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SUPER FIBONACCI GRACEFUL LABELING FOR (a, m)- SHELLGRAPH

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ABSTRACT

A (p,q)-graph G=(V, E) has super Fibonacci graceful labeling if there exists an injective map f:V(G) →{$F_0,F_1,F_2,\ldots,F_q$} where F_k is the k^{th} Fibonacci number of the Fibonacci series such that its induced map f^+: E(G) →{$F_1,F_2,\ldots,F_q$} defined by $f^+(xy) = |f(x) - f(y)|$ for all edge xy in G, is injective. In this paper, the existence of super Fibonacci graceful labeling for the various types of (a, m) – shell graph and all the above (a, m) - shell graph merged with a fan.

Keywords: *Graceful graph, Fibonacci graceful graph, super Fibonacci graceful graph, shell graph, (a, m)–shellgraph.*

SECTION 1 –INTRODUCTION

The notions of Fibonacci graceful labeling and super Fibonacci graceful labeling were introduced by Kathiraesan and Amutha [2006] and they proved [2010] that Path (P_n), caterpillar and bi-star ($B_{m,n}$) are Fibonacci graceful. Also they proved every fan (F_n) graph is super Fibonacci graceful. Jeba Jesintha and Ezhilarasi Hilda [2015] obtain the graceful labeling for Shell – Butterfly graph. Meena, Renugha and Sivasakthi [2015] found cordial Labeling for different types of shell graph, shell graph merging with path, shell graph merging with star, multiple shell graph. These results on super Fibonacci graceful labeling and other results are mentioned in Gallian survey [2011].

SECTION 2 - BASIC DEFINITIONS

Definition 2.1: Graceful graph: A (p, q)-graph G= (V, E) has graceful labeling if there exists an injection map f : V(G)→{0,1,2,…,q} such that its induced map f^+: E(G)→{1,2,3,…,q} defined by $f^+(xy)=|f(x) - f(y)|$ for each edge xy in G, is injective. Then the graph G is graceful.

Definition 2.2:Fibonacci graceful graph: A (p, q)-graph G = (V,E) has Fibonacci graceful labeling if there exists an injective map f : V(G) →{$0,1,2,\ldots,F_q$} where F_k is the k^{th} Fibonacci number in the Fibonacci series such that its induced map f^+: E(G) →{$F_1,F_2,\ldots,F_q$} defined by $f^+(xy) = |f(x) - f(y)|$ for every edge xy in G, is injective.

Definition 2.3: Super Fibonacci graceful graph: A (p, q)-graph G = (V, E) has super Fibonacci graceful labeling if there exists an injective map f : V(G)→{$F_0,F_1,F_2,\ldots,F_q$} where F_k is the k^{th} Fibonacci number in Fibonacci series such that its induced map f^+: E(G) →{$F_1,F_2,\ldots,F_q$} defined by $f^+(xy)=|f(x) - f(y)|$ for every edge xy in G, is injective.

Definition 2.4: Shell graph: A shell $S_{n,n-3}$ of width n is a graph obtained by taking (n-3) concurrent chords in a cycle C_n on n vertices. The vertex at which all the chords are concurrent is called apex. The two vertices adjacent to the apex have degree 2, apex has degree (n-1) and all the other vertices have degree 3.

Definition 2.5: (a, m) – Shell graph ($SH_{a,m}$): Let a ≥ 2 and m ≥ 2be integer and n = am + (a+2). An (a, m)–shell is a cycle C_n with verticesv$_1$,v$_2$, …., v_n together with m concurrent chords (edges) at v_1 (Apex) and other ends of chords are adjacent to the vertex v_{ai+2}, where i = 1, 2, 3,…, m. Here the degree of v_1 is (m + 2), and the degree of v_{ai+2}, (where i = 1, 2,…,m) is 3. All the remaining vertices are of degree 2.

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SECTION 3: SUPER FIBONACCI GRACEFUL LABELING FOR VARIOUS TYPES OF SHELL MODEL GRAPHS

Theorem 3.1: A (4, m)- shell graph is super Fibonacci graceful, for all integers $m \geq 2$.

Proof: Let G be the graph (a, m)-shell graph whose vertex set is $\{v_1,\dots, v_n\}$ and edge set is $\{\{v_i v_{i+1}\}_{i=1}^{n-1} \cup \{v_1 v_{4i+2}\}_{i=1}^{m+1}\}$ where n = 4m + 6andm ≥ 2 is any integers.

Construct an injective map f: V(G) → $\{F_0,F_1,F_2,\dots,F_q\}$ as follows:

Let i vary 1 to n.

$f(v_1) = F_0$;
$f(v_i) = F_{5k+(r-1)}$; where i = 4k + r; k = 0,1,2,…,(m +1); r = 2;
$f(v_i) = F_{5k+r}$; where i = 4k + r; k = 0,1,2,…,m; r = 3;
$f(v_{4k}) = F_{5k}$; where k = 1,2,…,(m +1);
$f(v_{4k+1}) = F_{(5k-1)}$; where k = 1,2,…,(m +1);

The induced map f^+ from f defined by $f^+(xy) = |f(x) - f(y)|$ for each edges xy in G, is injective. Thus the given graph has super Fibonacci graceful labeling from the injective maps f and f^+. Hence the graph (4, m)-shell graph is super Fibonacci graceful.

Theorem 3.2: A(7, m) – shell graph is super Fibonacci graceful, for all integers $m \geq 2$.

Proof: Let G be the graph (7,m)-shell graph whose vertex set is $\{v_1,\dots, v_n\}$ and edge set is $\{\{v_i v_{i+1}\}_{i=1}^{n-1} \cup \{v_1 v_{7i+2}\}_{i=1}^{m+1}\}$ where n = 7m + 6 and m ≥ 2 is any integers.

Define an injective map f: V(G) → $\{F_0,F_1,F_2,\dots,F_q\}$ as follows:

Let i be in {1, 2, …, n}.

$f(v_1) = F_0$;
$f(v_i) = F_{8k+(r-1)}$; where i = 7k + r; k = 0,1,2,…,(m+1); r = 2,5;
$f(v_i) = F_{8k+r}$; where i = 7k + r; k = 0,1,2,…,m; r = 3,6;
$f(v_i) = F_{8k+(r+1)}$; where i = 7k + r; k = 0,1,2,…,m; r = 4;
$f(v_{7k}) = F_{8k}$; where k = 1,2,…,(m+1);
$f(v_{7k+1}) = F_{(8k-1)}$; where k = 1,2,…,(m+1);

The induced map f^+ of f defined by $f^+(xy) = |f(x) - f(y)|$ for each edges xy in G, is injective. Thus the given graph has super Fibonacci graceful labeling from the injective maps f and f^+. Hence the graph (7, m)-shell graph is super Fibonacci graceful.

Theorem 3.3:A(10, m) – shell graph is super Fibonacci graceful, for all integers $m \geq 2$.

Proof: Let G be the graph (10, m)-shell graph whose vertex set is $\{v_1,\dots, v_n\}$ and edge set is $\{\{v_i v_{i+1}\}_{i=1}^{n-1} \cup \{v_1 v_{10i+2}\}_{i=1}^{m+1}\}$ where n = 10m + 12 and m ≥ 2 is any integers.

An injective map f: V(G) → $\{F_0,F_1,F_2,\dots,F_q\}$ is defined as follows:

Let i ∈ {1, 2,…, n}.

$f(v_1) = F_0$;
$f(v_i) = F_{11k+(r-1)}$; where i = 10k + r; k = 0, 1,2,…,(m+1); r = 2,5,8;
$f(v_i) = F_{11k+r}$; where i = 10k + r; k = 0,1,2,…, m; r = 3,6,9;
$f(v_i) = F_{11k+(r+1)}$; where i = 10k + r; k = 0,1,2,…, m; r = 4,7;
$f(v_{10k}) = F_{11k}$; where k = 1,2,…,(m+1);
$f(v_{10k+1}) = F_{(11k-1)}$; where k = 1,2,…,(m+1);

Its induced map f^+ defined by $f^+(xy) = |f(x) - f(y)|$ for each edges xy in G, is an injective. Thus the given graph has super Fibonacci graceful labeling from the injective maps f and f^+. Hence the graph (10, m)-shell graph is super Fibonacci graceful.

Theorem 3.4: A (13, m) – shell graph is super Fibonacci graceful, for all integers $m \geq 2$.

Proof: Let G be the graph (13, m)-shell graph whose vertex set is $\{v_1,\dots, v_n\}$ and edge set is $\{\{v_i v_{i+1}\}_{i=1}^{n-1} \cup \{v_1 v_{13i+2}\}_{i=1}^{m+1}\}$ where n = 13m +15and m ≥ 2is any integers.

An injective map f: $V(SH_{13,m}) \rightarrow \{F_0,F_1,F_2,\ldots,F_q\}$ is defined as follows:

Let i be in {1,2,…, n}.

$f(v_1) = F_0$;
$f(v_i) = F_{14k + (r - 1)}$; where i = 13k + r; k = 0,1,2,…,(m+1); r = 2,5,8,11;
$f(v_i) = F_{14k + r}$; where i = 13k + r; k = 0,1,2,…,m; r = 3,6,9,12;
$f(v_i) = F_{14k + (r + 1)}$; where i = 13k + r; k = 0,1,2,…,m; r = 4,7,10;
$f(v_{13k}) = F_{14k}$; where k = 1,2,…,(m +1);
$f(v_{13k+1}) = F_{14k - 1}$; where k = 1,2,…,(m +1);

Its induced map f^+ defined by $f^+(xy) = |f(x) - f(y)|$ for each edges xy in G is injective. Thus the given graph has super Fibonacci graceful labeling from the injective maps f and f^+. Hence the graph (13, m)-shell graph is super Fibonacci graceful.

Theorem 3.5: A (16, m) – shell graph is super Fibonacci graceful, for all integers m ≥ 2.

Proof: Let G be the graph (16, m)-shell graph whose vertex set is $\{v_1,\ldots, v_n\}$ and edge set is $\{\{v_i v_{i+1}\}_{i=1}^{n-1} \cup \{v_1 v_{16i+2}\}_{i=1}^{m+1}\}$ where n = 16m +18 and m ≥ 2 is any integers.

Define an injective map f: $V(G) \rightarrow \{F_0,F_1,F_2,\ldots,F_q\}$ as follows:

Let i vary from 1 to n.

$f(v_1) = F_0$;
$f(v_i) = F_{17k + (r - 1)}$; where i = 16k + r; k = 0,1,2,…,(m+1); r = 2,5,8,11,14;
$f(v_i) = F_{17k + r}$; where i = 16k + r; k = 0,1,2,…,m; r = 3,6,9,12,15;
$f(v_i) = F_{17k + (r + 1)}$; where i = 16k + r; k = 0,1,2,…, m; r = 4,7,10,13;
$f(v_{16k}) = F_{17k}$; where k = 1,2,…,(m+1);
$f(v_{16k+1}) = F_{17k - 1}$; where k = 1,2,…,(m+1);

Its induced map f^+ defined by $f^+(xy) = |f(x) - f(y)|$ for each edges xy in G, is injective. Thus the given graph has super Fibonacci graceful labeling from the injective maps f and f^+. Hence the graph (16, m)-shell graph is super Fibonacci graceful.

Theorem 3.6:A(19, m) – shell graph is super Fibonacci graceful, for all integers m ≥ 2.

Proof: Let G be the (19, m)-shellgraph, whose vertex set is $\{v_1,\ldots, v_n\}$ and edge set is $\{\{v_i v_{i+1}\}_{i=1}^{n-1} \cup \{v_1 v_{19i+2}\}_{i=1}^{m+1}\}$ where n =19m +21and m ≥ 2is any integers.

Construct an injective map f: $V(G) \rightarrow \{F_0,F_1,F_2,\ldots,F_q\}$ as follows:

Let i be in {1, 2,…, n}.

$f(v_1) = F_0$;
$f(v_i) = F_{20k + (r - 1)}$; where i = 19k + r; k= 0,1,2,…,(m +1); r = 2,5,8,11,14,17;
$f(v_i) = F_{20k + r}$; where i = 19k + r; k = 0,1,2,…,m; r = 3,6,9,12,15,18;
$f(v_i) = F_{20k + (r + 1)}$; where i = 19k + r; k = 0,1,2,…,m; r = 4,7,10,13, 16;
$f(v_{19k}) = F_{20k}$; where k = 1,2,…,(m +1);
$f(v_{19k+1}) = F_{20k - 1}$; where k = 1,2,…,(m +1);

Its induced map f^+ defined by $f^+(xy) = |f(x) - f(y)|$ for each edges xy in G, is injective. Thus the given graph has super Fibonacci graceful labeling from the injective maps f and f^+. Hence the graph (19, m)-shell graph is super Fibonacci graceful.

SECTION 4: (a, m) - SHELL GRAPH MERGING WITH A FAN GRAPH

The following definitions are given:

Definition 4.1: The join $G_1 + G_2$ of two graphs G_1 and G_2 with disjoint vertex sets V_1 and V_2 and edge sets X_1 and X_2 is the graph union $G_1 \cup G_2$ together with all edges joining each vertex ofV_1 to every vertex of V_2. A Fan graph F_n is $K_1 + P_n$

Definition 4.2: $SH_{a,m}^{F_t}$ is a graph having one copy of the (a, m) – shell graph ($SH_{a,m}$) and one copy of fan graph (F_t) such that the apex of the graph $SH_{a,m}$ is identified with the vertex of maximum degree in fan graph (F_t).

Theorem 4.3: The graph G $=SH_{4,m}^{F_t}$ is super Fibonacci graceful, for all integers m, t ≥ 2.

Proof: Let$\{u_0,u_1,u_2,\ldots,u_t\}$ be the vertex set of fan graph F_t and $\{v_1,v_2,\ldots,v_n\}$ be the vertex set of graph$SH_{4,m}$ .Then the graph $SH_{4,m}^{F_t}$ is obtained by the apex of the $SH_{4,m}$ graph v_1 is merged with the maximum degree of the fan graph u_0. $|V(G)| = n + t$ and edge set is $\{\{v_iv_{i+1}\}_{i=1}^{n-1} \cup \{v_1v_{4i+2}\}_{i=1}^{m+1} \cup \{v_1u_j\}_{j=1}^{t} \cup \{u_ju_{j+1}\}_{j=1}^{t-1}\}$ where n = (4m + 6) for all integers m, t ≥ 2.

An injective map f: $V(SH_{4,m}^{F_t}) \rightarrow \{F_0,F_1,F_2,\ldots,F_q\}$ is defined as follows:

Let i ∈ {1, 2, …, n}.

$f(v_1) = F_0$;
$f(v_i) = F_{5k+(r-1)}$; where i = 4k + r; k = 0,1,2,…,(m+1); r = 2;
$f(v_i) = F_{5k+r}$; where i = 4k + r; k = 0,1,2,…,m; r = 3;
$f(v_{4k}) = F_{5k}$; where k = 1,2,…,(m +1);
$f(v_{4k+1}) = F_{(5k-1)}$; where k = 1,2,…,(m+1);
$f(u_j) = F_{(5m+2j+5)}$; where j = 1,2,…,t;

Its induced map f^+ is defined by $f^+(xy) = |f(x) - f(y)|$ for each edges xy in G is injective. Thus the given graph has super Fibonacci graceful labeling from the injective maps f and f^+. Hence the graph $SH_{4,m}^{F_t}$ is super Fibonacci graceful.

Theorem 4.4: The graph $G=SH_{7,m}^{F_t}$ is super Fibonacci graceful for all integers m, t ≥ 2.

Proof: Let$\{u_0,u_1,u_2,\ldots,u_t\}$ be the vertex set of fan graph (F_t) and $\{v_1,v_2,\ldots,v_n\}$ be the vertex set of the graph $SH_{7,m}$ .Then the graph$SH_{7,m}^{F_t}$ is obtained by the apex of the $S_{7,m}$ graph v_1 is merged with the maximum degree of the fan graph u_0. $|V(G)| = n + t$ and edge set is $\{\{v_iv_{i+1}\}_{i=1}^{n-1} \cup \{v_1v_{7i+2}\}_{i=1}^{m+1}\} \cup \{v_1u_j\}_{j=1}^{t} \cup \{u_ju_{j+1}\}_{j=1}^{t-1}\}$ where n = 7m + 9 for all integers m, t ≥ 2.

An injective map f: $V(SH_{7,m}^{F_t}) \rightarrow \{F_0,F_1,F_2,\ldots,F_q\}$ is defined as follows:

Let i be an element in {1, 2,…, n}.

$f(v_1) = F_0$;
$f(v_i) = F_{8k+(r-1)}$; where i = 7k + r; k = 0,1,2,…,(m +1); r = 2,5;
$f(v_i) = F_{8k+r}$; where i = 7k + r; k = 0,1,2,…,m; r = 3,6;
$f(v_i) = F_{8k+(r+1)}$; where i = 7k + r; k = 0,1,2,…,m; r = 4;
$f(v_{7k}) = F_{8k}$; where k = 1,2,…,(m +1);
$f(v_{7k+1}) = F_{(8k-1)}$; where k = 1,2,…,(m +1);
$f(u_j) = F_{(8m+2j+8)}$; for j =1,2,3,…,t.

Its induced map f^+ is defined by $f^+(xy) = |f(x) - f(y)|$ for each edges xy in Gis injective. Thus the given graph has super Fibonacci graceful labeling from the injective maps f and f^+. Hence the graph $SH_{7,m}^{F_t}$ is super Fibonacci graceful.

Theorem 4.5: The graph G $=SH_{10,m}^{F_t}$ is super Fibonacci graceful, for all integers m, t ≥ 2.

Proof: Let$\{u_0,u_1,u_2,\ldots,u_t\}$ be the vertex set of fan graph (F_t) and $\{v_1,v_2,\ldots,v_n\}$ be the vertex set of graph $SH_{10,m}$.Then the graph $SH_{10,m}^{F_t}$ is obtained by the apex of the $SH_{10,m}$ graph v_1 is merged with the maximum degree of the fan graph u_0. $|V(G)| = n + t$ and edge set is$\{\{v_iv_{i+1}\}_{i=1}^{n-1} \cup \{v_1v_{10i+2}\}_{i=1}^{m+1} \cup \{v_1u_j\}_{j=1}^{t} \cup \{u_ju_{j+1}\}_{j=1}^{t-1}\}$where n = 10m +12for all integers m,t ≥ 2.

Define an injective map f: $V(SH_{10,m}^{F_t}) \rightarrow \{F_0,F_1,F_2,\ldots,F_q\}$ as follows:

Let i vary from 1 to n.

$f(v_1) = F_0$;
$f(v_i) = F_{11k+(r-1)}$; where i = 10k + r; k = 0,1,2,…,(m +1); r = 2,5,8;
$f(v_i) = F_{11k+r}$; where i = 10k + r; k = 0,1,2,…, m; r = 3,6,9;
$f(v_i) = F_{11k+(r+1)}$; where i = 10k + r; k = 0,1,2,…,m; r = 4,7;
$f(v_{10k}) = F_{11k}$; where k = 1,2,…,(m +1);
$f(v_{10k+1}) = F_{(11k-1)}$; where k = 1,2,…,(m +1);
$f(u_j) = F_{(11m+2j+11)}$; for j =1,2,3,…,t.

Its induced map f^+ from f is defined by $f^+(xy) = |f(x)–f(y)|$ for each edges xy in G is injective. Thus the given graph has super Fibonacci graceful labeling from the injective maps f and f^+. Hence the graph $SH_{10,m}^{F_t}$ is super Fibonacci graceful.

Theorem 4.6: The graph G $=SH_{13,m}^{F_t}$ is super Fibonacci graceful, for all integers m, t ≥ 2.

Proof: Let$\{u_0,u_1,u_2,\ldots,u_t\}$ be the vertex set of fan graph $(F_t)$and $\{v_1,v_2,\ldots,v_n\}$ be the vertex set of graph $SH_{13,m}$. Then the graph $SH_{13,m}^{F_t}$ is obtained by the apex of the $SH_{13,m}$ graph v_1 is merged with the maximum degree of the fan graph u_0. $|V(G)| = n + t$ and edge set is $\{\{v_iv_{i+1}\}_{i=1}^{n-1} \cup \{v_1v_{13i+2}\}_{i=1}^{m+1}\} \cup \{v_1u_j\}_{j=1}^{t} \cup \{u_ju_{j+1}\}_{j=1}^{t-1}\}$ where n = 13m +15 for all integers m, t ≥ 2.

An injective map f $V(SH_{13,m}^{F_t}) \rightarrow \{F_0,F_1,F_2,\ldots,F_q\}$ is found as follows:

Let i ∈ {1,2,…, n}.
$f(v_1) = F_0$;
$f(v_i) = F_{14k + (r - 1)}$; where i = 13k + r; k = 0,1,2,…,(m +1); r = 2,5,8,11;
$f(v_i) = F_{14k + r}$; where i = 13k + r; k = 0,1,2,…,m; r = 3,6,9,12;
$f(v_i) = F_{14k + (r + 1)}$; where i = 13k + r; k = 0,1,2,…,m; r = 4,7,10;
$f(v_{13k}) = F_{14k}$; where k = 1,2,…,(m +1);
$f(v_{13k +1}) = F_{14k - 1}$; where k = 1,2,…,(m +1);
$f(u_j) = F_{(14m +2j+14)}$; for j =1,2,3,…,t.

Its induced map f^+ from f is defined by $f^+(xy) = |f(x) – f(y)|$ for each edges xy in G is injective. Thus the given graph has super Fibonacci graceful labeling from the injective maps f and f^+. Hence the graph $SH_{13,m}^{F_t}$ is super Fibonacci graceful.

Theorem 4.7: The graph G $=SH_{16,m}^{F_t}$ is super Fibonacci graceful, for all integers m, t ≥ 2.

Proof: Let$\{u_0,u_1,u_2,\ldots,u_t\}$be the vertex set of fan graph $(F_t)$and $\{v_1,v_2,\ldots,v_n\}$ be the vertex set of the graph $S_{16,m}$. Then the graph $SH_{16,m}^{F_t}$ is obtained by the apex of the $S_{16,m}$graph v_1 is merged with the maximum degree of the fan graph u_0. $|V(G)| = n + t$ and edge set is $\{\{v_iv_{i+1}\}_{i=1}^{n-1} \cup \{v_1v_{16i+2}\}_{i=1}^{m+1} \cup \{v_1u_j\}_{j=1}^{t} \cup \{u_ju_{j+1}\}_{j=1}^{t-1}\}$ where n = 16m +18 for all integers m, t ≥ 2.

Define an injective map f: $V(SH_{16,m}^{F_t}) \rightarrow \{F_0,F_1,F_2,\ldots,F_q\}$ as follows:

Let i vary from 1 to n.
$f(v_1) = F_0$;
$f(v_i) = F_{17k + (r - 1)}$; where i = 16k + r; k = 0,1,2,…,(m +1); r = 2,5,8,11,14;
$f(v_i) = F_{17k + r}$; where i = 16k + r; k = 0,1,2,…,m; r = 3,6,9, 12, 15;
$f(v_i) = F_{17k + (r + 1)}$; where i = 16k + r; k = 0,1,2,…,m; r = 4,7,10,13;
$f(v_{16k}) = F_{17k}$; where k = 1,2,…,(m +1);
$f(v_{16k +1}) = F_{17k- 1}$; where k = 1,2,…,(m +1);
$f(u_j) = F_{(17m+2j+17)}$; for j =1,2,3,…,t.

Its induced map f^+ is defined by $f^+(xy) = |f(x) – f(y)|$ for each edges xy in G is injective. Thus the given graph has super Fibonacci graceful labeling from the injective maps f and f^+. Hence the graph $SH_{16,m}^{F_t}$is super Fibonacci graceful.

Theorem 4.8: The graph G= $SH_{19,m}^{F_t}$is super Fibonacci graceful, for all integers m, t ≥ 2.

Proof: Let$\{u_0,u_1,u_2,\ldots,u_t\}$ be the vertex set of fan graph (F_t) and $\{v_1,v_2,\ldots,v_n\}$ be the vertex set of the graph $SH_{19,m}$. Then the graph $SH_{19,m}^{F_t}$ is obtained by the apex of the $SH_{19,m}$ graph v_1 is merged with the maximum degree of the fan graph u_0. $|V(G)| = n + t$ and edge set is $\{\{v_iv_{i+1}\}_{i=1}^{n-1} \cup \{v_1v_{19i+2}\}_{i=1}^{m+1} \cup \{v_1u_j\}_{j=1}^{t} \cup \{u_ju_{j+1}\}_{j=1}^{t-1}\}$
where n = 19m +21for all integers m, t ≥ 2.

An injective map f: $V(SH_{19,m}^{F_t}) \rightarrow \{F_0,F_1,F_2,\ldots,F_q\}$ is constructed as follows:

Let i be in {1, 2, …, n}.
$f(v_1) = F_0$;

$f(v_i) = F_{20k + (r - 1)}$; where i = 19k + r; k = 0,1,…,(m+1); r = 2,5,8,11,14,17;
$f(v_i) = F_{20k + r}$; where i = 19k + r; k = 0,1,2,…,m; r = 3,6,9,12,15,18;
$f(v_i) = F_{20k + (r + 1)}$; where i = 19k + r; k = 0,1,2,…,m; r = 4,7,10,13,16;
$f(v_{19k}) = F_{20k}$; where k = 1,2,…,(m +1);
$f(v_{19k+1}) = F_{20k - 1}$; where k = 1,2,…,(m +1);
$f(u_j) = F_{(20m+2j+20)}$; for j =1,2,3,…,t.

Its induced map f^+ from f is defined by $f^+(xy) = |f(x)–f(y)|$ for each edges xy in G is injective. Thus the given graph has super Fibonacci graceful labeling from the injective maps f and f^+. Hence the graph $SH_{19,m}^{F_t}$ is super Fibonacci graceful.

CONCLUSIONS

The problem still remains open for various types of trees merged with the apex of the new types of shell graphs as super Fibonacci graceful.

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