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OSCILLATORY AND NON OSCILLATORY SOLUTIONS OF LINEAR HOMOGENEOUS DIFFERENCE EQUATIONS

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ABSTRACT

In this paper we study the Oscillatory and Non Oscillatory Solutions of Linear Homogeneous Difference equations of Existence and Uniqueness theorem.

Key Words: *Difference equation; homogeneous; linear; sequence; Oscillation and Non oscillation.*

Mathematics Subject Classification (2010): *39A10, 33A21.*

INTRODUCTION

Difference calculus forms the basis of difference equations. These equations arise in all situations in which sequential relation exists at various discrete values of the Independent variable. In this article we will explain the oscillatory properties of solution of difference equations by using graphical representation.

Definition: 1 An equation which expresses a value of a sequence as a function of the other terms in the sequence is called a difference equation. In particular, an equation which expresses the value a_n of a sequence $\{a_n\}$ as a function of the term a_{n-1} is called a first order difference equation.

Definition: 2 The difference between the largest and smallest arguments appearing in the difference equation is called its order.

Definition: 3 A solution of a difference equation is a relation between the independent variable and the dependent variable satisfying the equation.

Definition: 4 The sequence y is said to be oscillatory around a $(a \in R)$ if there exists an increasing sequence of integers $\{n_k\}_{k=1}^{\infty}$ such that

$$(y_{n_k} - a)(y_{n_{k+1}} - a) \leq 0 \text{ .for all } k \in N$$

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Theorem: 1 The linear difference equation of order n

$$f_0(x)\,y_{x+n-1} + ... + f_{n-1}(x)\,y_{x+1} + f_n(x)y_x = g(x) \qquad (1)$$

Over a set T of consecutive integral values of x has one, and only one, solution of y for which values at n consecutive x- values are arbitrarily prescribed.

Proof: By hypothesis, T is a set of one of types $a \le x \;\; or \;\; a \le x \le b$, a, b non negative integers. Suppose first that $y_a, y_{a+1, ...} y_{a+n-1}$ are n prescribed values of y. we shall now prove with the help of mathematical induction that the value of y at each point of T is uniquely determined.

The given values conclude the values y_{a+n} uniquely as from the equation (1) when x = a, we have

$$y_{a+n} = \tfrac{1}{f_0(a)}\left[g(a) - f_1(a)y_{a+n-1} ..... f_{n-1}(a)y_{a+1} - f_n(a)y_0\right], \quad f_0(a) \neq 0.$$

Which determines y_{a+n} uniquely.

We now set up the hypothesis that y is known for all x values in T up to and including y_{a+j} Where j$\ge$n.

Substituting $x = x_1 = a + j + 1 - n$ in (1), we obtain

$$f_0(x_1)y_{a+j-1} = g(x_1) - f_1(x_1)y_{a+j} - f_2(x_1)y_{a+j-1} + ... - f_n(x_1)y_{a+j-n+1}$$

Since $y_{a+j}, y_{a+j-1}, ...y_{a+j-n+1}$ are known and $f_0(x_1) \neq 0$ since f_0 is never zero in T, we conclude that y_{a+j+1} is uniquely determined. Thus we have proved by induction that y is uniquely determined for all x in T provided that the values $y_a, y_{a+1}, ...y_{a+n-1}$ are known or prescribed.

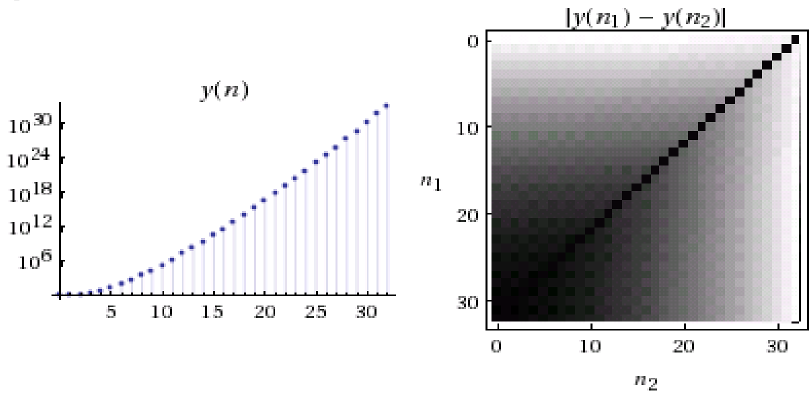
Now if $y_m(m > 0)$ is the first of the n consecutive prescribed values of y instead of y then we can successively determine unique values for $y_{m+1}, y_{m+2}, ...y_{a+1}, y_a$ and then show that all other values of y are uniquely determined. Substituting $x = m-1$ in (1) we obtain $f_n(m-1)y_{m-1} = g(m-1) - f_0(m-1)y_{m-1+n} .... + f_{n-1}(m-1)y_n$. Since $y_m, y_{m+1,} ....y_{m+n-1}$ are supposed to be the prescribed values of y, the right hand side and therefore y_{m-1} is determined as f_n is never zero.

In any case whether T is the finite set $a \le x \le b$ or infinite set $x \ge a$ and no matter n consecutive values of y is prescribed. We have a unique value of y determined for every x values in T. and, therefore have a unique function which satisfies the difference equation and assumes the prescribed value.

Example: 1 The difference equation $y_{n+2} - ny_{n+1} - y_n = 0$, $n = 0,1,2,......$ has no solution when $y_0 = 1$ and $y_1 = 1$ but there are infinitely many solutions if $y_0 = y_1 = 1$.

$$\{y(n) + ny(n+1) = y(n+2), y(0) = 1, y(1) = 1\}$$

Graph of the above equation is



The values are

n	0	1	2	3	4
$y(n)$	1	1	1	2	5

CONCLUSION

Based on some theorems and by definition of Oscillation the solution of the given equation is non oscillatory.

Theorem: 2 Let $y^{(1)}$ *and* $y^{(2)}$ be two solutions of the homogeneous difference equation.

$$y_{n+2} + a_1 y_{n+1} + a_2 y_n = 0 \quad (2)$$

and let $y = c_1 y^{(1)} + c_2 y^{(2)}$ where c_1 and c_2 are arbitrary constants.

If $$\begin{vmatrix} y_0^{(1)} & y_0^{(2)} \\ y_1^{(1)} & y_1^{(2)} \end{vmatrix} \neq 0 \quad (3)$$

then y is the general solution of equation (2).

Proof: Since $y^{(1)}$ and $y^{(2)}$ are the solutions of (2), it can be proved that

If $c_1 y^{(1)} + c_2 y^{(2)}$ is also a solution. Hence Y is a solution of (2).

Next we shall prove that if y is any solution of (2), then c_1 and c_2 may be determined and hence Y and y are identical.

By the uniqueness theorem, it is clear that we should prove that Y and y are equal at n=0 and n=1. Thus we should determine c_1 and c_2 so that $Y_0 = y_0$ and $Y_1 = y_1$ for any choice of y_0 and y_1. But $Y_0 = c_1 y_0^{(1)} + c_2 y_0^{(2)}$ and $Y_1 = c_1 y_1^{(1)} + c_2 y_1^{(2)}$ so c_1 and c_2 must satisfy the equations.

$$y_0^{(1)} c_1 + y_0^{(2)} c_2 = y_0 \ ; \ y_1^{(1)} c_1 + y_1^{(2)} c_2 = y_1$$

Since $\begin{vmatrix} y_0^{(1)} & y_0^{(2)} \\ y_1^{(1)} & y_1^{(2)} \end{vmatrix} \neq 0$, we find first a finite unique pair values of c_1 and c_2 for each choice of y_0 and y_1.

Definition: 5 Two solutions $y^{(1)}$ and $y^{(2)}$ of the above theorem satisfying conditions (2) are said to form a fundamental set or system of solution of (2).

Example: 2 let $y^{(1)} = \left(\frac{1}{2}\right)^n$ and $y^{(2)} = (-2)^n$ are the solutions of the following homogeneous equation and that they form fundamental set of solutions.

$$2y_{n+2} + 3y_{n+1} - 2y_n = 0 \quad (4)$$

$y^{(1)} = \left(\frac{1}{2}\right)^n$ is solution of (4) since we have on substituting $y_n^{(1)} = \left(\frac{1}{2}\right)^n$

$$2\left(\frac{1}{2}\right)^{n+2} + 3\left(\frac{1}{2}\right)^{n+1} - 2\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^n \left[2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 2\right] = \left(\frac{1}{2}\right)^n [0] = 0$$

Also $y^{(2)} = (-2)^n$ is a solution of (4).

For then LHS of (4) = $2(-2)^{n+2} + 3(-2)^{n+1} - 2(-2)^n = (-2)^n \left[2(-2)^2 + 3(-2) - 2\right] = 0$ thus $y^{(1)} = \left(\frac{1}{2}\right)^n$ and $y^{(2)} = (-2)^n$ are solutions of (4).

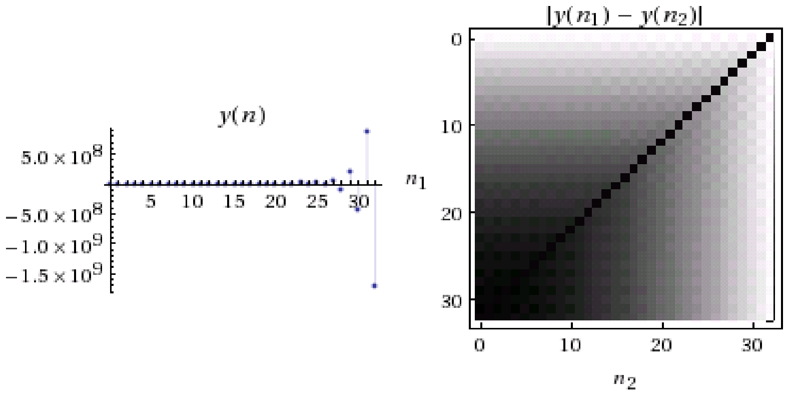
Further $\begin{vmatrix} y_0^{(1)} & y_0^{(2)} \\ y_1^{(1)} & y_1^{(2)} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ \frac{1}{2} & -2 \end{vmatrix} = (-2) - \left(\frac{1}{2}\right) = \frac{-5}{2} \neq 0$

Hence $y^{(1)}$ and $y^{(2)}$ form a fundamental set of (4).

Now we have to check the solutions of the equation (4) is oscillatory or not.

The given equation is $2y_{n+2} + 3y_{n+1} - 2y_n = 0$, y(0)=0, y(1)=1

The graph of the equation is



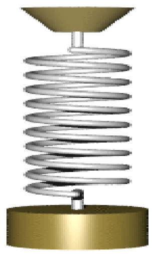
The values are

n	0	1	2	3	4
$y(n)$	0	1	−1.5	3.25	−6.375

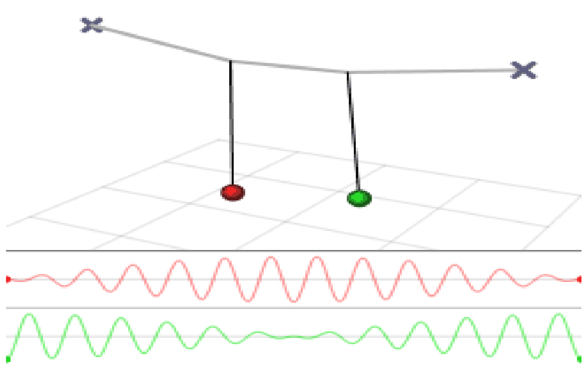
Conclusion: Based on some theorems and by definition of Oscillation the solution of the above equation is oscillatory.

Real time examples for Oscillation:

Example: i An undamped spring–mass system is an oscillatory system:



Example: ii Coupled oscillations

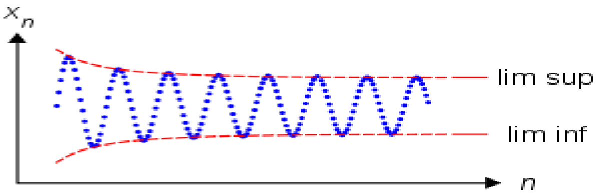


Two pendulums with the same period fixed on a string act as pair of coupled oscillators. The oscillation alternates between the two.

$$y^{(2)} = (-2)^n$$

Example: iii

Mathematics of oscillation



Oscillation of a sequence (shown in blue) is the difference between the limit superior and limit inferior of the sequence.

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