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EFFECTS OF THERMAL RADIATION AND HEAT SOURCE ON MHD FREE CONVECTION OVER A VERTICAL PLATE WITH THERMAL DIFFUSION AND DIFFUSION THERMO

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ABSTRACT

In the present paper, an analysis is carried out the thermal radiation and heat source effects on an unsteady magnetohydrodynamic free convection flow past an infinite vertical porous plate in the presence of thermal diffusion and diffusion thermo. A uniform magnetic field acts perpendicular to the porous surface. The Rosseland approximation has been used to describe the radiative heat flux in energy equation. The dimensionless governing equations are solved numerically using Galerkin finite element method. The effects of the different physical flow parameters on these respective flow fields are discussed through graphs and results are physically interpreted.

Keywords: *Thermal radiation, Heat source, Thermal diffusion, Diffusion thermo, MHD, Free convection, Vertical plate, Galerkin finite element method.*

NOMENCLATURE

A Small positive parameter
T_w' Wall temperature
T_∞' Reference temperature
U' Dimensional free stream velocity
t' Dimensional time
g Acceleration due to gravity
w_0' Dimensional suction velocity
(u', w') Dimensional velocity components
(x', y') Dimensional Cartesian coordinates
$H_0'^2$ Constant transverse magnetic field
K' Dimensional porosity parameter
c_p Specific heat capacity
c_s Concentration susceptibility
M^2 Non – dimensional Hartmann number
$\Pr$ Prandtl number
Gr Grashof number
Gc Modified Grashof number
Sc Schmidt number
Sr Soret number
Du Soret number
D Chemical diffusivity
D_m Molecular diffusivity
k_T Mean absorption coefficient
C' Concentration
C Dimensionless concentration
C_w' Concentration near the plate
C_∞' Concentration in the fluid far away from the plate
R^2 Radiation parameter
U_0 Mean velocity of $U'(t')$
q_z' Radiative heat flux
S Heat source

GREEK SYMBOLS

ε Small positive parameter
β Coefficient of Volume expansion
β^* Volumetric Coefficient of Expansion with Concentration
ν Kinematic viscosity

σ_C Electrical conductivity
μ Permeability
ρ Fluid density
ω' Dimensional free stream frequency of oscillation
k^* Thermal conductivity
α^2 Absorption coefficient
χ^2 Darcy number
δ Radiation absorption coefficient

1. INTRODUCTION

In recent years, the flows of fluid through porous media are of principal interest because these are quite prevalent in nature. Such flows have attracted the attention of a number of scholars due to their application in many branches of science and technology, viz., in the field of agriculture engineering to study the underground water resources, seepage of water in river –beds, in petroleum technology to study the movement of natural gas, oil and water through oil reservoirs, in chemical engineering for filtration and purification processes. The convection problem in porous medium has also important applications in geothermal reservoirs and geothermal energy extractions. A comprehensive review of the studies of convective heat transfer mechanism through porous media has been made by Nield and Bejan [1]. Hiremath and Patil [2] studied the effect on free convection currents on the oscillatory flow through a porous medium, which is bounded by vertical plane surface of constant temperature. Fluctuating heat and mass transfer on three dimensional flow through a porous medium with variable permeability has been discussed by Sharma *et al.* [3]. Magnetohydrodynamics is currently undergoing a period of great enlargement and differentiation of subject matter. The interest in these new problems generates from their importance in liquid metals, electrolytes and ionized gases. Unsteady hydromagnetic free convection flow of Newtonian fluid has been investigated by Helmy [4]. Chaudhary and Sharma [5] considered combined heat and mass transfer by laminar mixed convection flow from a vertical surface with induced magnetic field. Hydromagnetic unsteady mixed convection and mass transfer flow past a vertical porous plate immersed in a porous medium was investigated by Sharma and Chaudhary [6]. El – Amin [7] considered the MHD free convection and mass transfer flow in a micropolar fluid over a stationary vertical plate with constant suction. Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of water body, energy transfer in wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously.

Chemical reaction can be codified as either homogeneous or heterogeneous processes. A homogeneous reaction is one that occurs uniformly through a given phase. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. A reaction is said to be first order, if the rate of reaction is directly proportional to the concentration itself which has many applications in different chemical engineering processes and other industrial applications such as polymer production, manufacturing of ceramics or glassware and food processing [8]. Das *et al.* [9] considered the effects of first order chemical reaction on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer. Muthucumarswamy and Ganesan [10] and Muthucumarswamy [11] studied first order homogeneous chemical reaction on flow past infinite vertical plate. In the above mentioned studies the effects of heat sources/sinks and radiation have not been considered. Due to its great applicability to ceramic tiles production problems, the study of heat transfer in the presence of a source/sink has acquired newer dimensions. Actually, many processes in new engineering areas occur at high temperature and knowledge of radiation heat transfer becomes imperative for the design of the pertinent equipment. Kandasamy *et al.* [12] discussed heat and mass transfer effect along a wedge with heat source and concentration in the presence of suction/injection taking into account the chemical reaction of first order. Sharma *et al.* [13, 14] have reported on the radiation effect with simultaneous thermal and mass diffusion in MHD mixed convection flow from a vertical surface. Sharma *et al.* [15, 16] discussed radiation effect on free convective flow along a uniform moving porous vertical plate in the presence of heat source/sink and transverse magnetic field.

Due to the importance of Soret (thermal diffusion) and Dufour (diffusion thermo) effects for the fluids with very light molecular weight as well as medium molecular weight many investigators have studied and reported results for these flows of whom the names are Eckert and Drake [17], Dursunkaya and Worek [18], Anghel *et al.* [19], Postelnicu [20] are worth mentioning. Recently, Alam and Rahman [21] studied the Dufour and Soret effects on steady MHD free convective heat and mass transfer flow past a semi infinite vertical porous plate embedded in a porous medium. The effects of Hall currents, Soret and Dufour on an unsteady magnetohydrodynamic flow and heat transfer along a porous flat plate with mass transfer studied by Anand Rao and Srinivasa Raju [22]. Influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time – dependent suction studied by Israel – Cookey *et al.* [24]. Mansour *et al.* [25] investigated the effects of chemical reaction, thermal stratification, Soret number, and Dufour number on MHD free convective heat and mass transfer of a viscous, incompressible, and electrically conducting fluid on a vertical stretching surface embedded in a saturated porous medium. Motsa [26] investigated the effect of both the Soret, and Dufour effects on the onset of double diffusive convection. Hence, based on the mentioned investigations and applications, the present paper considers the effect of thermal radiation and heat source on an unsteady MHD free convective flow past an infinite vertical plate with thermal diffusion and diffusion thermo. Mukhopadhyay [27] performed an analysis to investigate the effects of thermal radiation on unsteady mixed convection flow and heat transfer over a porous stretching surface in porous medium. Osalusi *et al.* [28] investigated thermal diffusion and diffusion thermo effects on combined heat and mass transfer of a steady hydromagnetic convective and slip flow due to a rotating disk in the presence of viscous dissipation and Ohmic heating. Pal and Talukdar [29] analyzed the combined effect of mixed convection with thermal radiation and chemical reaction on MHD flow of viscous and electrically conducting fluid past a vertical permeable surface embedded in a porous medium is analyzed. Postelnicu [30] studied simultaneous heat and mass transfer by natural convection from a vertical plate embedded in electrically conducting fluid saturated porous medium, using Darcy – Boussinesq's model including Soret, and Dufour effects. The interaction of buoyancy with thermal radiation has increased greatly during the last decade due to its importance in many practical applications. The thermal radiation effect is important under many isothermal and non – isothermal situations. If the entire system involving the polymer extrusion process is placed in a thermally controlled environment, then thermal radiation could be important. The knowledge of radiation heat transfer in the system can, perhaps, lead to a desired product with sought characteristics.

Motivated by the above referenced work and the numerous possible industrial applications of the problem (like in isotope separation), it is of paramount interest in this study to investigate the effects of thermal radiation, heat source on unsteady MHD free convective flow past an infinite vertical plate in presence of thermal diffusion and diffusion thermo. None of the above investigations simultaneously studied the effects of thermal radiation, heat source, thermal diffusion and diffusion thermo unsteady MHD free convective flow past an infinite vertical plate. Hence, the purpose of this paper is to extend Israel – Cookey *et al.* [24] to study the more general problem which includes thermal radiation, heat source, thermal diffusion and diffusion thermo unsteady MHD free convective flow past an infinite vertical plate. The momentum, thermal, and solutal boundary layer equations are transformed into a set of ordinary differential equations and then solved using Galerkin finite element method**.** The analysis of the results obtained in the present work shows that the flow field is appreciably influenced by Dufour and Soret numbers, heat source and thermal radiation parameters. To reveal the tendency of the solutions, selected results for the velocity components, temperature, and concentration are graphically depicted. The rest of the paper is structured as follows. In Section 2, we formulate the problem in Section 3, we give the method of solution. Our results are presented and discussed in Section 5 and in Section 6, we present some brief conclusions.

2. MATHEMATICAL FORMULATION

We consider the unsteady flow of an incompressible viscous, radiating hydro magnetic fluid past an infinite porous heated vertical plate with time – dependent suction in an optically thin environment. The physical model and the coordinate system are shown in figure 1. The $x'-$ axis is taken along the vertical infinite porous plate in the upward direction and the $y'-$ axis normal to the plate.

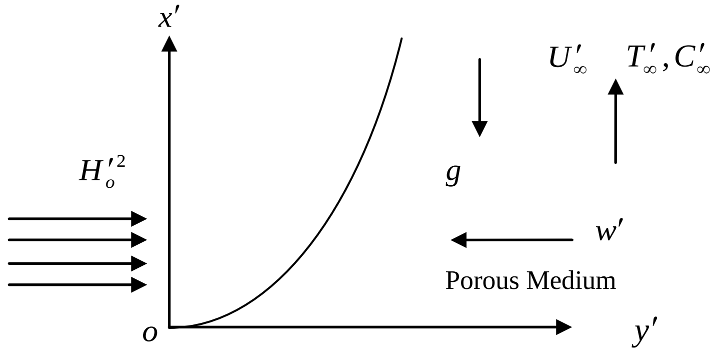

Figure 1: The physical model and coordinate system of the problem

At time $t'=0$, the plate is maintained at a temperature T'_w, which is high enough to initiate radiative heat transfer. A constant magnetic field $H_o'^2$ is maintained in the y' direction and the plate moves uniformly along the positive x' direction with velocity U_0. Under Boussinesq's approximation the flow is governed by the following equations:

$$\frac{\partial w'}{\partial y'}=0 \tag{1}$$

$$\frac{\partial u'}{\partial t'}+w'\frac{\partial u'}{\partial y'}=\nu\frac{\partial^2 u'}{\partial y'^2}+\frac{\partial U'}{\partial t'}-\left(\frac{\mu^2\sigma_C H_0'^2}{\rho}+\frac{\nu}{K'}\right)(u'-U')+g\beta\left(T'-T'_\infty\right)+g\beta^*\left(C'-C'_\infty\right) \tag{2}$$

$$\frac{\partial T'}{\partial t'}+w'\frac{\partial T'}{\partial y'}=\frac{k}{\rho\, c_P}\left(\frac{\partial^2 T'}{\partial y'^2}-\nabla q_z'\right)-\frac{Q_o}{\rho c_P}(T'-T_\infty')+\frac{D_m k_T}{c_S c_P}\frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

$$\frac{\partial^2 q_z'}{\partial y'^2}-3\alpha^2 q_z'-16\alpha\,\sigma\, T_\infty{}^3\frac{\partial T'}{\partial y'}=0 \tag{4}$$

$$\frac{\partial C'}{\partial t'}+w'\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T'}{\partial y'^2} \tag{5}$$

The boundary conditions are

$$\left.\begin{aligned} &u'=0,\ T'=T_w',\ C'=C_w'\ \ on\ \ y'=0\\ &u'=U'(t')=w_0'\left(1+\varepsilon\, e^{i\omega' t'}\right),\ T'=T_\infty',\ C'=C_\infty'\ \ as\ \ y'\to\infty\end{aligned}\right\} \tag{6}$$

Since the medium is optically thin with relatively low density and $\alpha << 1$ the radiative heat flux given by equation (4) in the spirit of Cogley *et al.* [23] becomes

$$\frac{\partial q_z'}{\partial y'}=4\alpha^2\left(T'-T_\infty'\right) \tag{7}$$

Where $\alpha^2=\int_0^\infty \delta\lambda\frac{\partial B}{\partial T'}$ (8)

Here λ is a frequency. Further, from equation (1) it is clear that w' is a constant or a function of time only and so we assume $w'=-w_0'\left(1+\varepsilon A e^{i\omega' t'}\right)$ (9)

Such that $\varepsilon A << 1$, and the negative sign indicates that the suction velocity is towards the plate. In view of equations (4), (8) and (9), equations (2), (3) and (5) become

$$\frac{1}{4}\frac{\partial u}{\partial t}-\left(1+\varepsilon A\, e^{i\omega t}\right)\frac{\partial u}{\partial y}=\frac{1}{4}\frac{\partial U}{\partial t}+\frac{\partial^2 u}{\partial y^2}-(M^2+\chi^2)(u-U)+Gr\theta+GcC \tag{10}$$

$$\frac{1}{4}(\mathrm{Pr})\frac{\partial\theta}{\partial t}-(\mathrm{Pr})\left(1+\varepsilon A\, e^{i\omega t}\right)\frac{\partial\theta}{\partial y}=\left(\frac{\partial^2}{\partial y^2}-R^2\right)\theta-(\mathrm{Pr})(S)\theta+(\mathrm{Pr})(Du)\left(\frac{\partial^2 C}{\partial y^2}\right) \tag{11}$$

$$\frac{1}{4}(Sc)\frac{\partial C}{\partial t}-(Sc)\left(1+\varepsilon A\, e^{i\omega t}\right)\frac{\partial C}{\partial y}=\frac{\partial^2 C}{\partial y^2}+(Sc)(Sr)\left(\frac{\partial^2\theta}{\partial y^2}\right) \tag{12}$$

Where we have used the following dimensionless variables:

$$\left.\begin{aligned} &t=\frac{w_0'^2 t'}{4\nu},\ y=\frac{w_0' y'}{\nu},\ u=\frac{u'}{U_0},\ w=\frac{4\nu\omega'}{w_0'^2},\ U=\frac{U'}{U_0},\ \theta=\frac{T'-T_\infty'}{T_w'-T_\infty'},\ \chi^2=\frac{\nu^2}{K' w_0'^2},\ \mathrm{Pr}=\frac{\mu\, c_p}{k^*}\\ &Gr=\frac{g\beta\,\nu\left(T_w'-T_\infty'\right)}{U_0 w_0'^2},\ \ Gc=\frac{g\beta^*\,\nu\left(C_w'-C_\infty'\right)}{U_0 w_0'^2},\ R^2=\frac{4\alpha^2}{\rho\, c_P k^* w_0'^2}(T_w'-T_\infty'),\ S=\frac{\nu Q_o}{\rho c_p w_0'^2},\\ &M^2=\frac{\mu^2\sigma_C H_0'^2}{\rho w_0'^2},\ Sr=\frac{D_m k_T(T_w'-T_\infty')}{w_o T_m\left(C_w'-C_\infty'\right)},\ Du=\frac{D_m k_T\left(C_w'-C_\infty'\right)}{c_S c_P\left(T_w'-T_\infty'\right)},\ Sc=\frac{w_o}{D},\ C=\frac{\left(C'-C_\infty'\right)}{\left(C_w'-C_\infty'\right)}\end{aligned}\right\} \tag{13}$$

Equations (10), (11) and (12) are now subject to the boundary conditions

$$\left.\begin{aligned} &u=0,\ \ \theta=1,\ C=1\ \ on\ \ y=0\\ &u\to 1+\varepsilon\, e^{i\omega t},\ \ \theta\to 0,\ C\to 0\ \ as\ \ y\to\infty\end{aligned}\right\} \tag{14}$$

The mathematical statement of the problem is now complete and embodies the solution of equations (10), (11) and (12) subject to boundary conditions (14).

3. METHOD OF SOLUTION

By applying Galerkin finite element method for equation (10) over the element (e), $\left(y_j \le y \le y_k\right)$ is:

$$\int_{y_j}^{y_k}\left\{N^{(e)T}\left[4\frac{\partial^2 u^{(e)}}{\partial y^2}-\frac{\partial u^{(e)}}{\partial t}+4B\frac{\partial u^{(e)}}{\partial y}-Du^{(e)}+P\right]\right\}dy=0 \tag{15}$$

Where $P=\frac{\partial U}{\partial t}+4(Gr)\theta+4(Gc)C+DU$, $B=1+\varepsilon Ae^{i\omega t}$, $D=4(M^2+\chi^2)$; Integrating the first term in equation (15) by parts one obtains

$$4N^{(e)T}\left\{\frac{\partial u^{(e)}}{\partial y}\right\}_{y_j}^{y_k}-\int_{y_j}^{y_k}\left\{4\frac{\partial N^{(e)T}}{\partial y}\frac{\partial u^{(e)}}{\partial y}+N^{(e)T}\left(\frac{\partial u^{(e)}}{\partial t}-4B\frac{\partial u^{(e)}}{\partial y}+Du^{(e)}-P\right)\right\}dy=0 \tag{16}$$

Neglecting the first term in equation (16), one gets:

$$\int_{y_j}^{y_k}\left\{4\frac{\partial N^{(e)T}}{\partial y}\frac{\partial u^{(e)}}{\partial y}+N^{(e)T}\left(\frac{\partial u^{(e)}}{\partial t}-4B\frac{\partial u^{(e)}}{\partial y}+Du^{(e)}-P\right)\right\}dy=0$$

Let $u^{(e)}=N^{(e)}\phi^{(e)}$ be the linear piecewise approximation solution over the element (e) $(y_j \le y \le y_k)$ where $N^{(e)}=\begin{bmatrix}N_j & N_k\end{bmatrix}$, $\phi^{(e)}=\begin{bmatrix}u_j & u_k\end{bmatrix}^T$ and $N_j=\frac{y_k-y}{y_k-y_j}$, $N_k=\frac{y-y_j}{y_k-y_j}$ are the basis functions. One obtains:

$$4\int_{y_j}^{y_k}\left\{\begin{bmatrix}N_j^{'}N_j^{'} & N_j^{'}N_k^{'}\\ N_j^{'}N_k^{'} & N_k^{'}N_k^{'}\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}\right\}dy+\int_{y_j}^{y_k}\left\{\begin{bmatrix}N_jN_j & N_jN_k\\ N_jN_k & N_kN_k\end{bmatrix}\begin{bmatrix}\dot{u}_j\\ \dot{u}_k\end{bmatrix}\right\}dy-4B\int_{y_j}^{y_k}\left\{\begin{bmatrix}N_jN_j^{'} & N_jN_k^{'}\\ N_j^{'}N_k & N_k^{'}N_k\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}\right\}dy$$

$$+D\int_{y_j}^{y_k}\left\{\begin{bmatrix}N_jN_j & N_jN_k\\ N_jN_k & N_kN_k\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}\right\}dy=P\int_{y_j}^{y_k}\begin{bmatrix}N_j\\ N_k\end{bmatrix}dy$$

Simplifying we get

$$\frac{4}{l^{(e)^2}}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}+\frac{1}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}\dot{u}_j\\ \dot{u}_k\end{bmatrix}-\frac{4B}{2l^{(e)}}\begin{bmatrix}-1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}+\frac{D}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}=\frac{P}{2}\begin{bmatrix}1\\ 1\end{bmatrix}$$

Where prime and dot are denotes differentiation w.r.t y and time t respectively. Assembling the element equations for two consecutive elements $\left(y_{i-1} \le y \le y_i\right)$ and $\left(y_i \le y \le y_{i+1}\right)$ following is obtained:

$$\frac{4}{l^{(e)^2}}\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}+\frac{1}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}\dot{u}_{i-1}\\ \dot{u}_i\\ \dot{u}_{i+1}\end{bmatrix}-\frac{4B}{2l^{(e)}}\begin{bmatrix}-1 & 1 & 0\\ -1 & 0 & 1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}$$

$$+\frac{D}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}=\frac{P}{2}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix} \tag{17}$$

Now put row corresponding to the node i to zero, from equation (17) the difference schemes with $l^{(e)}=h$ is:

$$\frac{4}{h^2}\left[-u_{i-1}+2u_i-u_{i+1}\right]+\frac{1}{6}\left[\dot{u}_{i-1}+4\dot{u}_i+\dot{u}_{i+1}\right]-\frac{4B}{2h}\left[-u_{i-1}+u_{i+1}\right]+\frac{D}{6}\left[u_{i-1}+4u_i+u_{i+1}\right]=P \tag{18}$$

Applying the trapezoidal rule, following system of equations in Crank – Nicholson method are obtained:

$$A_1 u_{i-1}^{n+1} + A_2 u_i^{n+1} + A_3 u_{i+1}^{n+1} = A_4 u_{i-1}^{n} + A_5 u_i^{n} + A_6 u_{i+1}^{n} + P^* \tag{19}$$

Now from equations (11) and (12) following equations are obtained:

$$B_1\theta_{i-1}^{n+1} + B_2\theta_i^{n+1} + B_3\theta_{i+1}^{n+1} = B_4\theta_{i-1}^{n} + B_5\theta_i^{n} + B_6\theta_{i+1}^{n} + Q^* \tag{20}$$

$$C_1 C_{i-1}^{n+1} + C_2 C_i^{n+1} + C_3 C_{i+1}^{n+1} = C_4 C_{i-1}^{n} + C_5 C_i^{n} + C_6 C_{i+1}^{n} + R^* \tag{21}$$

Where $A_1 = 2 - 12Brh - Dk - 24r,\ A_2 = 8 + 4Dk + 48r,\ A_3 = 2 + 12Brh + Dk - 24r,$

$A_4 = 2 - 12Brh - Dk + 24r,\ A_5 = 8 - 4Dk - 48r,\ A_6 = 2 + 12Brh + Dk + 24r,$

$B_1 = 2(\mathrm{Pr}) - 12(\mathrm{Pr})Brh - 4R^2k - 24r - 4(\mathrm{Pr})Sk,\ B_2 = 8(\mathrm{Pr}) + 48r + 16R^2k + 16(\mathrm{Pr})Sk,$

$B_3 = 2(\mathrm{Pr}) + 12(\mathrm{Pr})Brh + 4R^2k - 24r + 4(\mathrm{Pr})Sk,$

$B_4 = 2(\mathrm{Pr}) - 12(\mathrm{Pr})Brh - 4R^2k + 24r - 4(\mathrm{Pr})Sk,$

$B_5 = 8(\mathrm{Pr}) - 48r - 16R^2k - 16(\mathrm{Pr})Sk,\ B_6 = 2(\mathrm{Pr}) + 12(\mathrm{Pr})Brh - 4R^2k + 24r - 4(\mathrm{Pr})Sk,$

$C_1 = 2(Sc) - 12(\mathrm{Pr})Brh - 24r,\ C_2 = 8(Sc) + 48r,\ C_3 = 2(Sc) + 12(Sc)Brh - 24r,$

$C_4 = 2(Sc) - 12(Sc)Brh + 24r,\ C_5 = 8(Sc) - 48r,\ C_6 = 2(Sc) + 12(Sc)Brh + 24r,$

$P^* = 12Pk = 12k\left(\frac{\partial U}{\partial t} + 4(Gr)\theta + DU\right),\ Q^* = 12Qk = 48(\mathrm{Pr})k(Du)\left(\frac{\partial^2 C}{\partial y^2}\right),$

$R^{**} = 12R^*k = 48(Sc)(Sr)k\left(\frac{\partial^2\theta}{\partial y^2}\right);$

Here r= $\frac{k}{h^2}$ and h, k are mesh sizes along y – direction and time – direction respectively. Index i refers to space and j refers to the time. In the equations (19), (20) and (21), taking $i = 1(1)n$ and using boundary conditions (14), then the following system of equations are obtained:

$$A_i X_i = B_i \quad i = 1(1)n \tag{22}$$

where $A_i^{'}$s are matrices of order n and $X_i, B_i^{'}$s are column matrices having n – components. The solutions of above system of equations are obtained by using Thomas algorithm for velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by C – programme. In order to prove the convergence and stability of Galerkin finite element method, the same C – programme was run with smaller values of h and k no significant change was observed in the values of $u,\ \theta$ and C. Hence the Galerkin finite element method is stable and convergent.

4. RESULTS AND DISCUSSIONS

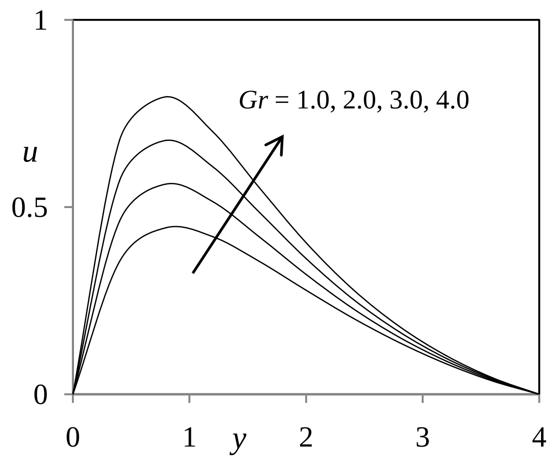

Figure 2: Velocity profiles for different values of *Gr*

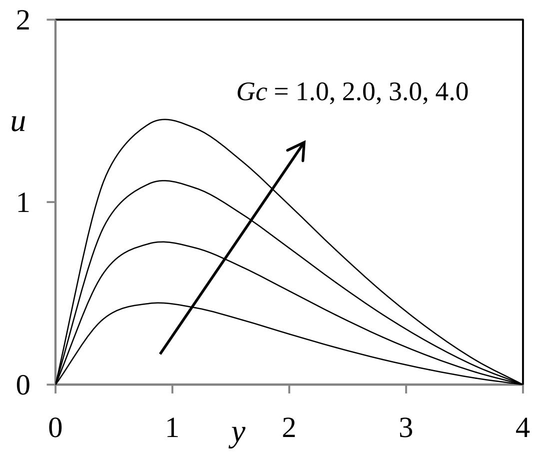

Figure 3: Velocity profiles for different values of *Gc*

In the previous sections, we have formulated and solved the problem of an unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with thermal diffusion, diffusion thermo, radiation and heat source. By invoking, the optically thin differential approximation for the radiative heat flux in the energy equation. In the numerical computation, the Prandtl number ($\Pr = 0.71$) which corresponds to air and various values of the material parameters are used. In addition, the boundary condition $y \to \infty$ is approximated by $y_{\max} = 4$, which is sufficiently large for the velocity to approach the relevant stream velocity. The temperature and the species concentration are coupled to the velocity via Grashof number (Gr) and Modified Grashof number (Gc) as seen in equation (10). For various values of Grashof number and Modified Grashof number, the velocity profiles u are plotted in figures (2) and (3). The Grashof number (Gr) signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Also, as Gr increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity. The Modified Grashof number (Gc) defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value. It is noticed that the velocity increases with increasing values of Modified Grashof number (Gc) .

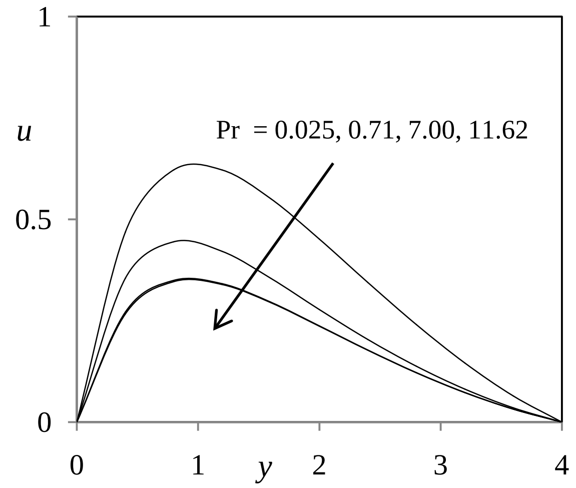

Figure 4: Velocity profiles for different values of Pr

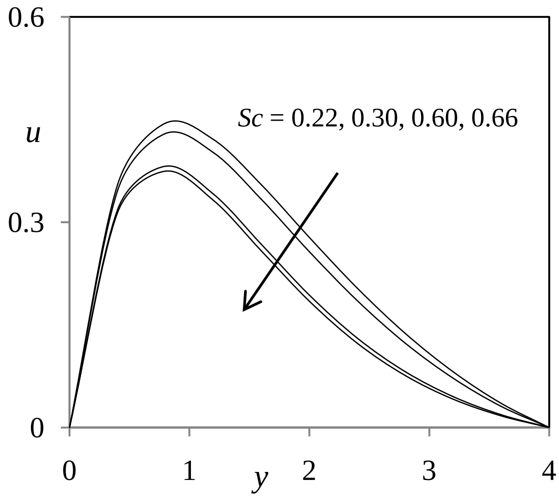

Figure 5: Velocity profiles for different values of *Sc*

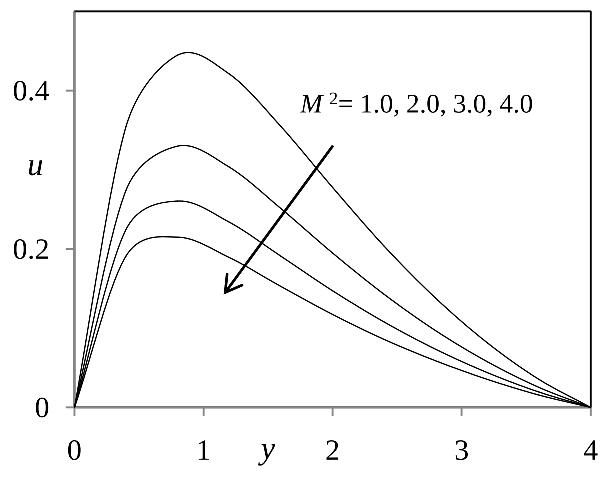

Figure 6: Velocity profiles for different values of M^2

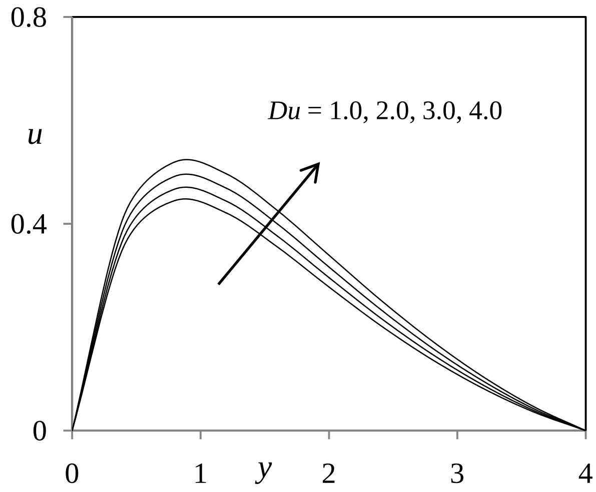

Figure 7: Velocity profiles for different values of Du

Figure (4) illustrate the velocity profiles for different values of Prandtl number $\Pr$. The numerical results show that the effect of increasing values of Prandtl number result in decreasing velocity. The nature of velocity profiles in presence of foreign species such as Hydrogen (Sc = 0.22), Helium (Sc = 0.22), Oxygen (Sc = 0.60) and Water vapour (Sc = 0.66) are shown in figure (5). The flow field suffers a decrease in primary velocity at all points in presence of heavier diffusing species. The effect of the magnetic field parameter M^2 is shown in figure (6) in case of cooling of the plate.

It is observed that the velocity of the fluid decreases with the increase of the magnetic field parameter values. The decrease in the velocity as the Hartmann number M^2 increases is because the presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present study. This resistive force slows down the fluid velocity component as shown in figure (6).

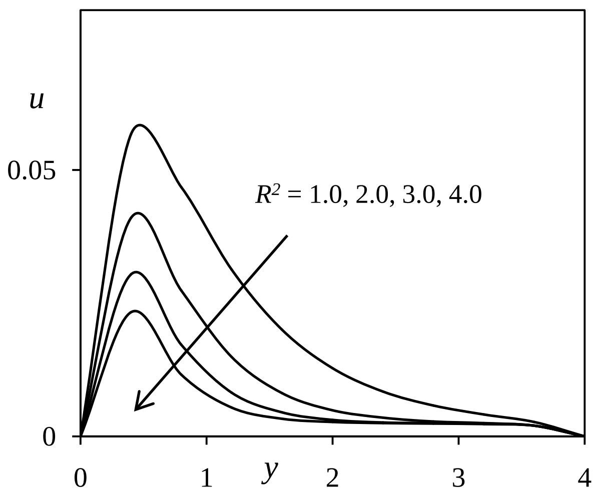

Figure 8: Velocity profiles for different values of R^2

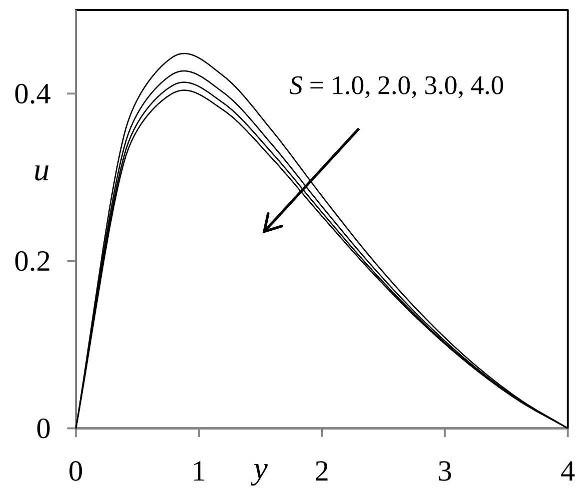

Figure 9: Velocity profiles for different values of *S*

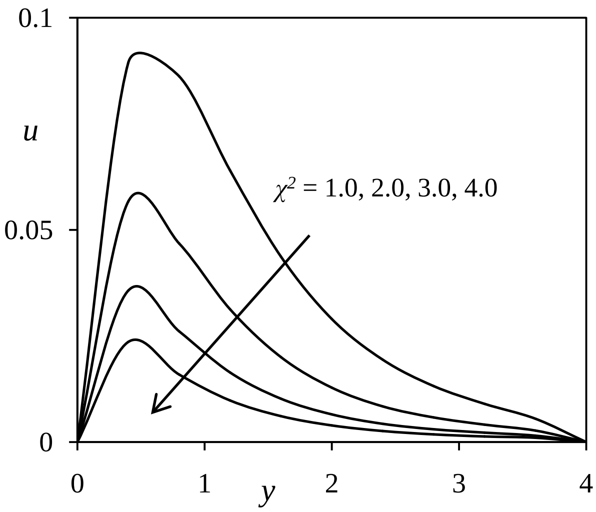

Figure 10: Velocity profiles for different values of χ^2

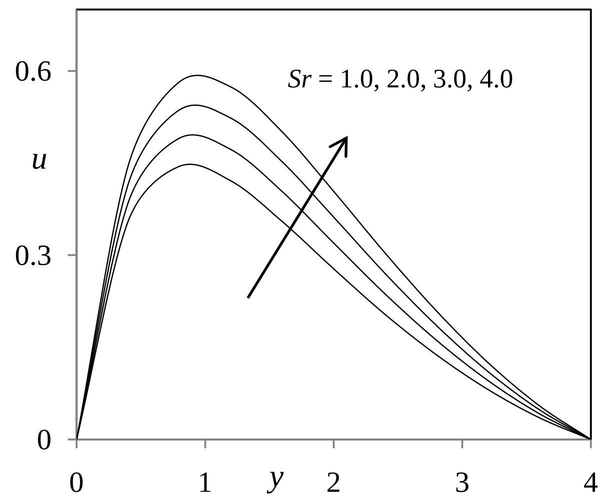

Figure 11: Velocity profiles for different values of *Sr*

The variations of velocity distribution with y for different values of the Dufour number (Du) and Soret number (Sr) are shown in figure (7) and (11). It can be clearly seen that the velocity distribution in the boundary layer increases with the Dufour and Soret numbers. The effect of the thermal radiation parameter R^2 on the primary velocity and temperature profiles in the boundary layer are illustrated in figures (8) and (15) respectively. Increasing the thermal radiation parameter R^2 produces significant increase in the thermal condition of the fluid and its thermal boundary layer. This increase in the fluid temperature induces more flow in the boundary layer causing the velocity of the fluid there to increase. Figure (9) and (14) has been plotted to depict the variation of velocity and temperature profiles against y for different values of heat source parameter S by fixing other physical parameters. From this Graph we observe that velocity and temperature decrease with increase in the heat source parameter S because when heat is absorbed, the buoyancy force decreases the temperature profiles.

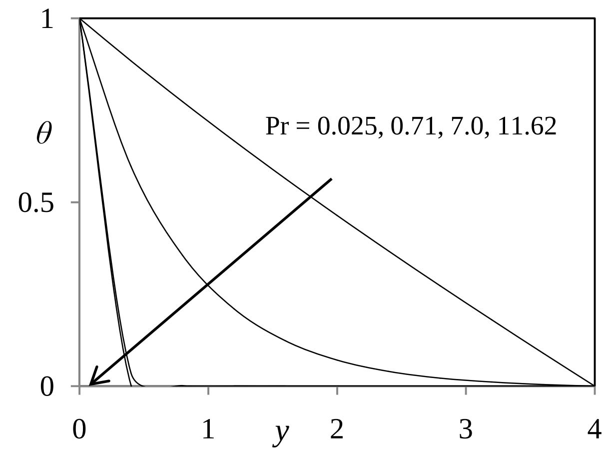

Figure 12: Temperature profiles for different values of *Pr*

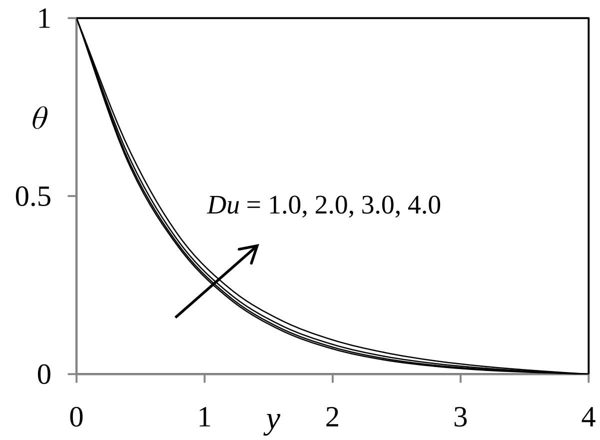

Figure 13: Temperature profiles for different values of Du

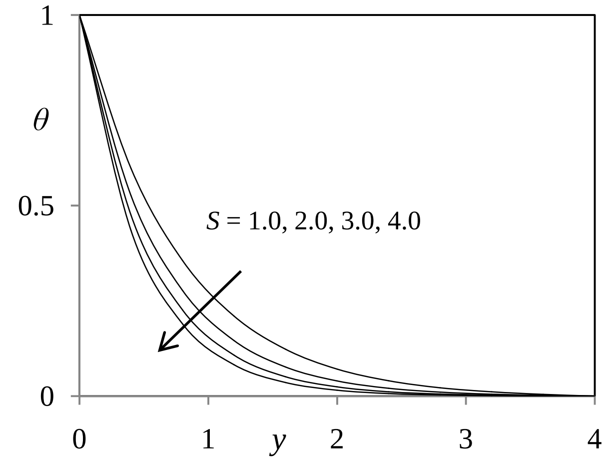

Figure 14: Temperature profiles for different values of *S*

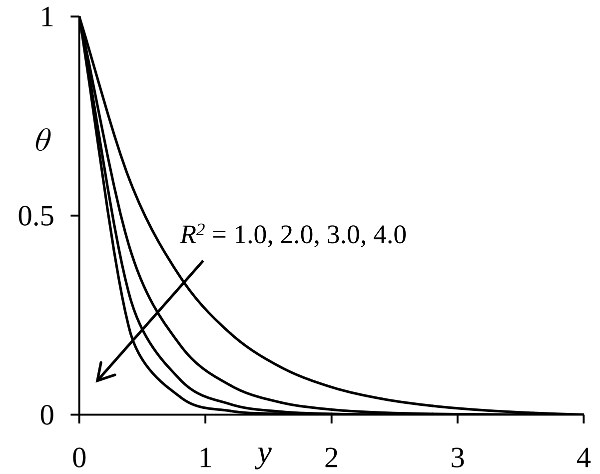

Figure 15: Temperature profiles for different values of R^2

Figure (10) shows the effects of Darcy number χ^2 on the velocity profiles for cooling as well as heating of the plate. For a cooling plate fluid velocity increases, whereas for a heating plate it decreases with increase of χ^2. Darcy number is the measurement of the porosity of the medium. As the porosity of the medium increases, the value of χ^2 increases. For large porosity of the medium fluid gets more space to flow as a consequence its velocity increases. Figure (12) illustrate the temperature profiles for different values of Prandtl number $\Pr$. It is observed that the temperature decrease as an increasing the Prandtl number. The reason is that smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of Pr. Hence in the case of smaller Prandtl number the thermal boundary layer is thicker and the rate of heat transfer is reduced. The Dufour number (Du) does not enter directly into the momentum and mass equations. Thus the effect of Dufour number on velocity and mass profiles is not apparent.

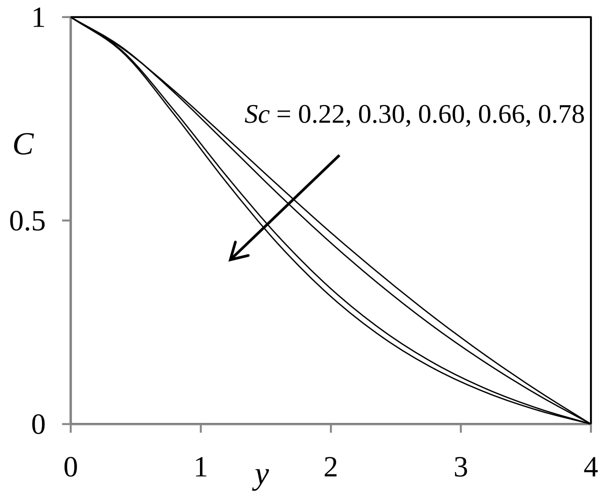

Figure 16: Concentration profiles for different values of *Sc*

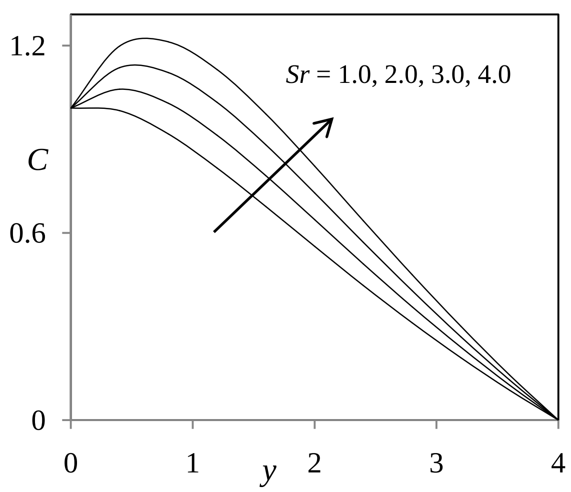

Figure 17: Concentration profiles for different values of *Sr*

Figure (13) shows the variation of temperature profiles for different values of Du. The parameter Du has marked effects on the temperature profiles. It is observed that the temperature profiles increase with the increasing values of Du. It is also observed from this figure that when Du = 1.0, that is, when the ratio between temperature and concentration gradient is very small the temperature profile shows its usual trend of gradual decay. As Dufour number Du becomes large the profiles overshoot the uniform temperature close to the boundary. The effects of Schmidt number (Sc) and Soret number (Sr) on the concentration field are presented in figures (16) and (17). Figure (16) shows the concentration field due to variation in Schmidt number (Sc) for the gasses Hydrogen, Helium, Water – vapour, Oxygen and Ammonia. It is observed that concentration field is steadily for Hydrogen and falls rapidly for Oxygen and Ammonia in comparison to Water – vapour. Thus Hydrogen can be used for maintaining effective concentration field and Water – vapour can be used for maintaining normal concentration field. In figure (17), it is observed that an increase in the Soret number (Sr) leads to increase in the concentration field.

5. CONCLUSIONS

In conclusion therefore, the flow of an unsteady MHD free convection past an infinite heated vertical plate in a porous medium under the simultaneous effects of thermal diffusion, diffusion thermo, radiation and heat source is affected by the material parameters. The governing equations are approximated to a system of linear partial differential equations by using Galerkin finite element method. The results are presented graphically and we can conclude that the flow field and the quantities of physical interest are significantly influenced by these parameters.

1. The velocity increases as Grashof number Gr, Modified Grashof number Gc, Dufour number Du and Soret number Sr increases. However, the velocity was found to decreases as the Hartmann number M, Prandtl number $\Pr$, Schmidt number Sc, Thermal radiation parameter R^2 Heat source parameter S and Darcy parameter χ^2 increases.
2. The fluid temperature was found to decreases as the thermal radiation parameter R^2, Heat source parameter S and Prandtl number $\Pr$ increases and found to increase as Dufour number Du increases.
3. The fluid concentration was found to decreases as the Schmidt number Sc and increases as the Soret number Sr increases.

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