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SOLVING FUZZY SEQUENCING PROBLEM USING GENERALIZED TRAPEZOIDAL FUZZY NUMBERS

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ABSTRACT

In this paper, we present the different methods to solve fuzzy sequencing problem using fuzzy technological values like generalized fuzzy numbers. The procedure adopted was the fuzzy sequencing problems are defuzzified using ranking functions and hence solving the crisp sequencing problem by standard sequencing algorithm for obtaining the optimal sequence and minimum completion time in terms of fuzzy values which is illustrated with numerical examples and solutions.

Keywords: *Fuzzy sequencing problem, generalized fuzzy numbers, fuzzy arithmetic and ranking functions.*

1. INTRODUCTION

Operations research is a problem solving and decision making Science. Modeling is the essence of operations research. Formulating a model help us to convert the complexities and uncertainties of a decision making problem to a logical model which is open to formal analysis. It also involves the application of scientific tools for finding optimum solution to the problem involving the operations of system.

A sequencing problem is to determine the optimal sequence in which 'n' jobs to be performed by 'm' machine and various optimality criteria like minimum elapsed time, minimum idle time, minimum inventory cost with the given conditions i) the order of the machine in which each job should be performed ii) the actual or expected time required by the jobs on each of the machine.

Sequencing have been most commonly encountered in production shops where different products are to be processed over various combinations of machines.

The complicated real life situations are defined in terms of impressions which was overcame by **Zadeh** with a powerful tool of fuzzy data. Thus fuzzy sequencing problem plays a vital role in formulating the uncertainty in actual environment.

This paper is framed as follows section2: Basic definitions and preliminary results, section 3: Application of ranking functions to solve Fuzzy sequencing problem (FSP), section 4: Numerical examples illustrated with solutions and section 5: conclusion.

2. PRELIMINARIES

Basic notations and preliminary results are referred from [1, 2, 3]

Definition 2.1: A fuzzy set **A**= $\{(x, \mu_{\tilde{A}}(x)): x \varepsilon X\}$ is defined for $x \varepsilon X$ with respect to the membership function $\mu_{\tilde{A}}$ where $\boldsymbol{\mu_{\tilde{A}}}$ is defined by $\boldsymbol{\mu_{\tilde{A}}}$**:** $X \rightarrow [0, 1]$.

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Definition 2.1.1: A generalized fuzzy number Ã where Ã = (a, b, c, d; w) is called **generalized trapezoidal fuzzy number** if its membership function is given by

$$\mu_{\tilde{A}} = \begin{cases} \frac{w(x-a)}{(b-a)}, & a \le x \le b \\ w, & b \le x \le c \\ \frac{w(x-c)}{(c-d)}, & c \le x \le d, \text{ where } 0 < w \le 1. \end{cases}$$

Definition 2.1.2: The **ranking function of generalized trapezoidal fuzzy number** Ã = (a, b, c, d; w) is given by

$$\Re(\tilde{A}) = \frac{w(2a+2b+d-c)}{4}, \quad \text{where } \tilde{A} = (a, b, c, d; w)$$

Fuzzy arithmetic operations 2.1.3:

Let $\tilde{A}$ = (a, b, c, d; w_1) and $\tilde{B}$ = (e, f, g, h; w_2) then

1. $\tilde{A} \oplus \tilde{B}$ = (a+e, b+f, c+g, d+h; min(w_1, w_2))
2. $\tilde{A} \ominus \tilde{B}$ = (a-e, b-f, c-g, d-h; min(w_1, w_2))

3. APPLICATION OF RANKING FUNCTIONS TO SOLVE FUZZY SEQUENCING PROBLEM

3.1 Algorithm for solving fuzzy sequencing problem

3.1.1 Processing 'n' jobs through two machines

The simplest possible fuzzy sequencing decision problem is that of 'n' jobs two machine fuzzy sequencing problem is to determine the sequence in which 'n' jobs should be processed through two machines so as to minimize the total elapsed time T. This type of problem can be completely described as:

i) only two machines Ã and B̃ are involved
ii) each job is processed in the order AB, and
iii) the expected fuzzy processing time $\tilde{A}_i$ = (a_i, b_i, c_i, d_i: w_i) and $\tilde{B}_i$ = (x_1, y_1, z_1, u_1: w_1) where i= 1,2,3…n are known as given below:

Job	J1	J2	……	J̃n
$\tilde{A}_1$	(a_1, b_1, c_1, d_1; w_1)	(a_2, b_2, c_2, d_2; w_2)	……	(a_n, b_n, c_n, d_n; w_n)
$\tilde{B}_1$	(x_1, y_1, z_1, u_1: w_1)	(x_2, y_2, z_2, u_2; w_2)	…….	(x_n, y_n, z_n, u_n:w_n)

The procedure for the solution of the above problem is described as follows:

Step-1: using ranking function the fuzzy sequencing problem is defuzzified into crisp sequencing problem

Step-2: the optimal sequence for the crisp sequencing problem is determined using crisp sequencing algorithm

Step-3: After finding the optimal sequence as stated above, the total elapsed fuzzy time and also the fuzzy idle times on machines Ã and B̃ are determined as follows

Step-4: Total elapsed fuzzy time = the fuzzy time between starting the first job in the optimal sequence on machine Ã and completing the last job in the optimal sequence on machine B̃.

Step-5: Fuzzy idle time on machine Ã = (fuzzy time when the last job in the optimal sequences is completed on Machine B̃) – (fuzzy time when the last job in the optimal sequence completed on machine Ã)

Step-6: Fuzzy idle time on machine B̃ = (fuzzy time when the first job in the optimal sequence completed on machine Ã) + $\sum_{k=2}^{n}[(fuzzy\ time\ when$ k^{th} job starts on machine B̃) – (fuzzy time k – 1)st job finished on machine B̃)]

3.1.2 Processing 'n' jobs through three machines

This type of problem can be completely described as:

i) only two machines Ã, B̃ and C̃ are involved
ii) each job is processed in the order ÃB̃C̃
iii) no passing of jobs is permitted, and
iv) the expected fuzzy processing time $\tilde{A}_i$ = (a_i, b_i, c_i, d_i; w_i), $\tilde{B}_i$ = (x_i, y_i, z_i, u_i: w_i) C̃= (e_i, f_i, g_i, h_i : w_i) where i= 1, 2, 3…n are known as given below:

Job:	J1	J2	……	Jn
$\tilde{A}_1$	$(a_1, b_1, c_1, d_{1,}; w_1)$	$(a_2, b_2, c_2, d_{2,}; w_2)$	…….	$(a_n, b_n, c_n, d_{n,}; w_n)$
$\tilde{B}_1$	$(x_1, y_1, z_1, u_1: w_1)$	$(x_2, y_2, z_2, u_2: w_2)$	…….	$(x_n, y_n, z_n, u_n: w_n)$
$\tilde{C}_1$	$(e_n, f_n, g_n, h_{n:} w_n)$	$(e_n, f_n, g_n, h_{n:} w_n)$	………	$(e_n, f_n, g_n, h_{n:} w_n)$

The procedure for the solution of the above problem is described as follows:

Step-1: using ranking function the fuzzy sequencing problem is defuzzified into crisp sequencing problem

Step-2: the optimal sequence for the crisp sequencing problem of Processing 'n' jobs through three machines is determined using crisp sequencing algorithm.

The resulting optimal sequence will also be optimal for the original problem of 3 machines and n jobs. The total elapsed fuzzy time and also the fuzzy idle times on machines $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ are determined as follows:

Step-3: Total elapsed fuzzy time = the fuzzy time between starting the first job in the optimal sequence on machine $\tilde{A}$ and completing the last job in the optimal sequence on machine $\tilde{C}$.

Step-4: Fuzzy idle time on machine $\tilde{A}$ = (fuzzy time when the job J1 is completed on Machine $\tilde{C}$) – (fuzzy time when the last job is completed on machine $\tilde{A}$)

Step-5: Fuzzy idle time on machine $\tilde{B}$ =(fuzzy time when the first job J3is completed on machine $\tilde{A}$) + $\sum_{k=2}^{n}[(fuzzy\ time\ when$ k^{th} job on machine $\tilde{B}$) – (fuzzy time out for $(k – 1)^{st}$ job on machine $\tilde{B}$)] + [fuzzy time last job is completed on $\tilde{C}$ – fuzzy time last job is completed on $\tilde{B}$]

Step-6: Fuzzy idle time on machine C= (fuzzy time when the first job $\tilde{J}$3is completed on machine $\tilde{B}$) + $\sum_{k=2}^{n}[(fuzzy\ time\ when$ k^{th} job on machine $\tilde{C}$) – (fuzzy time out for $(k – 1)^{st}$ job on machine $\tilde{C}$)]

3.1.3 Processing 'n' jobs through m machines

This type of problem can be completely described as:

i) there are n jobs denoted by J1,J2,J3,….Jn to be performed
ii) each job is processed through m machines $\tilde{A}_1$, $\tilde{A}_2$, $\tilde{A}_3$,..$\tilde{A}_m$ in the order $\tilde{A}_1\tilde{A}_2$ $\tilde{A}_3$,..$\tilde{A}_m$
iii) no passing of jobs is permitted
iv) the expected fuzzy processing time are known as given below:

Job	machine fuzzy time for n jobs and m machines			
	Machines			
	$\tilde{A}_1$	$\tilde{A}_2$	…………	$\tilde{A}_m$
J1	$(a_{11}, b_{11}, c_{11}, d_{11;} w_{11})$	$(a_{12}, b_{12}, c_{12}, d_{12;} w_{12})$	…………	$(a_{1m}, b_{1m}, c_{1m}, d_{1m;} w_{1m})$
J2	$(a_{21}, b_{21}, c_{21}, d_{21;} w_{21})$	$(a_{22}, b_{22}, c_{22}, d_{22;} w_{22})$	…………	$(a_{2m}, b_{2m}, c_{2m}, d_{2m;} w_{2m})$
J3	$(a_{31}, b_{31}, c_{31}, d_{31;} w_{31})$	$(a_{32}, b_{32}, c_{32}, d_{32;} w_{32})$	…………	$(a_{3m}, b_{3m}, c_{3m}, d_{3m;} w_{3m})$
	………………	………………	………………	………………
Ji	$(a_{i1}, b_{i1}, c_{i1}, d_{i1;} w_{i1})$	$(a_{i2}, b_{i2}, c_{i2}, d_{i2;} w_{i2})$	…………	$(a_{im}, b_{im}, c_{im}, d_{im,;} w_{im})$
	………………	………………	………………	………………
$\tilde{J}$n	$(a_{n1}, b_{n1}, c_{n1}, d_{n1;} w_{n1})$	$(a_{n2}, b_{n2}, c_{n2}, d_{n2;} w_{n2})$	…………	$(a_{nm}, b_{nm}, c_{nm}, d_{nm;} w_{nm})$

The procedure for the solution of the above problem is described as follows:

Step-1: using ranking function the fuzzy sequencing problem is defuzzified into crisp sequencing problem

Step-2: the optimal sequence for the crisp sequencing problem of Processing 'n' jobs through m machines is determined using crisp sequencing algorithm.

The resulting optimal sequence will also be optimal for the original problem. The total elapsed fuzzy time and also the fuzzy idle times on machines are also determined

4. NUMERICAL EXAMPLES ILLUSTRATED WITH SOLUTIONS

4.1 Type I: Fuzzy Sequencing for 5 jobs on 2 machines

Ex-1: A book binder has one printing press, one binding machine and manuscripts of a number of different books. The time required to perform the printing and binding operation for each book are shown below. We wish to determine the order in which books should be processed, in order to minimize the total time (in hrs) required to turn out all the books

Books:	B1	B2	B3	B4	B5
Printing time:	(8,10,12,16;1)	(2,2,2,2;1)	(16,18,20,24;1)	(4,6,8,12;1)	(18,20,22,26;1)
Binding time:	(2,4,6,10;1)	(10,12,16,20;1)	(12,14,16,20;1)	(14,16,18,22;1)	(6,8,10,14;1)

Solution: Using ranking function for trapezoidal fuzzy number given fuzzy sequencing problem is converted in crisp problem and the optimal sequence is obtained by standard sequencing algorithm

The sequence of the books is given by

B2	B4	B3	B5	B1

The fuzzy optimum time for completing all the books and the idle time for printing and binding is given in the following table:

Books	Printing	Printing	Binding	Binding	Idle time for printing	Idle time For binding
	In time	Out time	In time	Out time		
B2	(0,0,0,0;1)	(2,2,2,2;1)	(2,2,2,2;1)	(12,14,18,22;1)	-	(2,2,2,2;1)
B4	(2,2,2,2;1)	(6,8,10,14;1)	(12,14,18,22;1)	(26,30,36,44;1)	-	-
B3	(6,8,10,14;1)	(22,26,30,38;1)	(26,30,36,44;1)	(38,44,52,64;1)	-	-
B5	(22,26,30,38;1)	(40,46,52,64;1)	(40,46,52,64;1)	(46,54,62,78;1)	-	(2,2,0,0;1)
B1	(40,46,52,64;1)	(48,56,64,80;1)	(48,56,64,80;1)	(50,60,70,90;1)	(2,4,6,10;1)	(2,2,2,2;1)
				Total	(2,4,6,10;1)	(6,6,4,4;1)

The optimum (minimum) time required to turn out all the books is (50, 60, 70, 90; 1) hrs

The idle time for printing is (2, 4, 6, 10; 1) hours

The idle time for binding is (6, 6, 4, 4; 1) hours

4.2Type II : Fuzzy Sequencing for 5 jobs on 3 machines

Ex 2:

Job:	J1	J2	J3	J4	J5
Machine $\tilde{A}$:	(2,6,8,12;1)	(4,8,10,14;1)	(4,6,8,12;1)	(6,10,12,14;1)	(2,6,8,12;1)
Machine $\tilde{B}$:	(2,2,2,2;1)	(1,1,1,1;1)	(2,4,6,10;1)	(2,6,8,12;1)	(3,3,3,3;1)
Machine $\tilde{C}$:	(3,3,3,3;1)	(4,8,10,14;1)	(2,6,8,12;1)	(4,6,8,12;1)	(4,8,10,14;1)

Solution: Using ranking function for trapezoidal fuzzy number given fuzzy sequencing problem is converted in crisp problem and the optimal sequence is obtained by standard sequencing algorithm

The sequence of the jobs is given by

J2	J5	J4	J3	J1

The fuzzy optimum time for completing all the jobs and the idle time for the two machines is given in the following table:

Job		Machine $\tilde{A}$		Machine $\tilde{A}$	Machine $\tilde{B}$	Machine $\tilde{B}$	Machine $\tilde{C}$	Machine $\tilde{C}$
		In time		Out time	In time	Out time	In time	Out time
J2		(0,0,0,0;1)		(4,810,14;1)	(4,8,10,14;1)	(5,91115;1)	(5,9,11,15;1)	(9,17,21,29;1)
J5		(4,8,10,14;1)		(7,11,14,16;1)	(6,14,18,26;1)	(9,17,21,29;1)	(9,17,21,29;1)	(13,25,31,43;1)
J4		(6,14,18,26;1)		(12,24,30,42;1)	(12,24,30,42;1)	(14,30,38,54;1)	(14,30,38,54;1)	(18,36,46,66;1)
J3		(12,24,30,42;1)		(16,30,38,54;1)	(16,30,38,54;1)	(18,34,44,64;1)	(18,34,44,64;1)	(20,42,54,78;1)
J1		(16,30,38,54;1)		(18,36,46,66;1)	(18,36,46,66;1)	(20,38,48,68;1)	(20,38,48,68;1)	(23,45,57,81;1)

	Idle time	Idle time	Idle time
	$\tilde{A}$	$\tilde{B}$	C
	-	(4,8,10,14;1)	(5,9,11,15;1)
	-	(1,5,7,11;1)	-

	-	(3,7,9,13;1)	(1,5,7,11;1)
	-	(2,0,0,0;1)	-
	(5,9,11,15;1)	(0,2,2,2;1)	-
		(3,7,11,15;1)	
Total	(5,9,11,15;1)	(13,29,39,55;1)	(6,14,18,26;1)

The optimum (minimum) time required to finish all the jobs is (23, 45, 57, 81; 1) hrs

The idle time for machines Ã is (5, 9, 11, 15; 1) hours

The idle time for machines B̃ is (13, 29, 39, 55; 1) hours

The idle time for machines C̃ is (6, 14, 18, 26; 1) hours

4.3 Type III: Fuzzy Sequencing for 4 jobs on 5 machines

Ex: 3

		Machine				
		Ã	B̃	C̃	D̃	Ẽ
	J1	(6,10,12,16;1)	(4,8,10,14;1)	(2,4,6,10;1)	(2,6,812;1)	(8,12,14,18;1)
Job	J2	(6,8,10,14;1)	(6,810,14;1)	(4,6,8,12;1)	(4,8,10,14;1)	(10,12,16,20;1)
	J3	(4,8,1014;1)	(4,6,8,12;1)	(4,8,10,14;1)	(6,810,14;1)	(8,10,12,16;1)
	J4	(8,10,12,16;1)	(2,6,8,12;1)	(2,4,6,10;1)	(2,4,6,10;1)	(6,8,10,14;1)

Solution: Using ranking function for trapezoidal fuzzy number given fuzzy sequencing problem is converted in crisp problem and the optimal sequence is obtained by standard sequencing algorithm

The sequence of the jobs is given by

J1	J3	J2	J4

The fuzzy optimum time for completing all the jobs and the idle time for the two machines is given in the following table:

Job	Machine Ã	Machine Ã	Machine B̃	Machine B̃	Machine C̃	Machine C̃
	In time	Out time	In time	Out time	In time	Out time
J1	(0,0,0,0)	(6,10,12,16;1)	(6,10,12,16;1)	(10,18,22,30;1)	(10,18,22,30;1)	(12,2228,40;1)
J3	(6,10,12,16;1)	(10,18,22,30;1)	(10,18,22,30;1)	(14,24,30,42;1)	(14,24,30,42;1)	(18,32,40,56;1)
J2	(10,18,22,30;1)	(16,26,32,44;1)	(16,26,32,44;1)	(22,3,42,58;1)	(22,34,42,58;1)	(26,40,50,70;1)
J4	(16,26,32,44;1)	(24,36,44,60;1)	(24,36,44,60;1)	(26,42,52,72;1)	(26,42,52,72;1)	(28,48,60,84;1)

Job	Machine D̃	Machine D̃	Machine Ẽ	Machine Ẽ
	In time	Out time	In time	Out time
J1	(12,22,28,40;1)	(14,28,36,52;1)	(14,28,36,52;1)	(22,40,50,70;1)
J3	(18,32,40,56;1)	(24,40,50,70;1)	(24,40,50,70;1)	(32,50,62,86;1)
J2	(26,40,50,70;1)	(30,48,60,84;1)	(32,50,62,86;1)	(42,62,78,106;1)
J4	(30,48,60,84;1)	(32,52,66,94;1)	(42,62,78,106;1)	(48,70,88,120;1)

Job	Idle time	Idle time	Idle time	Idle time	Idle time
	Ã	B̃	C̃	D̃	Ẽ
J1	-	(6,10,12,16;1)	(10,18,22,30;1)	(12,22,28,40;1)	(14,28,36,52;1
J3	-	-	(-26,-4,8,30;1)	(-34,-4,12,42;1)	(-46,-10,10,48;1)
J2	-	(-26,-4,8,30;1)	(-34,-6,10,40;1)	(-44,-10,10,46;1)	-
J4	(24,34,44,60;1)	(-34,-6,10,38;1)	(-44,-8,8,46;1)	-	-
		(-24,18,46,94;1)	(-36,10,40,94;1)	(-46,4,36,88;1)	
Total	(24,34,44,60;1)	(-78,18,76,78;1)	(-130,10,88,240;1)	(-112,12,86,216;1)	(-32,18,46,100;1)

The optimum (minimum) time required to finish all the jobs is (48, 70, 88,120; 1) hrs

The idle time for machines Ã is (24, 34, 44, 60; 1) hours

The idle time for machines B̃ is (-78, 18, 76, 78; 1) hours

The idle time for machines $\tilde{C}$ is (-130, 10, 88,240; 1) hours

The idle time for machines $\tilde{D}$ is (-112, 12, 86,216; 1) hours

The idle time for machines $\tilde{E}$ is (-32, 18, 46,100; 1) hours

5. CONCLUSION

Fuzzy sequencing problem is solved by classical approach after defuzzification, which is easy to understand, helps to formulate uncertainty in actual environment and also serves as application for the decision makers in real life situation.

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