International Journal of Mathematical Archive-7(6), 2016, 91-97
Available online through www.ijma.info ISSN 2229 – 5046

SEMI REGULAR WEAKLY CLOSED SETS IN TOPOLOGICAL SPACES

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(Received On: 19-05-16; Revised & Accepted On: 16-06-16)

ABSTRACT

In this paper we introduce and study the new class of sets, namely semi regular weakly closed (briefly srw-closed) sets in topological spaces. This new class of sets lies between the class of α-regular weakly closed (briefly αrw-closed) set and the class of generalized semi closed (briefly gs-closed) sets. We study the some properties of this class of sets.

Mathematics Subject Classification 2000: *54C05.*

Keywords: *Semi-closed set, αrw-closed set, gs-closed set, srw-closed set and topological space.*

1. INTRODUCTION

The notation of closed set is fundamental in the study of topological spaces. In 1970, Levine [1] introduced the concept of generalized closed sets in the topological space by comparing the closure of subset with its open supersets. The investigation on generalization of closed set has lead to significant contribution to the theory of separation axiom, covering properties and generalization of continuity. Kong *et al.* [2] shown some of the properties of generalized closed set have been found to be useful in computer science and digital topology. Caw *et al.* [3,4] has shown that generalization of closed set is also useful to characterize certain classes of topological spaces and there variations, for example the class of extremely disconnected spaces and the class of sub maximal spaces. In 1990, Arya and Nour [5] define generalized semi-open sets, generalized semi closed sets and use them to obtain some characterization of s-normal spaces. In 2007, S.S. Benchalli and R.S. Wali [6] introduced the new class of the set called regular w-closed (briefly rw-closed) sets in topological spaces. Recently, R. S. Wali and Mendalgeri [7] introduced and studied the concepts of α-regular w-closed (briefly αrw-closed) sets in topological spaces. In this paper we define new generalization of closed set called Semi regular weakly closed (briefly srw-closed) set which lies between αrw-closed set and gs-closed sets. We also study their fundamental properties.

2. PRELIMINARIES

Definition 2.1: A subset A of X is called regular open (briefly r-open) [8] set if A = int(cl(A)) and regular closed (briefly r-closed) [8] set if A = cl(int(A)).

Definition 2.2: A subset A of X is called pre-open set [9] if $A\subseteq int(cl(A))$ and pre-closed [9] set if $A\subseteq cl(int(A))$.

Definition 2.3: A subset A of X is called semi-open set [10] if $A\subseteq cl(int(A))$ and pre-closed [10] set if $A\subseteq int(cl(A))$.

Definition 2.4: A subset A of X is called α-open [11] if $A\subseteq int(cl(int(A)))$ and α-closed [11] if $cl(int(cl(A))) \subseteq A$.

Definition 2.5: A subset A of X is called β-open [12] if $A\subseteq cl(int(cl(A)))$ and β-closed [12] if $int(cl(int(A))) \subseteq A$.

Definition 2.6: A subset A of X is called θ-closed [13] if $A = cl_\theta(A)$, where $cl_\theta(A) = \{x \in X: cl(U) \cap A \neq \emptyset, U\in A\}$.

Definition 2.7: A subset A of X is called δ-closed [13] if $A = cl_\delta(A)$, where $cl_\delta(A) = \{x \in X: int(cl(U)) \cap A \neq \emptyset, U\in A\}$.

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Definition 2.8: A subset A of a space (X, τ) is called regular semi-open [14] if there is a regular open set U such thatU ⊆ A ⊆ cl(U). The family of all regular semi-open sets of X is denoted by RSO(X).

Definition 2.9: A subset A of a space (X, τ) is said to be semi-regular open [15] if it is both semi-open and semi-closed set.

Definition 2.10: A subset A of a space (X, τ) is said to be regular α-open (briefly rα-open) [16] if there is a regular open set U such that U ⊆ A ⊆ αcl(U).

Definition 2.11: A subset of a topological space (X, τ) is called

1. Generalized closed (briefly g-closed) [1] if cl(A) ⊆U whenever A⊆U and U is open in X.
2. Semi-generalized closed (briefly sg-closed) [17] if scl(A) ⊆U whenever A⊆U and U is semi- open in X.
3. Generalized semi closed (briefly gs-closed) [18] if scl(A) ⊆U whenever A⊆U and U is open in X.
4. Generalized α-closed (briefly gα-closed) [19] if α-cl(A) ⊆U whenever A⊆U and U is α-open in X.
5. α-generalized closed (briefly αg-closed) [20] if α-cl(A) ⊆U whenever A⊆U and U is open in X.
6. Generalized semi-preclosed (briefly gsp-closed) [21] if spcl(A) ⊆U whenever A⊆U and U is open in X.
7. Regular generalized closed (briefly rg-closed) [22] if cl(A) ⊆U whenever A⊆U and U is regular open in X.
8. Generalized preclosed (briefly gp-closed) [23] if pcl(A) ⊆U whenever A⊆U and U is open in X.
9. Generalized pre regular closed (briefly gpr-closed) [24] if pcl(A)⊆U whenever A⊆U and U is regular open in X.
10. θ-generalized closed (briefly θ-g-closed) [25] if $cl_\theta(A)$ ⊆U whenever A⊆U and U is open in X.
11. δ-generalized closed (briefly δ-g-closed) [26] if $cl_\delta(A)$ ⊆U whenever A⊆U and U is open in X.
12. Weakly generalized closed (briefly wg-closed) [27] if cl(int(A)) ⊆U whenever A⊆U and U is open in X.
13. Strongly generalized closed (briefly g*-closed)[28] if cl(A) ⊆U whenever A⊆U and U is g-open in X.
14. π-generalized closed (briefly π-g-closed) [26] if cl(A) ⊆U whenever A⊆U and U is π-open in X.
15. Weakly closed (briefly w-closed) [29] if cl(A) ⊆U whenever A⊆U and U is semi-open in X.
16. Mildly generalized closed (briefly mildly g-closed) [30] if cl(int(A)) ⊆U whenever A⊆U and U is g-open in X.
17. Semi weakly generalized closed (briefly swg-closed) [27] if cl(int(A)) ⊆U whenever A⊆U and U is semi-open in X.
18. Regular weakly generalized closed (briefly rwg-closed) [27] if cl(int(A)) ⊆U whenever A⊆U and U is regular open inX.
19. α-regular w-closed (brieflyαrw-closed) [7] if αcl(A) ⊆ U whenever A⊆U and U is regular weakly open in X.
20. Regular weakly closed (briefly rw-closed) [6] if cl(A) ⊆U whenever A⊆U and U is regular semi-open in X. We denotethe set of all r-closed sets in X by RWC(X).

The complements of the above mentioned closed sets are their respective open sets.

3. SEMI REGULAR WEAKLY CLOSED SETS (briefly srw-closed sets)

Definition 3.1: A subset A of a space X is called Semi regular weakly closed (briefly srw-closed) if scl(A) ⊆U whenever A⊆U and U is regular weakly open in X. We denote the collection of all srw-closed sets in X by SRWC(X).

Theorem 3.2: Every αrw-closed set in X is srw-closed set in X, but not conversely.

Proof: Let A be an αrw-closed set in space X. Suppose U is regular weakly open set in X such that A⊆U. Since A is αrw-closed set i.e. A ⊆U and U is rw-open in X. By definition of αrw-closed set we have αcl(A) ⊆U but scl(A) ⊆ αcl(A)implies scl(A) ⊆U. Therefore scl(A) ⊆U whenever A⊆U and U is rw-open in X. Hence A is srw-closed set.

The converse of above theorem need not be true.

Example 3.3: Let X = {a, b, c, d} be with topology τ = {X, φ, {a}, {b}, {a, b}, {a, b, c}}. Then the setA = {a} is srw-closed but not αrw-closed in X.

Theorem 3.4: Every srw-closed set in X is gs-closed set in X, but not conversely.

Proof: Let A be an arbitrary srw-closed set in space X. Suppose U is open set in X such that A⊆U. Since every open set is rw-open in X[31] and A is srw-closed set in X. So, we have scl(A) ⊆Uwhenever A ⊆U and U is open in X. Hence A is gs-closed set.

The converse of above theorem need not be true.

Example 3.5: Let X = {a, b, c, d} be with topology τ = {X, φ, {a}, {b}, {a, b}, {a, b, c}}, then A = {b} is gs-closed but not srw-closed in X.

Theorem 3.6: Every semi closed set is srw-closed set but not conversely.

Proof: Let A be arbitrary semi closed set in space X. Suppose U is regular weakly open set in X such thatA ⊆ U. Since A is semi closed set in X i.e. scl(A)=A and A ⊆ U then we have scl(A) ⊆ U whenever A ⊆ U and U is rw-open in X. Finally by definition 3.1 A is srw-closed.

The converse of above theorem need not be true.

Example 3.7: Let X = {a, b, c, d} be space with topology τ = {X, φ, {a}, {b, c}, {a, b, c}}. Here, A = {a, c, d} is srw-closed set but not semi closed set in X.

Corollary 3.8:
1) Every α-closed set is srw-closed but not conversely.
2) Every closed set is srw -closed but not conversely.
3) Every δ-closed set is srw-closed but not conversely.
4) Every π-closed set is srw -closed but not conversely.
5) Every regular closed set is srw-closed but not conversely.

Proof:
1) Every α-closed set is αrw-closed set from Wali and Mendalgeri [7] and follows from Theorem 3.2.
2) Every closed set is αrw-closed set from Wali and Mendalgeri [7] and follows from Theorem 3.2.
3) Every δ-closed set is αrw-closed set from Wali and Mendalgeri [7] and follows from Theorem 3.2.
4) Every π-closed set is αrw-closed set from Wali and Mendalgeri [7] and follows from Theorem 3.2.
5) Every regular closed set is closed from Stone[8] and follows from Corollary 3.8.2.

The converse of Corollary 3.8 is not true as shown in below examples.

Example 3.9: Let X = {a, b, c, d} space be with topology τ = {X, φ, {a}, {b, c}, {a, b, c}}.
1) Let A = {a} is a srw-closed set but not α-closed in X.
2) Let A = {b, c} is a srw-closed set but not closed in X.
3) Let A = {b, c} is a srw-closed set but not δ-closed in X.
4) Let A = {a, b, d} is a srw-closed set but not π-closed in X.
5) Let A = {a, c, d} is a srw-closed set but not regular closed in X.

Theorem 3.10: Every srw-closed set is gsp-closed set but not conversely.

Proof: Let A be an arbitrary srw-closed set in space X. Suppose U is open set in X such that A⊆U. Since every open set is rw-open in X [31] and A is srw-closed set in X. So, we have scl(A) ⊆U but spcl(A) ⊆scl(A) ⊆U i.e. spcl(A) ⊆U whenever A⊆U and U is open in X. Therefore A is gsp-closed set in X.

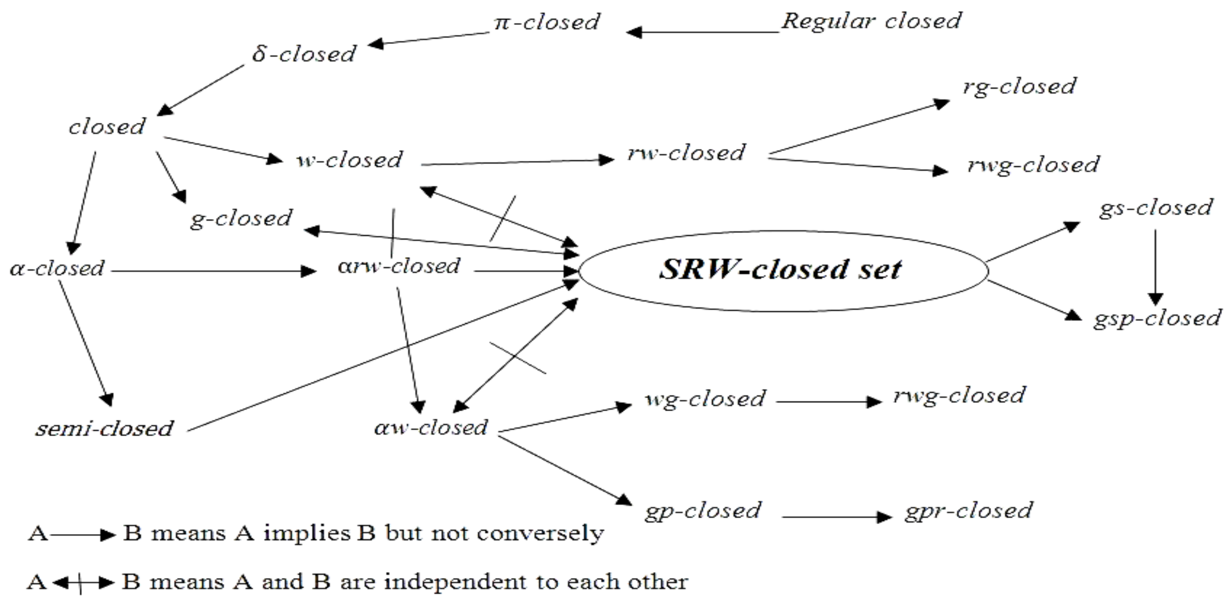
The converse of the above theorem need not be hold.

Example 3.11: Let X = {a, b, c, d} space be with topology τ = {X, φ, {a}, {b, c}, {a, b, c}}. Here A = {b} is a gsp-closed set but not srw-closed in X.

Remark 3.12: The following examples show that srw-closed sets are independent of αg-closed sets, wg-closed sets, rwg-closed sets, gp-closed sets, gpr-closed sets, g-closed sets, w-closed sets, rw-closed sets, rg-closed sets, rwg-closed sets, wα-closed sets and αgr-closed sets.

Example 3.13: Let X = {a, b, c, d} be with the topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$, then
1) Closed sets in X are $X, \emptyset, \{d\}, \{a, d\}, \{b, c, d\}$.
2) SRW-closed sets in X are $X, \emptyset, \{a\}, \{d\}, \{b, c\}, \{a, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}$.
3) g-closed sets in X are$X, \emptyset, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}$.
4) w-closed sets in X are $X, \emptyset, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}$.
5) rw-closed sets in X are $X, \emptyset, \{d\}, \{a, b\}, \{c, d\}, \{a, d\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}$.
6) rg-closed sets in X are $X, \emptyset, \{d\}, \{a, b\}, \{c, d\}, \{a, d\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}$.
7) rwg-closed sets in X are $X, \emptyset, \{b\}, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, d\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}$.
8) wα-closed sets in X are $X, \emptyset, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}$.
9) αgr-closed sets in X are $X, \emptyset, \{d\}, \{a, b\}, \{c, d\}, \{a, d\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}$.
10) αg-closed sets in X are $X, \emptyset, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}$.
11) wg-closed sets in X are $X, \emptyset, \{b\}, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}$.
12) rwg-closed sets in X are $X, \emptyset, \{b\}, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, d\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}$.
13) gpr-closed sets in X are $X, \emptyset, \{b\}, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, d\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}$.
14) gp-closed sets in X are $X, \emptyset, \{b\}, \{c\}, \{d\}, \{c, d\}, \{a, d\}, \{b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}$.

Example 3.14: Let X = {a, b, c, d} be with the topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$then
1) regular closed sets in X are $X, \emptyset, \{a, d\}, \{b, c, d\}$.
2) π-closed sets in X are $X, \emptyset, \{d\}, \{a, d\}, \{b, c, d\}$.
3) δ-closed sets in X are $X, \emptyset, \{d\}, \{a, d\}, \{b, c, d\}$.
4) semi-closed sets in X are $X, \emptyset, \{a\}, \{d\}, \{b, c\}, \{a, d\}, \{b, c, d\}$.
5) gs-closed sets in X are $X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{b, c\}, \{c, d\}, \{a, d\}, \{b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}$.
6) αrw-closed sets in X are $X, \emptyset, \{d\}, \{a, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}$.
7) gsp-closed sets in X are $X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, d\}, \{a, c\}, \{b, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}$.



Remark 3.15: The union of two srw-closed subsets of X is need not be a srw-closed set in X.

Example 3.16: Let X = {a, b, c, d} be with topology τ = {X, φ, {a}, {b, c},{a, b, c}}. Let A = {b} and B = {b, c} be two srw-closed subsets of X. But A∪B = {a, b, c} which is not contained in srw-closed set in X. Hence union of two srw-closed sets is not srw-closed set in X.

Remark 3.17: The intersection of two srw-closed sets in X is generally not an srw-closed set in X.

Example 3.18: Let X = {a, b, c, d} be with topology τ = {X, φ, {a}, {b, c}, {a, b, c}}. Let A = {b, c, d} and B = {a, c, d} be two srw-closed subsets of X. But $A \cap B = \{c, d\} \notin RSWC(X)$. Hence intersection of two srw-closed sets is not srw-closed set in X.

Theorem 3.19: If a subset A of X is srw-closed in X, then $scl(A)\backslash A$ does not contain any non empty regular weakly closed set in X.

Proof: Suppose that A is srw-closed set in X. Let F be a rw-closed subset of $scl(A)\backslash A$. Then $F \subseteq scl(A)\backslash A \Rightarrow F \subseteq scl(A) \cap (X\backslash A) \subseteq X\backslash A$and so $A \subseteq X\backslash F$. But A is srw-closed. Since $X\backslash A$ is rw-open, $scl(A) \subseteq X\backslash F$thatimplies$F \subseteq X\backslash scl(A)$. As we have already $F \subseteq scl(A)$, it follows that $F \subseteq scl(A) \cap (X\backslash scl(A)) = \emptyset$. Thus $F = \emptyset$. Therefore $scl(A)\backslash A$ does not contain a non-empty rw-closed set.

Example 3.20: If $scl(A)\backslash A$ contains no non-empty rw-closed subset in X, thenA neednotbe srw-closed. Consider $X = \{a, b, c, d\}$ and τ = {X, φ, {a}, {b, c}, {a, b, c}} and A = {b} then $scl(A)\backslash A = \{b, c\}\backslash\{b\} = \{c\}$ does not contain any non-empty rw-closed set, but A is not a srw-closed set in X.

Corollary 3.21: If a subset A of X is an srw-closed set in X then $scl(A)\backslash A$ does not contain any non-empty regular open set in X but not conversely.

Proof: Follows from Theorem 3.19 and the fact that every regular open set is rw-open in X.

Theorem 3.22: Let A be srw-closed. Then A is semi-closed if and only if $scl(A)\backslash A$ is rw-closed.

Proof: Let A is semi-closed in X then $scl(A)\backslash A = A$ and so $scl(A)\backslash A = \emptyset$ which is rw-closed.

Conversely, suppose that $\mathrm{scl}(A)\backslash A$ is rw-closed. Since A is srw-closed, then by Theorem 3.19, $\mathrm{scl}(A)\backslash A = \emptyset$ that is $\mathrm{scl}(A) = A$ and hence A is semi-closed.

Theorem 3.23: If A is srw-closed subset of X such that $A \subseteq B \subseteq \mathrm{scl}(A)$, then B is asrw-closed set in X.

Proof: Let A be a srw-closed set in X such that $A \subseteq B \subseteq \mathrm{scl}(A)$. If $A \subseteq B \subseteq \mathrm{scl}(A) \Rightarrow \mathrm{scl}(A) \subseteq \mathrm{scl}(B) \subseteq \mathrm{scl}(\mathrm{scl}(A))$ $\Rightarrow \mathrm{scl}(A) \subseteq \mathrm{scl}(B) \subseteq \mathrm{scl}(A) \Rightarrow \mathrm{scl}(A) = \mathrm{scl}(B)$. Let U is an rw-open in X such that $B \subseteq U$ then $A \subseteq U$ because $A \subseteq B$. Since A is srw-closed, we have $\mathrm{srw}(A) \subseteq U$. Implies $\mathrm{scl}(A) \subseteq U$ because $\mathrm{srw}(B) = \mathrm{scl}(A)$. Thus every rw-open set U containing B contains $\mathrm{scl}(B)$. Therefore B is srw-closed.

The converse of the above Theorem need not be true as seen from the following example.

Example 3.24: Let $X = \{a, b, c, d\}$ with the topology τ = {X, ∅, {a}, {b, c}, {a, b, c}}. Let A={d} and B={a, d}. Then A and B are srw-closed sets in (X, τ), but $A \subseteq B$ is not subset in scl(A). $\because \mathrm{scl}(A) = \{d\}$.

Theorem 3.25: For an element $x \in X$, the set $X - \{x\}$ is srw-closed or rw-open in X.

Proof: Suppose $x \in X$, $X - \{x\}$ is not regular weakly open. Then X is the only rw-open set containing $\{x\}(\because X - \{x\} \subseteq X)$. This impliesscl$(X - \{x\}) \subseteq X$. Hence $X - \{x\}$ is a srw-closed set in X.

Theorem 3.26: If a subset A of a topological space X is both regular semi open and rw-closed then it is semi regular weakly closed in X.

Proof: Suppose a subset A of a topological space X is both regular semi open and rw-closed. Let $A \subseteq U$ and U is rw-open in X. NowA⊆A then by definition of rw-closed we have cl(A) ⊆A but we know $\mathrm{scl}(A) \subseteq \mathrm{cl}(A)$
$\Rightarrow \mathrm{scl}(A) \subseteq \mathrm{cl}(A) \subseteq A \subseteq U$. Thus A is semi regular weakly closed.

Remark 3.27: If A is both regular semi open and srw-closed, then A need not be rw-closed in general, as seen from the following example.

Example 3.28: Let $X = \{a, b, c, d\}$ with the topology τ = {X,∅,{a},{b, c}, {a, b, c}}. Let A={b, c} is both regular semi open and srw-closed but not rw-closed in X.

Theorem 3.29: If a subset A of a topological space X is both regular semi open and rgw-closed then it is srw-closed in X.

Proof: Suppose a subset A of a topological space X is both regular semi open and rgw-closed set in X. Let $A \subseteq U$ and U be rw-open in X. Now $A \subseteq A$ by hypothesis $\mathrm{cl}(\mathrm{int}(A)) \subseteq A$ then we know that $\mathrm{cl}(\mathrm{int}(A)) \subseteq \mathrm{cl}(\mathrm{int}(\mathrm{cl}(A))) \subseteq A$, hence $\mathrm{scl}(A) \subseteq U$. Therefore A is srw-closed set in X.

Remark 3.30: If A is both regular semi open and srw-closed, then A need not be rgw-closed in general, as seen from the following example.

Example 3.31: Let $X = \{a, b, c, d\}$ with the topology τ = {X, ∅, {a}, {b, c}, {a, b, c}}. Let A={a} is both regular semi open and srw-closed but not rgw-closed in X.

Theorem 3.32: If a subset A of a topological space X is both regular open and srw-closed then it is semi-closed in X.

Proof: Suppose a subset A of a topological space X is regular open and srw-closed. As every regular open is rw-open in X. Now A⊆A then by definition of srw-closed scl(A) ⊆ A and also A⊆ scl(A) then we have scl(A)=A. Hence A is semi-closed in X.

Corollary 3.33: If A be regular open and srw-closed in X. Suppose that F is semi-closed in X, then $A \cap$ Fis an srw-closed.

Proof: Let A is regular open and srw-closed in X, by Theorem 3.25 A is semi-closed. As given F is semi-closed in X and we know every semi-closed set is srw-closed i.e. both A and F are semi-closed, so $A \cap F$ is also semi-closed and hence $A \cap F$ is srw-closed in X

Theorem 3.34: In a topological space X, if regular weakly open sets of X are $\{X, \emptyset\}$, then every subset of X is an srw-closed set.

Proof: Let X be a topological space and RWO(X) = {X, ∅}. Suppose A be any arbitrary subset of X, if A = ∅then X is an srw-closed set in X. If A ≠ ∅then X is the only regular weakly open set containing Aand so scl(A) ⊆X. Hence by definition A is an srw-closed in X.

Definition 3.35: The intersection of all rw-open subsets of (x, τ) containing A is called the rw-kernel of A and is denoted by rwker (A).

Lemma 3.36: Let X be a topological space and A be a subset of X. If A is regular weakly open in X, then rwker(A)=A, but converse is not true.

Proof: Follows from Definition 3.35.

Lemma 3.37: For any subset A of (x, τ), $A \subset \mathrm{rwker}(A)$.

Proof: Follows from Definition 3.35.

Theorem 3.38: A subset A of X is srw-closed if and only if $\mathrm{scl}(A) \subseteq \mathrm{rwker}(A)$.

Proof: Suppose that A is srw-closed: $\mathrm{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is rw-open. Let $x \in \mathrm{scl}(A)$ and suppose $x \notin \mathrm{rwker}(A)$, then there is a regular weakly open set U containing A such that x is not in U. Since A is srw-closed, $\mathrm{scl}(A) \subset U$. We have x is not in scl(A), which is contradiction. Hence $x \in \mathrm{rwker}(A)$ and so $\mathrm{scl}(A) \subset \mathrm{rwker}(A)$.

Conversely, let $\mathrm{scl}(A) \subset \mathrm{rwker}(A)$. If U is any regular weakly open set containing A, then $\mathrm{rwker}(A) \subset U$. Yhat is $\mathrm{scl}(A) \subset \mathrm{rwker}(A) \subset U$. Therefore A is srw-closed in X.

Definition 3.39: A subset A in (x, τ) is called Semi regular weakly open(briefly srw-open)set in X if A^c is srw-closed in (x, τ).

Theorem 3.40: Every singleton point set in a space is either srw-open or regular weakly open in X.

Proof: Let X be a topological space. Let $x \in X$. We prove {x} is either srw-open or regular weakly open, i.e. X\{x} is either srw-closed or regular weakly open. From Theorem 3.25 we have X\{x} is srw-closed or regular weakly open. Thus {x} is either srw-open or regular weakly open in X.

CONCLUSION

In this article we have focused on semi regular weakly closed sets in topological space which is lies between αrw-closed set and gs-closed set. This set is also shows the properties between semi-closed set and gs-closed sets in topological space and also we got some important results to extend some more research works on different topological spaces.

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Source of support: Nil, Conflict of interest: None Declared