)$, we have the R-dominating set $\{v_1\}$ and therefore $\gamma_R(SCS'(C_5)) = 1$; C-dominating set $\{v_1, v_2\}$and therefore $\gamma_C(SCS'(C_5)) = 2$.

Theorem: 3.3 Let G be a complete graph. Then R dominating does not exist for SCS′(G) and C dominating is totally disconnected for SCS′(G).

Proof: As, G is a complete graph, any two vertices in G are adjacent. Therefore we have the splitting graph S′(G) such that any two vertices are adjacent except the vertex v_i and their corresponding clone vertex v_i' which are not adjacent. Thus the corresponding SCS′(G) of S′(G), only the real vertices are adjacent with their corresponding clone vertices. Hence no two vertices in R are R – adjacent that is R – dominating does not exist for SCS′(G). Also all the real vertices are C – adjacent only with their corresponding clone vertices only that is v_i is C – adjacent with v_i' only. Therefore C dominating for SCS′(G) is totally disconnected.

Theorem: 3.4 Let G be a complete graph. Then $\gamma_R(S'(G)) + \gamma(G) = \gamma_c(G)$.

Proof: As, G is a complete graph, any two vertices in G are adjacent. Therefore, we have $D = \{x\}$ is a minimal dominating set for G. Obviously; $< D >$ is not connected. Thus D does not form a connected dominating set for G. Therefore, there exists at least one vertex $v \in V(G) - D$ such that $D \cup \{v\}$ forms a minimal connected dominating set for G. Also, $F = \{v\}$is a minimal dominating set for G, since G is a complete Graph. Suppose $F \neq \{v\}$, which is a contradiction to the definition of dominating set for G. Hence, $| D | \cup | F |$ forms a minimal connected dominating set for G. Therefore, $| D | \cup | F | = \gamma_c(G)$. That is $\gamma_R(S'(G)) + \gamma(G) = \gamma_c(G)$.

Theorem: 3.5 Let G be any connected graph and it has at most one vertex of degree $p-1$. Then R domination number for SCS′(G) does not exist.

Proof: Let G be a connected graph. Then every two vertices in G are connected. Let $u \in R$ be the vertex of degree $p - 1$. Then $N[u] = V(G)$. Thus we have the Semi – Complementary Splitting graph G such that u is adjacent with u′ only. Therefore, u is R– adjacent with no vertex in real vertices of SCS′(G). Hence, R – dominating set for SCS′(G) does not exist and so that R – domination number for SCS′(G) does not exist.

Observation: 3.6 For all the Wheel graph (W_n) with $n \geq 5$ vertices, we have R – domination for SCS′(G) does not exist and $\gamma_C(SCS'(G)) = \delta(G)$.

4. REAL AND CLONE DOMINATION IN SCS′(G) – SPECIAL GRAPHS

Initially, we give the $\gamma_R(S'(G))$, $\gamma_C(S'(G))$ and $\gamma_R(SCS'(G))$, $\gamma_C(SCS'(G))$ domination number for some **s**pecial graphs, which are straight forward in the following Theorem.

Definition: 4.1 Any cycle with a pendant edge attached at each vertex is called **Crown graph** and is denoted by $\mathbf{C_n^+}$.

Example: 4.2 Consider the crown graph s C_3^+, C_4^+ as shown in figure 4.1 and 4.2.

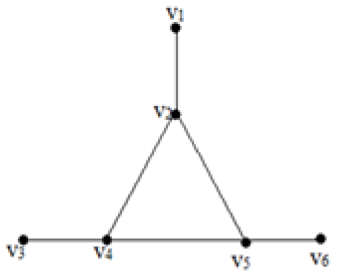


Figure : 4.1 - C_3^+

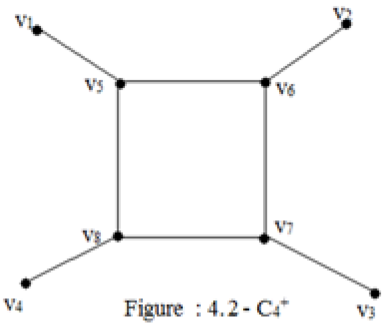


Figure : 4.2 - C_4^+

For C_3^+, we have $\gamma_R(S'(C_3^+)) = 1$; $\gamma_C(S'(C_3^+)) = 3$; $\gamma_R(SCS'(C_3^+)) = 1$; $\gamma_C(SCS'(C_3^+)) = 2$.

For C_4^+, we have $\gamma_R(S'(C_4^+)) = 2$; $\gamma_C(S'(C_4^+)) = 4$; $\gamma_R(SCS'(C_4^+)) = 1$; $\gamma_C(SCS'(C_4^+)) = 2$.

Observation: 4.3 Let G be a Crown Graph C_n^+ ($n \geq 3$). Then

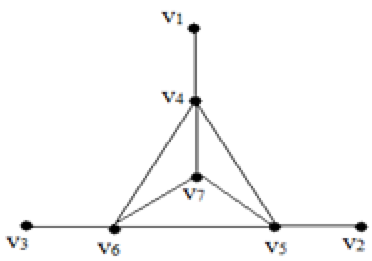
a) $\gamma_R(S'(C_n^+)) = \begin{cases} \frac{n}{3} & if\ n \equiv o\ (mod\ 3) \\ \left\lceil \frac{n}{3} \right\rceil & otherwise \end{cases}$

b) $\gamma_C(S'(C_n^+)) =$ ĕ

c) $\gamma_R(SCS'(C_n^+)) = 1$

d) $\gamma_C(SCS'(C_n^+)) = 2$.

Definition: 4.4 A Helm Graph ($\mathbf{H_n}$) is the graph obtained from an n -wheel graph by adjoining a pendant edge at each vertex of the cycle.



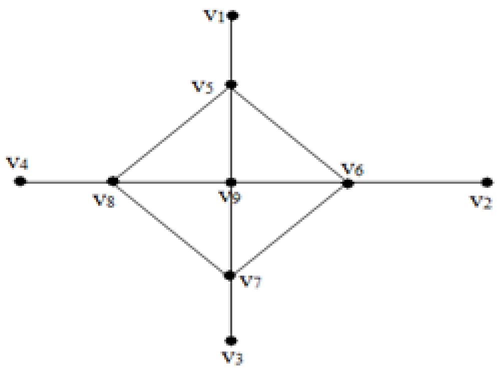


Figure 4.3 – H_3

Figure 4.4 – H_4

Example: 4.5 From Figure: 4.3, we have $\gamma_R(S'(H_3)) = 1$; $\gamma_C(S'(H_3)) = 3$; $\gamma_R(SCS'(H_3)) = 1$; $\gamma_C(SCS'(H_3)) = 2$.

From Figure: 4.4,we have $\gamma_R(S'(H_4)) = 2$; $\gamma_C(S'(H_4)) = 4$; $\gamma_R(SCS'(H_4)) = 1$; $\gamma_C(SCS'(H_4)) = 2$.

Observation: 4.6 Let G be a Helm graph H_n ($n \geq 3$). Then

a) $\gamma_R(S'(H_n)) = 1 = \gamma_R(SCS'(H_n))$

b) $\gamma_C(S'(H_n)) =$ ĕ

c) $\gamma_C(SCS'(H_n)) = 2$.

Definition: 4.7 An **n – Barbell Graph** is the simple graph obtained by connecting two copies of a complete graph K_n by a bridge.

Example: 4.8 Consider the 3 and 4-barbell graphs as shown in figure: 4.5 and 4.6

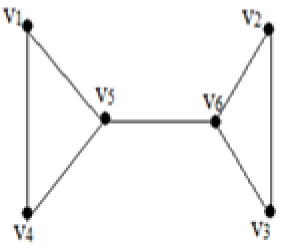


Figure : 4.5 – n = 3

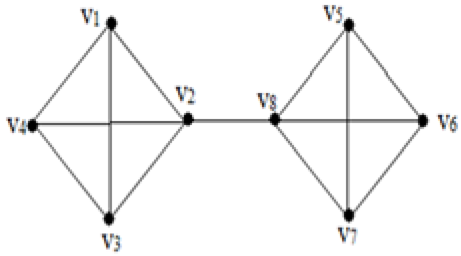


Figure 4.6 – n = 4

For 3 – barbell graphs, we have $\gamma_R(S'(G)) = 1$; $\gamma_C(S'(G)) = 2$; $\gamma_R(SCS'(G)) = 1$; $\gamma_C(SCS'(G)) = 2$.

For 4 – barbell graphs, we have $\gamma_R(S'(G)) = 1$; $\gamma_C(S'(G)) = 2$; $\gamma_R(SCS'(G)) = 1$; $\gamma_C(SCS'(G)) = 2$.

Observation: 4.9 Let G be a barbell graph ($n \geq 3$). Then $\gamma_R(S'(G)) = 1 = \gamma_R(SCS'(G))$; $\gamma_C(S'(G)) = 2 = \gamma_C(SCS'(G))$.

Definition: 4.10 The graph $\mathbf{C_m^{(t)}}$ denote the one - point union of t cycles of length m. If m = 3, then $C_m^{(t)}$ is called the **Dutch t – windmill** graph .

Example: 4.11 For t = 2, 3, we have the following Dutch t – windmill graphs.

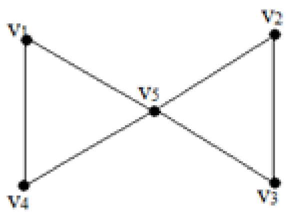


Figure 4.7 : $C_3^{(2)}$

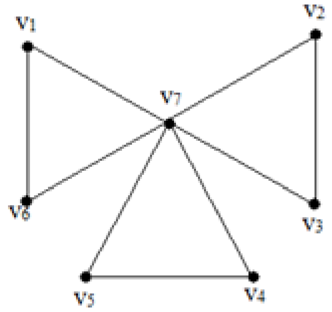


Figure 4.8: $C_3^{(3)}$

For $C_3^{(2)}$, we have $\gamma_R(S'(C_3^{(2)})) = 1$; $\gamma_C(S'(C_3^{(2)})) = 2$. Also $\gamma_R(SCS'(C_3^{(2)}))$ does not exist and $\gamma_C(SCS'(C_3^{(2)})) = 3$.

For $C_3^{(3)}$, we have $\gamma_R(S'(C_3^{(3)})) = 1$; $\gamma_C(S'(C_3^{(3)})) = 2$. Also $\gamma_R(SCS'(C_3^{(3)}))$ does not exist and $\gamma_C(SCS'(C_3^{(3)})) = 3$.

Observation: 4.12 Let G be $C_m^{(t)}$ - Dutch t - windmill graph and $t \geq 2$. Then
i) $\gamma_R(S'(G)) = 1$
ii) $\gamma_C(S'(G)) = 2$
iii) $\gamma_R(SCS'(G))$ does not exist and $\gamma_C(SCS'(G)) = 3$.

Definition: 4.13 The **Friendship graph** F_n can be constructed by joining n copies of the cycle graph C_3 with a common vertex.

Example: 4.14 Consider the Friendship graphs F_3 and F_4 as shown in fig 4.9 and 4.10.

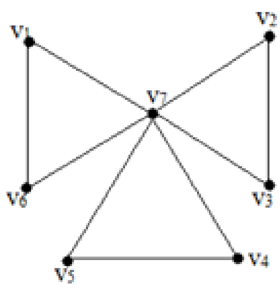


Figure 4.9 : F_3

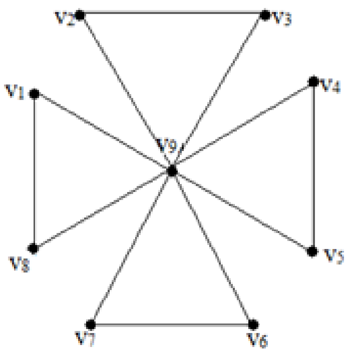


Figure 4.10 : F_4.

For F_3, we have $\gamma_R(S'(F_3)) = 1$; $\gamma_C(S'(F_3)) = 2$. Also $\gamma_R(SCS'(F_3))$ does not exist and $\gamma_C(SCS'(F_3)) = 3$.

For F_4, we have $\gamma_R(S'(F_4)) = 1$; $\gamma_C(S'(F_4)) = 2$. Also $\gamma_R(SCS'(F_4))$ does not exist and $\gamma_C(SCS'(F_4)) = 3$.

Observation: 4.15 Let G be Friendship graph F_n ($n \geq 2$). Then

i) $\gamma_R(S'(G)) = 1$
ii) $\gamma_C(S'(G)) = 2$
iii) $\gamma_R(SCS'(G))$ does not exist and $\gamma_C(SCS'(G)) = 3$

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