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STRONGLY PSEUDO IRREGULAR FUZZY GRAPHS

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ABSTRACT

In this paper, strongly pseudo irregular fuzzy graphs and strongly pseudo totally irregular fuzzy graphs are defined. Comparative study between strongly pseudo irregular fuzzy graph and strongly pseudo totally irregular fuzzy graph is done. A necessary and sufficient condition under which they are equivalent is provided. Also, we discussed some of their properties.

Key words: *2-degree, pseudo degree of a vertex in graph, irregular fuzzy graph, totally irregular fuzzy graph.*

AMS subject classification: *05C12, 03E72, 05C72.*

1. INTRODUCTION

In this paper, we consider only finite, simple, connected graphs. We denote the vertex set and the edge set of a graph G by $V(G)$ and $E(G)$ respectively. The degree of a vertex v is the number of edges incident at v, and it is denoted by $d(v)$. A graph G is irregular if there is a vertex which is adjacent to the vertices with distinct degree. The 2-degree of v [4] is the sum of the degrees of the vertices adjacent to v and it is denoted by t(v). We call $\frac{t(v)}{d(v)}$, the pseudo degree of v. A graph is called strongly pseudo-irregular if there is a vertex which is adjacent to the vertices with distinct pseudo degree [2].

The notion of fuzzy sets was introduced by Zadeh as a way of representing uncertainity and vagueness. The first definition of fuzzy graph was introduced by Haufmann in 1973. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas. Irregular fuzzy graphs plays a central role in combinatorics and theoretical computer science.

2. REVIEW OF LITERATURE

Nagoorgani and Radha introduced the concept of degree, total degree, regular fuzzy graphs in 2008 [7]. Nagoorgani and Latha introduced the concept of irregular fuzzy graphs, neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs in 2008 [6]. Mathew, Sunitha and Anjali introduced some connectivity concepts in bipolar fuzzy graphs [16]. Akram and Dudek introduced the notions of regular bipolar fuzzy graphs [1] and also introduced intuitionistic fuzzy graphs [2]. Samanta and Pal introduced the concept of irregular bipolar fuzzy graphs [14]. N.R.S. Maheswari and C.Sekar introduced (2, k)-regular fuzzy graphs and totally (2, k)-regular fuzzy graphs [9]. N.R.S. Maheswari and C. Sekar introduced m-neighbourly irregular fuzzy graphs [13]. N.R.S. Maheswari and C.Sekar introduced neighbourly edge irregular fuzzy graphs [10]. N.R.S.Maheswari and C.Sekar introduced neighbourly edge irregular bipolar fuzzy graphs [11]. Pal and Hossein introduced irregular interval-valued fuzzy graphs [17]. Sunitha and Mathew discussed about growth of fuzzy graph theory [15]. N.R.S. Maheswari and C.Sekar introduced pseudo degree and total pseudo degree in fuzzy graphs and pseudo regular fuzzy graphs and discussed some of its properties [12]. These motivate us to introduce pseudo irregular fuzzy graphs, and highly pseudo irregular fuzzy graphs discussed some of their properties.

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2. PRELIMINARIES

Definition 2.1: A fuzzy graph $G:(\sigma,\mu)$ is a pair of functions (σ,μ), where $\sigma: V \rightarrow [0,1]$ is a fuzzy subset of a non-empty set V and $\mu: VXV \rightarrow [0,1]$ is a symmetric fuzzy relation on σ such that for all u,v in V, the relation $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ is satisfied. A fuzzy graph G is called complete fuzzy graph if the relation $\mu(uv) = \sigma(u) \wedge \sigma(v)$ is satisfied.

Definition 2.2: Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. The degree of a vertex u in G is denoted by $d(u)$ and is defined as $d(u) = \sum \mu(uv)$ for all $uv \in E$.

Definition 2.3: Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. The total degree of a vertex u in G is denoted by $td(u)$ and is defined as $td(u) = d(u) + \sigma(u)$, for all $u \in V$.

Definition 2.4: Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. Then G is irregular, if there is a vertex which is adjacent to the vertices with distinct degrees.

Definition 2.5: Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. Then G is said to be a totally irregular fuzzy graph if there is a vertex which is adjacent to the vertices with distinct total degrees.

Definition 2.6: Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. Then G is said to be a neighbourly irregular fuzzy graph if every two adjacent vertices of G have distinct degree.

Definition 2.7: Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. Then G is said to be a neighbourly total irregular fuzzy graph if every two adjacent vertices have distinct total degrees.

Definition 2.8: Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. Then G is said to be a highly irregular fuzzy graph if every vertex of G is adjacent to the vertices with distinct degree.

Definition 2.9: Let G be a graph. The 2-degree of a vertex is defined as the sum of degrees of the vertices incident at v and it is denoted by $t(v)$. ie) $t(v) = \sum_{uv \in E} d_G(u)$, where $d_G(u)$ is the degree of the vertex u in fuzzy graph G which is adjacent to the vertex v.

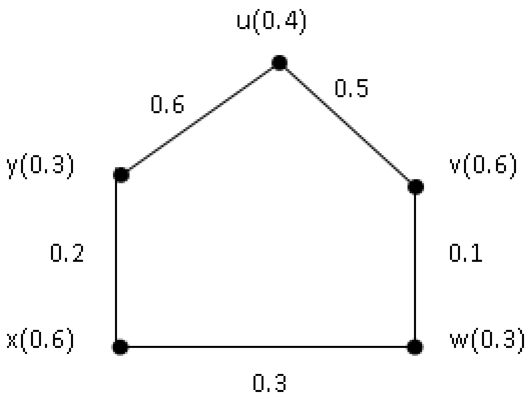
A pseudo degree of a vertex v is defined as $d_a(v) = \frac{t(v)}{d_{G^*(v)}}$, where $d_{G^*(v)}$ is the number of edges incident at v in underlying graph G.

Definition 2.10: Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. The total pseudo degree of a vertex v in G is denoted by $td_a(v)$ and is defined as $td_a(v) = d_a(v) + \sigma(v)$, for all $v \in V$.

3. STRONGLY PSEUDO IRREGULAR FUZZY GRAPHS AND STRONGLY PSEUDO TOTALLY IRREGULAR FUZZY GRAPHS

Definition 3.1: Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. Then G is said to be a strongly pseudo irregular fuzzy graph if no pair of vertices have the same pseudo degree.

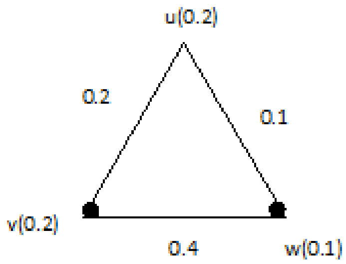
Example 3.2: Consider a graph on $G^*(V,E)$.



Now, $d_a(u)$= 0.7, $d_a(v) = 0.75$, $d_a(w)$= 0.55, $d_a(x) =$ 0.6, $d_a(y) = 0.8$. Here, all the vertices have distinct pseudo degree. Hence the graph is strongly pseudo irregular fuzzy graph.

Definition 3.4: Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. G is said to be a strongly pseudo totally irregular fuzzy graph if no pair of vertices have the same total pseudo degree.

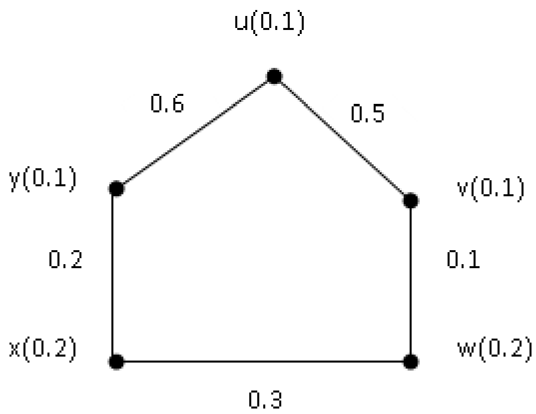
Example 3.5: Consider a graph on $G^*(V,E)$.



Now, $td_a(u)$=$d_a(u)+\sigma(u)=$ 0.55 + 0.3 = 0.85, $td_a(v)$= 0.6, $td_a(w)$= 0.55. Here, all the vertices have distinct total pseudo degree. Hence the graph is strongly pseudo totally irregular fuzzy graph.

Remark 3.6: A strongly pseudo irregular fuzzy graph need not be a strongly pseudo totally irregular fuzzy graph.

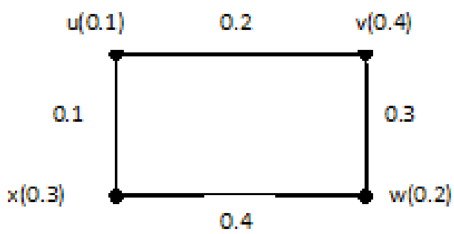
Example 3.7: Consider a graph on $G^*(V,E)$.



Now, $d_a(u)$= 0.7, $d_a(v)=$ 0.75, $d_a(w)$= 0.55, $d_a(x)=$ 0.6, $d_a(y)=$ 0.8 and $td_a(u)$= $td_a(x)$= 0.8, $td_a(v)$=0.85, $td_a(w)$= 0.75, $td_a(y)$= 0.9. Here, all the vertices have distinct pseudo degree but, the vertices u and x have the same total pseudo degree. Hence the graph is strongly pseudo irregular fuzzy graph but it is not a strongly pseudo totally irregular fuzzy graph.

Remark 3.8: A strongly pseudo totally irregular fuzzy graph need not be a strongly pseudo irregular fuzzy graph.

Example 3.9: Consider a graph on $G^*(V,E)$.



Now, $d_a(u)$= 0.5, $d_a(v)=$ 0.5, $d_a(w)$= 0.5, $d_a(x)=$ 0.5 and $td_a(u)$=0.6, $td_a(v)$=0.9, $td_a(w)$= 0.7, $td_a(x)$= 0.8.

Here, all the vertices have same pseudo degree and distinct total pseudo degree. Hence the graph is strongly pseudo totally irregular fuzzy graph but it is not a strongly pseudo irregular fuzzy graph.

Theorem 3.10: Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. If σ is a constant function then the following conditions are equivalent.

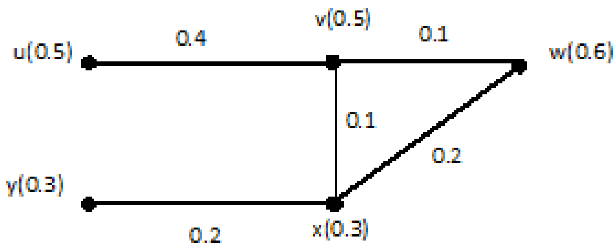
(i) G is strongly pseudo irregular fuzzy graph.
(ii) G is strongly pseudo totally irregular fuzzy graph.

Proof: Assume that σ is a constant function. Let $\sigma(u) = c,$ for all $u \in V$. Suppose G is a strongly pseudo irregular fuzzy graph. Then no pair of vertices have same pseudo degree. Then $d_a(u_i) = k_i$, for i=1, 2,... n $and\ k_1 \neq k_2 \neq k_3 \dots \neq k_n$. Now, $td_a(u_i) = d_a(u_i) + \sigma(u_i) = k_i + c$. Hence G is strongly pseudo totally irregular fuzzy graph. Thus (i) $\Longrightarrow$ (ii) is proved.

Now, suppose G is strongly pseudo totally irregular fuzzy graph. Then no pair of vertices have same total pseudo degree. Then $td_a(u_i) = k_i, for\ i = 1,2, \dots n\ and\ k_1 \neq k_2 \neq k_3 \dots \neq k_n \Longrightarrow d_a(u_i) + \sigma(u) = k_i$, for i=1,2,…n. $d_a(u_i) + c = k_i$, for i=1,2,…n $\Longrightarrow$ $d_a(u_i) = k_i - c$, for i=1,2,…n. Therefore, no pair of vertices have same pseudo degree. Hence G is neighbourly strongly pseudo irregular fuzzy graph. Thus (ii) $\Longrightarrow$ (i) is proved.
Hence (i) and (ii) are equivalent.

Remark 3.11: The converse of the above theorem need not be true.

Example 3.12: Consider a graph on $G^*(V, E)$.



Now, $d_a(u)$=0.6, $d_a(v) = 0.4$, $d_a(w)$= 0.55, $d_a(x)$ = 0.366, $d_a(y)$ = 0.5 and $td_a(u)$= 1.1, $td_a(v)$= 0.9, $td_a(w)$= 1.15, $td_a(x)$= 0.666, $td_a(y)$= 0.8. Here, every pair of adjacent vertices have distinct pseudo degree and distinct total pseudo degree. Hence the graph is strongly pseudo irregular fuzzy graph and strongly pseudo total irregular fuzzy graph. But σ is not a constant function.

Theorem 3.13: [12] If $G: (\sigma, \mu)$ is a fuzzy graph on $G^*(V, E)$, a cycle of length n and μ is a constant function then G is not a strongly pseudo irregular fuzzy graph.

Proof: Assume that μ is a constant function, say $\mu(u_i u_j) = c, i \neq j$ for all $u_i u_j$ ∈E.

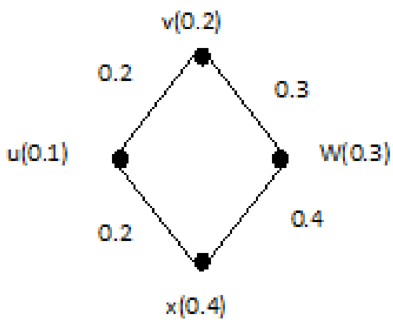
Then $d_a(u_i) = 2c$ for all $u_i \in V$. Thus $d_a(u_i)$ is constant for all $u_i \in V$. Hence G is not a strongly pseudo irregular fuzzy graph.

Theorem 3.14: Let $G: (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$, a cycle of length n. If μ is constant and σ is distinct then G is a strongly pseudo totally irregular fuzzy graph.

Proof: Assume that μ is a constant and σ is distinct. (i.e) $\mu(u_i u_j) = c,\ i \neq j$ for all $u_i u_j$ ∈E and $\sigma(u_i) = k_i$ for all $u_i \in V$ $and\ k_1 \neq k_2 \neq k_3 \dots \neq k_n$. Then $d_a(u_i) = 2c$, for all $u_i \in V$. Now $td_a(u_i)$=$d_a(u_i) + \sigma(u_i) = 2c + k_i$, for i=1,2,…n. Hence G is a strongly pseudo totally irregular fuzzy graph.

Remark 3.15: The converse of the above theorem need not be true.

Example 3.16: Consider a graph on $G^*(V, E)$.



Now, $td_a(u)$= 0.65, $td_a(v)$= 0.75, $td_a(w)$= .85, $td_a(x)$= 0.95. Here, every vertex of G has distinct total pseudo degree. Hence G is strongly pseudo totally irregular fuzzy graph. But μ is not a constant function.

Theorem 3.17: Let G be a fuzzy graph on G^*, an even cycle of length n and σ is distinct. If alternate edges have the same membership values then G is an strongly pseudo totally irregular fuzzy graph.

Proof: Assume that alternate edges takes the same membership values and $\sigma(u_i)$=k_i, for i=1, 2,. . .n and $k_1 \neq k_2 \neq k_3 \ldots \neq k_n$.

Let $e_1, e_2, \ldots, e_n$ be the edges of G^*

$$u(e_i) = \begin{cases} c_1 \; if \; i \; is \; odd \\ c_2 \; if \; i \; is \; even \end{cases}$$

$d_a(u_i) = c_1 + c_2$, i=1,2,…n.

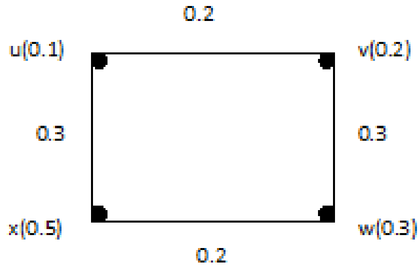
$d_a(u_i)$ = constant

$$td_a(u_i) = d_a(u_i) + \sigma(u_i)$$
$$= d_a(u_i) + k_i, \text{ i=1,2,…n and } k_1 \neq k_2 \neq k_3 \ldots \neq k_n.$$

So, every pair of vertices have distinct total pseudo degree. Hence G is strongly pseudo totally irregular fuzzy graph.

Remark 3.18: The above theorem does not hold for strongly pseudo irregular fuzzy graph.

Example 3.19: Consider a graph on $G^*(V, E)$.



Now, $d_a(u)$= $d_a(v)$= $d_a(w)$= $d_a(x)$= 0.5. Here every vertex of G has same pseudo degree. Hence G is not strongly pseudo irregular fuzzy graph.

Theorem 3.20: Every strongly pseudo irregular fuzzy graph is highly pseudo irregular fuzzy graph.

Proof: Let G be strongly pseudo irregular fuzzy graph.

⟹No two vertices have same pseudo degree.

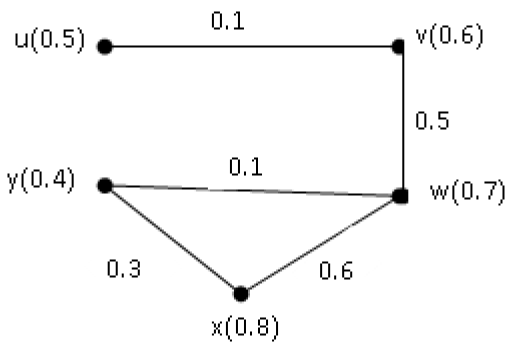
⟹Every pair of vertices have distinct pseudo degree.

⟹Each vertex is adjacent to the vertices having distinct pseudo degree.

⟹G is a highly irregular fuzzy graph.

Remark 3.21: Highly pseudo irregular fuzzy graphs need not be strongly pseudo irregular fuzzy graph.

Example 3.22: Consider a graph on $G^*(V, E)$.



Now, $d_a(u)$= 0.6, $d_a(v) = 0.6$, $d_a(w)$= 0.766, $d_a(x) = 0.8$, $d_a(y) = 1.05$. Here, every vertex of G is adjacent to the vertices with distinct pseudo degree. Hence the graph is highly pseudo irregular fuzzy graph. But the vertices u and v have same pseudo degree. Hence the graph is not strongly pseudo irregular fuzzy graph.

Theorem 3.23: Every strongly pseudo totally irregular fuzzy graph is highly pseudo irregular fuzzy graph.

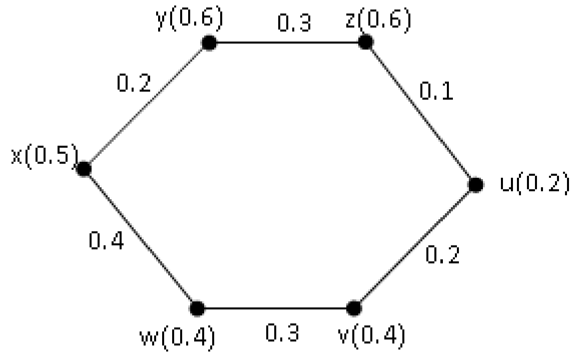
Proof: Proof is similar to the above theorem 3.20.

Theorem 3.24: Every strongly pseudo irregular fuzzy graph is neighbourly pseudo irregular fuzzy graph.

Proof: Let G be strongly pseudo irregular fuzzy graph.
⇒No two vertices have same pseudo degree.
⇒No two adjacent vertices have same pseudo degree
⇒G is neighbourly pseudo irregular fuzzy graph.

Remark 3.25: Neighbourly pseudo irregular fuzzy graphs need not be strongly pseudo irregular fuzzy graph.

Example 3.26: Consider a graph on $G^*(V, E)$.



Now, $d_a(u)$ =0.45, $d_a(v)$= 0.5, $d_a(w)$= 0.55, $d_a(x)$= 0.6, $d_a(y)$=0.5, $d_a(z)$= 0.4. Here, the graph is neighbourly pseudo irregular graph. But the vertices v and y have same pseudo degree. Hence the graph is not strongly pseudo irregular fuzzy graph.

Theorem 3.27: Every strongly pseudo totally irregular fuzzy graph is neighbourly pseudo irregular fuzzy graph.

Proof: Proof is similar to the above theorem 3.24.

Theorem 3.28: Every strongly pseudo irregular fuzzy graph is pseudo irregular fuzzy graph.

Proof: Let G be strongly pseudo irregular fuzzy graph.
⇒No two vertices have same pseudo degree.
⇒There is a vertex which is adjacent to the vertices having distinct pseudo degree.
⇒G is pseudo irregular fuzzy graph.

Remark 3.29: Pseudo irregular fuzzy graphs need not be strongly pseudo irregular fuzzy graph.

Theorem 3.30: Every strongly pseudo totally irregular fuzzy graph is pseudo totally irregular fuzzy graph.

Proof: Proof is similar to the above theorem 3.28.

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