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GENERAL MANPOWER –MODIFIED ERLANG TWO PHASE MACHINE SYSTEM WITH EXPONENTIAL PRODUCTION AND GENERAL RECRUITMENT, SALES AND REPAIR

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ABSTRACT

Two Manpower planning models with different recruitment patterns are studied. In this paper, the machine attended by manpower has modified Erlang phase 2 life time distribution with two phases. Two models are treated. In model 1, when the system fails the vacancies caused by the departure of employees are filled up one by one, when the manpower machine system is in operation, products are produced one at a time when inter production time has exponential distribution and sales time has general distribution. In Model 2, when the operation time is more than a threshold time, the recruitments are done all together and when it is less than the threshold, the recruitments are done one by one. Joint transforms of the variables, their means and the covariance of operation time and recruitment time with numerical results are presented.

Mathematics Subject Classification: *91B70.*

Keywords: *Departure and Recruitments, Failure and Repairs, Production and Sale times, Joint transforms.*

I. INTRODUCTION

Manpower Planning models have been studied by Grinold and Marshall [3]. For statistical approach one may refer to Bartholomew [1]. Lesson [6] has given methods to compute shortages (Resignations, Dismissals, Deaths). Markovian models are designed for shortage and promotion in Man Power System by Vassiliou [14]. Subramanian. [13] has made an attempt to provide optimal policy for recruitment, training, promotion and shortages in manpower planning models with special provisions such as time bound promotions, cost of training and voluntary retirement schemes. For other manpower models one may refer Setlhare [11]. For three characteristics system in manpower models one may refer to Mohan and Ramanarayanan [10]. Esary *et al.* [2] have discussed cumulative damage processes. Stochastic analysis of manpower levels affecting business with varying recruitment rate are presented by Hari Kumar, Sekar and Ramanarayanan [4]. Manpower System with Erlang departure and one by one recruitment is discussed by Hari Kumar [5]. For the study of Semi Markov Models in Manpower planning one may refer Meclean [9]. General production and Sales by Markovian Manpower and Machine System were analyzed by Madhusoodhanan.P, Sekar.P and Ramanarayanan.R[7]. General Manpower-SCBZ Machine System with Markovian Production General Sales and General Recruitment have been discussed by Snehalatha.M., Sekar.P and Ramanarayanan.R[12]. Madhusoodhanan.P., Sekar.P and Ramanarayanan.R discussed on General Production And Sales Time With Two-Units System and Manpower[8].

So far various manpower models have been discussed. The random threshold of employee’s departure has not been treated in depth, with modified Erlang two phase systems. In this paper, the machine attended by manpower has modified Erlang phase 2 life time distribution with two phases. Two models are treated. In model 1, when the system fails the vacancies caused by the departure of employees are filled up one by one , when the manpower machine system is in operation, products are produced one at a time where inter production time has exponential distribution and sales time has general distribution. In Model 2, when the operation time is more than a threshold time, the recruitments are done all together and when it is less than the threshold, the recruitments are done one by one. Joint transforms of the variables, their means and the covariance of operation time and recruitment time with numerical results are presented.

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II. MODEL 1

2.1 ASSUMPTIONS

(1) Inter departure time of employees are i.i.d random variables with Cdf F(x) and pdf f(x). The manpower system collapses with probability p and survives with probability q when an employee departs.
(2) The machine attended by the manpower has modified Erlang phase 2 life time distribution with parameter λ in the two phases. The machine is in phase 1 for a random time with exponential distribution whose parameter is λ after which it fails with probability α or moves to phase 2 with probability β where α+β=1. In phase 2 it has exponential life time distribution with parameter λ.
(3) The man power machine system fails when one of them fails.
(4) When the system fails the vacancies caused by the departure of employees are filled up one by one with recruitment time V, with Cdf V(y) and pdf v(y).
(5) The repair time of the system when the machine is sent for repair from phase I is R_i with Cdf $R_i(z)$ and pdf $r_i(z)$. When the manpower system fails and the machine is in phase 2 it is given for maintenance for time R_3 with Cdf $R_3(z)$ and pdf $r_3(z)$. If the machine is in phase 1 when manpower system fails, there is no maintenance or repair.
(6) When the manpower machine system is in operation, products are produced one at a time where inter production time has exponential distribution with parameter μ.
(7) The sale time of the product is general G with Cdf G(w) and pdf g(w).
(8) When the system fails, the repairs / maintenance, recruitments and sales are undertaken.

2.2 ANALYSIS:

The machine has3 states which are namely phase 1, phase 2 and repair/maintenance state. It starts from phase 1 and its transition rate matrix is given by

$Q=\begin{bmatrix}-\lambda & \lambda\beta & \lambda\alpha \\ 0 & -\lambda & \lambda\end{bmatrix}$ We note here when β=1 we get Erlang phase 2 life time distribution for the machine and when β = 0 we get the distribution as exponential single phase machine system.

The probability that the machine remains at phase 1 at time t is

$$P_1(t)=e^{-\lambda t} \tag{1}$$

The probability that the machine remains at phase 2 at time t starting at time 0 in phase 1 is

$$P_2(t)=\int_0^t \lambda\beta e^{-\lambda u}e^{-\lambda(t-u)}du \tag{2}$$

The transition from phase 1 to phase 2 is considered for writing equation (2), and this gives

$$P_2(t)=\lambda\beta te^{-\lambda t} \tag{3}$$

The Cdf of service time of the machine is

$$H(t)=1-e^{-\lambda t}-\lambda\beta te^{-\lambda t} \tag{4}$$

The pdf of its service time is

$$h(t)=H^{'}(t)=\lambda\alpha e^{-\lambda t}+\lambda^2\beta te^{-\lambda t} \tag{5}$$

The pdf has the structure

$$h(t)=\lambda\alpha P_1(t)+\lambda P_2(t) \tag{6}$$

Here the first term is the part of pdf that the machine fails in phase 1 and the second term is the part of the pdf that it fails in phase 2.

To study the model we require the joint probability density function of the four dimensional random variable $\left(X,\widehat{V},\widehat{R},\widehat{S}\right)$ where (i) X is the operation time of the manpower-machine system which is the minimum of life times of machine and manpower, (ii) $\widehat{V}$ is the sum of recruitment times of the employees to fill up the vacancies caused by the departure of employees, (iii) $\widehat{R}$ is the repair time/maintenance time of the machine and (iv) $\widehat{S}$ is the total sales time of the products. If n employees have left and k products have been produced during the life time of the system then

$$\widehat{V} = V_1 + V_2 + ...V_n,$$
$$\widehat{S} = G_1 + G_2 + ... + G_k,$$

Here V_i, G_i are random variables with Cdf V(y) and G(w) respectively for i=1, 2,…

We also note that $\widehat{V} = R_1, R_2, R_3$, according as the machine fails in phase 1, the machine fails in phase 2 or machine is sent for maintenance from phase 2 when the manpower system fails respectively. The pdf of $\left(X, \widehat{V}, \widehat{R}, \widehat{S}\right)$ is given by

$$f(x,y,z,w) = \begin{cases} (\lambda\alpha P_1(x)V_1(z) + \lambda P_2(x)V_2(z))[\sum_{n=0}^{\infty}(F_n(x) - F_{n+1}(x))q^n v_n(y)] \\ +(P_1(x) + P_2(x)V_3(z))(\sum_{n=1}^{\infty} f_n(x) q^{n-1} p v_n(y)) \end{cases} [\sum_{k=0}^{\infty} e^{-\mu x}\frac{(\mu x)^k}{k!} g_k(w)] \quad (7)$$

Here $v_n(y)$, $f_n(x)$ and $g_k(w)$ are n fold/k fold convolution of v(y), f(x) and g(w) respectively. $F_n(x)$ is the Stieltjes convolution of Cdf F(x) with itself. $P_i(x)$ for I = 1, 2 are as given in equations (1) and (3). To write down equation (7) the two cases, namely (i) the machine fails before the manpower and (ii) the manpower fails before the machine are noted. They are considered to write down the two terms inside the flower bracket besides considering respective repair in phase 1, repair in phase 2 or maintenance and recruitments of employees. The last bracket indicates that k products are produced during life time and are sold.

The quadruple Laplace transform of the pdf is

$$f^*(\xi,\eta,\varepsilon,\delta) = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-\xi x - \eta y - \varepsilon z - \delta w} f(x,y,z,w)\,dx\,dy\,dz\,dw \quad (8)$$

Here * indicates Laplace Transform. Using the structure of equation (7), Equation (8) becomes a single integral as follows.

$$f^*(\xi,\eta,\varepsilon,\delta) = \int_0^{\infty} e^{-\xi x} e^{-\mu x(1-g^*(\delta))} \begin{Bmatrix} [\lambda\alpha r_1^*(\varepsilon)e^{-\lambda x} + \lambda^2 r_2^*(\varepsilon)\beta x e^{-\lambda x}] \\ [\sum_{n=0}^{\infty}(F_n(x) - F_{n+1}(x))q^n v_n^*(\eta)] + [e^{-\lambda x} + \lambda\beta x e^{-\lambda x} r_3^*(\varepsilon)] \\ [\sum_{n=1}^{\infty} f_n(x) q^{n-1} p v_n^*(\eta)] \end{Bmatrix} dx \quad (9)$$

Using derivative of Laplace transforms

$$f^*(\xi,\eta,\varepsilon,\delta) = \frac{\lambda\alpha r_1^*(\varepsilon)(1-f^*(\chi))}{\chi(1-qv^*(\eta)f^*(\chi))} - \lambda^2\beta r_2^*(\varepsilon)\frac{\partial}{\partial\xi}\left(\frac{(1-f^*(\chi))}{\chi(1-qv^*(\eta)f^*(\chi))}\right)$$
$$+\frac{pv^*(\eta)f^*(\chi)}{(1-qv^*(\eta)f^*(\chi))} - \lambda\beta r_3^*(\varepsilon)\frac{\partial}{\partial\xi}\left(\frac{pv^*(\eta)f^*(\chi)}{(1-qv^*(\eta)f^*(\chi))}\right) \quad (10)$$

Here $\chi = \xi + \mu(1-g^*(\delta)) + \lambda$ (11)

On simplification we get,

$$f^*(\xi,\eta,\varepsilon,\delta) = \frac{(1-f^*(\chi))}{(1-qv^*(\eta)f^*(\chi))}\frac{1}{\chi^2}[\chi\lambda\alpha r_1^*(\varepsilon) + \lambda^2\beta r_2^*(\varepsilon)] + \lambda^2\beta\frac{r_2^*(\varepsilon)(1-qv^*(\eta))f^{*'}(\chi)}{\chi(1-qv^*(\eta)f^*(\chi))^2}$$
$$+\frac{pv^*(\eta)f^*(\chi)}{(1-qv^*(\eta)f^*(\chi))} - \lambda\beta r_3^*(\varepsilon)\frac{pv^*(\eta)f^{*'}(\chi)}{(1-qv^*(\eta)f^*(\chi))^2} \quad (12)$$

The expected value of repair/maintenance time is

$$E(\hat{R}) = -\frac{\partial}{\partial \varepsilon} f^*(\xi,\eta,\varepsilon,\delta) \mid \xi=\eta=\varepsilon=\delta=0$$

We get

$$E(\hat{R}) = \frac{(1-f^*(\lambda))}{(1-qf^*(\lambda))}(\alpha E(R_1)+\beta E(R_2)) + \frac{\lambda\beta p f^{*'}(\lambda)}{(1-qf^*(\lambda))^2}(E(R_2)-E(R_3)) \tag{13}$$

The expected sales time is

$$E(\hat{S}) = -\frac{\partial}{\partial \delta} f^*(\xi,\eta,\varepsilon,\delta) \mid \xi=\eta=\varepsilon=\delta=0$$

We get

$$E(\hat{S}) = E(S)\left[\frac{(1-f^*(\lambda))}{(1-qf^*(\lambda))}\frac{\mu}{\lambda}(1+\beta) + \frac{p\beta\mu f^{*'}(\lambda)}{(1-qf^*(\lambda))^2}\right] \tag{14}$$

Now expected time to failure of the manpower-machine system is

$$E(X) = -\frac{\partial}{\partial \xi} f^*(\xi,\eta,\varepsilon,\delta) \mid \xi=\eta=\varepsilon=\delta=0$$

$$E(X) = \left[\frac{(1-f^*(\lambda))}{(1-qf^*(\lambda))}\frac{1}{\lambda}(1+\beta) + \frac{p\beta\mu f^{*'}(\lambda)}{(1-qf^*(\lambda))^2}\right] \tag{15}$$

The Laplace transforms of the joint p.d.f of X and $\hat{V}$ is given by

$$f^*(\xi,\eta,0,0) = \frac{(1-f^*(\chi_1))(\xi\lambda\alpha+\lambda^2)}{(1-qv^*(\eta)f^*(\chi_1))\chi_1^2} + \frac{\lambda^2\beta}{\chi_1}\frac{(1-qv^*(\eta))f^{*'}(\chi_1)}{(1-qv^*(\eta)f^*(\chi_1))^2}$$
$$+\frac{pv^*(\eta)f^*(\chi_1)}{(1-qv^*(\eta)f^*(\chi_1))} - \frac{\lambda\beta pv^*(\eta)f^{*'}(\chi_1)}{(1-qv^*(\eta)f^*(\chi_1))^2} \tag{16}$$

Here $\chi_1 = \xi + \lambda$ (17)

Expected recruitment time is

$$E(\hat{V}) = -\frac{\partial}{\partial \eta} f^*(\xi,\eta,0,0) \mid \xi=\eta=0.$$

This becomes

$$E(\hat{V}) = E(V)\left[\frac{f^*(\lambda)}{(1-qf^*(\lambda))} - \frac{\lambda\beta f^{*'}(\lambda)}{(1-qf^*(\lambda))^2}\right] \tag{18}$$

The product moment of X and $\hat{V}$ is given by

$$E(X\hat{V}) = \frac{\partial^2}{\partial\xi\partial\eta} f^*(\xi,\eta,0,0) \mid \xi=\eta=0$$

This gives

$$E(X\hat{V}) = \frac{E(V)}{(1-qf^*(\lambda))^2}\left\{\begin{aligned}&\left[\left(\frac{1+\beta}{\lambda}\right)qf^*(\lambda)(1-f^*(\lambda)) - f^{*'}(\lambda)\right]\\ &-\frac{\beta q}{(1-qf^*(\lambda))} f^{*'}(\lambda)(1-f^*(\lambda)) - pf^*(\lambda)\\ &+\frac{\lambda\beta}{(1-qf^*(\lambda))}(f^{*''}(\lambda)(1-qf^*(\lambda)) + 2q(f^{*'}(\lambda))^2)\end{aligned}\right\} \tag{19}$$

Using the formula

$$Cov(X\hat{V}) = E(X\hat{V}) - E(X)E(\hat{V})$$

and equations (15), (18) and (19) cov ariacne of X and $\hat{V}$ can be written.

III. MODEL 2

In this section, the previous model 1 with all assumptions 1, 2, 3,5,6,7 & 8 except the assumption 4 concerning the recruitment pattern is treated.

3.1 ASSUMPTIONS FOR MANPOWER RECRUITMENT

4.1 When the operation time X is more than a threshold time U, the recruitments are done all together. It is assigned to an agent whose service time V_1 to fill up all vacancies has Cdf $V_1(y)$ and pdf $v_1(y)$.

4.2 When the operation time X is less than the threshold U, the recruitments are done one by one and the recruitment time V_2 for each has Cdf $V_2(y)$ and pdf $v_2(y)$.

4.3 The threshold U has exponential distribution with parameter θ.

3.2 ANALYSIS:

Using the arguments given for model 1 the joint pdf of $\left(X,\hat{V},\widehat{R},\hat{S}\right)$(operation time, recruitment time, repair/maintenance time of the machine, sales time) may be obtained as follows .

$$f(x,y,z,w) = \left\{ \begin{array}{l} (\lambda\alpha P_1(x)r_1(z) + \lambda P_2(x)r_2(z)) \\ \sum_{n=0}^{\infty}(F_n(x) - F_{n+1}(x))q^n[(1-e^{-\theta x})v_1(y) + e^{-\theta x}v_{2,n}(y)] + \\ (P_1(x) + P_2(x)r_2(z))\sum_{n=1}^{\infty} f_n(x)q^{n-1}p[(1-e^{-\theta x})v_1(y) + e^{-\theta x}v_{2,n}(y)] \end{array} \right\} (\sum_{k=0}^{\infty} e^{-\mu x}\frac{(\mu x)^k}{k!}g_k(w)) \quad (20)$$

We use the same arguments given for model 1 for all terms in equation (20) except the square brackets appearing in the first and second terms of the flower bracket. The pdf $v_n(y)$ appearing in equation (7) is replaced by $(1-e^{-\theta x})v_1(y)+e^{-\theta x}v_{2,n}(y)$ indicating the cases, namely the operation time X is greater than the threshold U, (X>U) and the operation time X is less than the threshold U, (X<U). The function v_2,n(y) is the n-fold convolution of $v_2(y)$ with itself. The quadruple Laplace transform of the pdf of $\left(X,\hat{V},\widehat{R},\hat{S}\right)$ is given by

$$f^*(\xi,\eta,\varepsilon,\delta) = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-\xi x-\eta y-\varepsilon z-\delta w} f(x,y,z,w)dxdydzdw \quad (21)$$

Using the structure of equation (20), equation (21) becomes a single integral as follows.

$$f^*(\xi,\eta,\varepsilon,\delta) = \int_0^{\infty} e^{-\xi x}e^{-\mu x(1-g^*(\delta))} \left\{ \begin{array}{l} (\lambda\alpha P_1(x)r_1^*(\varepsilon) + \lambda P_2(x)r_2^*(\varepsilon)) \\ \sum_{n=0}^{\infty}(F_n(x) - F_{n+1}(x))q^n[(1-e^{-\theta x})v_1^*(\eta) + e^{-\theta x}v_2^{*n}(\eta)] \\ +(P_1(x) + P_2(x)r_3^*(z))\sum_{n=1}^{\infty} f_n(x)q^{n-1}p[(1-e^{-\theta x})v_1^*(\eta) + e^{-\theta x}v_2^{*n}(\eta)] \end{array} \right\} dx \quad (22)$$

Here $P_1(x)$ and $P_2(x)$ are given by equations (1) and (3)

$$f^*(\xi,\eta,\varepsilon,\delta) = \lambda\alpha r_1^*(\varepsilon)\left\{ \frac{(1-f^*(\chi))v_1^*(\eta)}{\chi(1-qf^*(\chi))} - \frac{(1-f^*(\chi+\theta))v_1^*(\eta)}{(\chi+\theta)(1-qf^*(\chi+\theta))} + \frac{(1-f^*(\chi+\theta))}{(\chi+\theta)(1-qv_2^*(\eta)f^*(\chi+\theta))} \right\}$$

$$-\lambda^2\beta r_2^*(\varepsilon)\frac{\partial}{\partial\xi}\left\{\frac{(1-f^*(\chi))v_1^*(\eta)}{\chi(1-qf^*(\chi))}-\frac{(1-f^*(\chi+\theta))v_1^*(\eta)}{(\chi+\theta)(1-qf^*(\chi+\theta))}+\frac{(1-f^*(\chi+\theta))}{(\chi+\theta)(1-qv_2^*(\eta)f^*(\chi+\theta))}\right\}$$

$$+p\left\{\frac{v_1^*(\eta)f^*(\chi)}{(1-qf^*(\chi))}-\frac{v_1^*(\eta)f^*(\chi+\theta)}{(1-qf^*(\chi+\theta))}+\frac{v_2^*(\eta)f^*(\chi+\theta)}{(1-qv_2^*(\eta)f^*(\chi+\theta))}\right\}$$

$$-\lambda\beta pr_3^*(\varepsilon)\frac{\partial}{\partial\xi}\left\{\frac{v_1^*(\eta)f^*(\chi)}{(1-qf^*(\chi))}-\frac{v_1^*(\eta)f^*(\chi+\theta)}{(1-qf^*(\chi+\theta))}+\frac{v_2^*(\eta)f^*(\chi+\theta)}{(1-qv_2^*(\eta)f^*(\chi+\theta))}\right\} \quad (23)$$

Here χ is as given by equation (11) . Finding the partial derivatives with respect to ξ and simplifying equation (23), it is seen that

$$f^*(\xi,\eta,\varepsilon,\delta)=\lambda\alpha r_1^*(\varepsilon)\left\{\frac{(1-f^*(\chi))v_1^*(\eta)}{\chi(1-qf^*(\chi))}-\frac{(1-f^*(\chi+\theta))v_1^*(\eta)}{(\chi+\theta)(1-qf^*(\chi+\theta))}+\frac{(1-f^*(\chi+\theta))}{(\chi+\theta)(1-qv_2^*(\eta)f^*(\chi+\theta))}\right\}$$

$$+\lambda^2\beta r_2^*(\varepsilon)\left\{\begin{array}{l}\dfrac{(1-f^*(\chi))v_1^*(\eta)}{\chi^2(1-qf^*(\chi))}+\dfrac{pf^{*'}(\chi)v_1^*(\eta)}{\chi(1-qf^*(\chi))^2}\\ -\dfrac{(1-f^*(\chi+\theta))v_1^*(\eta)}{(\chi+\theta)^2(1-qf^*(\chi+\theta))}-\dfrac{pf^{*'}(\chi+\theta)v_1^*(\eta)}{(\chi+\theta)(1-qf^*(\chi+\theta))^2}\\ +\dfrac{(1-f^*(\chi+\theta))}{(\chi+\theta)^2(1-qv_2^*(\eta)f^*(\chi+\theta))}+\dfrac{f^{*'}(\chi+\theta))(1-qv_2^*(\eta))}{(\chi+\theta)(1-qv_2^*(\eta)f^*(\chi+\theta))^2}\end{array}\right\}$$

$$+p\left\{\frac{v_1^*(\eta)f^*(\chi)}{(1-qf^*(\chi))}-\frac{v_1^*(\eta)f^*(\chi+\theta)}{(1-qf^*(\chi+\theta))}+\frac{v_2^*(\eta)f^*(\chi+\theta)}{(1-qv_2^*(\eta)f^*(\chi+\theta))}\right\}$$

$$-\lambda\beta pr_3^*(\varepsilon)\left\{\frac{v_1^*(\eta)f^{*'}(\chi)}{(1-qf^*(\chi))^2}-\frac{v_1^*(\eta)f^{*'}(\chi+\theta)}{(1-qf^*(\chi+\theta))^2}+\frac{v_2^*(\eta)f^{*'}(\chi+\theta)}{(1-qv_2^*(\eta)f^*(\chi+\theta))^2}\right\} \quad (24)$$

Since there is only change in the recruitment pattern when compared to model 1, it is noted that

$$E(X),E(\hat{R})\,and\,E(\hat{S})\,remain\ the\ same\ as\ that\ of\ model\ 1.$$

$$Now\ E(\hat{V})=-\frac{\partial}{\partial\eta}f^*(\xi,\eta,\varepsilon,\delta)\,|\,\xi=\eta=\varepsilon=\delta=0,$$

This gives

$$E(\hat{V})=$$

$$E(V_1)-E(V_1)\left[\frac{(1-f^*(\lambda+\theta))}{(1-qf^*(\lambda+\theta))}\left(\frac{\lambda}{\lambda+\theta}\right)\left(\alpha+\frac{\beta\lambda}{\lambda+\theta}\right)+\frac{pf^*(\lambda+\theta)}{(1-qf^*(\lambda+\theta))}+\frac{\lambda\beta pf^{*'}(\lambda+\theta)}{(1-qf^*(\lambda+\theta))^2}\left(\frac{\theta}{\lambda+\theta}\right)\right]$$

$$+\frac{E(V_2)}{(1-qf^*(\lambda+\theta))^2}\left[f^*(\lambda+\theta)(1-f^*(\lambda+\theta))q\left(\frac{\lambda}{\lambda+\theta}\right)\left(\alpha+\frac{\beta\lambda}{\lambda+\theta}\right)+pf^*(\lambda+\theta)-\frac{\lambda^2\beta}{\lambda+\theta}f^{*'}(\lambda+\theta)\right]$$

$$-\frac{\lambda\beta\theta pE(V_2)f^{*'}(\lambda+\theta)(1+qf^*(\lambda+\theta))}{(\lambda+\theta)(1-qf^*(\lambda+\theta))^3} \quad (25)$$

Here θ=0 gives the equation (18) of model 1

$$E(X\hat{V})\ and\ \frac{\partial^2}{\partial\xi\partial\eta}f^*(\xi,\eta,\varepsilon,\delta)\,|\,\xi=\eta=\varepsilon=\delta=0$$

.

After simplification

$$E(X\hat{V}) = \frac{E(V_1)}{(1-qf^*(\lambda))^2}\left[p\beta f^{*'}(\lambda) + (\frac{1+\beta}{\lambda})(1-qf^*(\lambda))(1-f^*(\lambda))\right]$$

$$-\frac{E(V_1)}{(1-qf^*(\lambda+\theta))^2}\left[\begin{array}{l}(\lambda^2\beta-\lambda\theta(2-\alpha)-\theta^2)\dfrac{pf^{*'}(\lambda+\theta)}{(\lambda+\theta)^2} \\ +\dfrac{\lambda(\lambda+\lambda\beta+\theta\alpha)}{(\lambda+\theta)^3}(1-qf^*(\lambda+\theta))(1-f^*(\lambda+\theta)) \\ +\theta\lambda\beta pf^{*'}(\lambda+\theta)+\dfrac{2\theta\lambda\beta pq(f^{*'}(\lambda+\theta))^2}{(1-qf^*(\lambda+\theta))}\end{array}\right]$$

$$+E(V_2)\left[\begin{array}{l}\dfrac{\lambda qf^*(\lambda+\theta)(1-f^*(\lambda+\theta))(\lambda(1+\beta)+\alpha\theta)}{(\lambda+\theta)^3(1-qf^*(\lambda+\theta))^2} \\ -\dfrac{f^{*'}(\lambda+\theta)[(1+qf^*(\lambda+\theta))(\lambda p+\lambda\alpha q+\theta p)-2q\lambda\alpha f^*(\lambda+\theta)]}{(1-qf^*(\lambda+\theta))^3(\lambda+\theta)} \\ -\dfrac{2f^{*'}(\lambda+\theta)\beta\lambda^2 q(1-f^*(\lambda+\theta)-pf^*(\lambda+\theta))}{(1-qf^*(\lambda+\theta))^3(\lambda+\theta)^2} \\ +\dfrac{\beta\lambda(f^{*'}(\lambda+\theta))^2[2\lambda q(1-qf^*(\lambda+\theta)+2\theta pq(2+qf)]}{(\lambda+\theta)(1-qf^*(\lambda+\theta))^4} \\ +\dfrac{\lambda\beta f^{*'}(\lambda+\theta)[\lambda(1-qf^*(\lambda+\theta))+\theta p(1+qf^*(\lambda+\theta))]}{(\lambda+\theta)(1-qf^*(\lambda+\theta))^3}\end{array}\right] \quad (26)$$

Using the formula

$$Cov(X\hat{V}) = E(X\hat{V}) - E(X)E(\hat{V})$$

The Co-variance may be written using equations (15),(25) and (26).

IV. NUMERICAL EXAMPLES

NUMERICAL EXAMPLES FOR MODELS 1 & 2:

To highlight the usefulness of the results obtained so far numerical examples are presented. Here the two models 1 and 2 are treated together. Since there is only change in the manpower recruitment pattern, to fill up the manpower loss, the expected operation time E(X), expected repair time and expected sales time coincide in the two models.

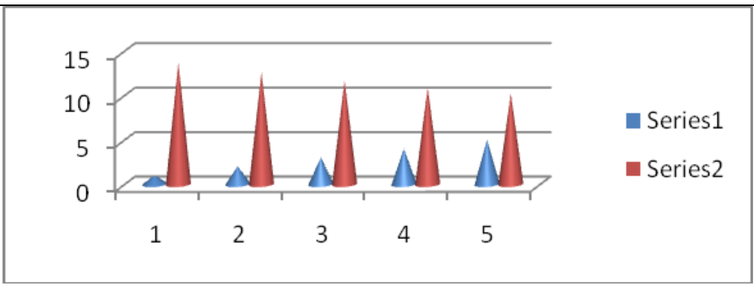
Numerical values for Model 1:

α=0.5, β=0.5, p=0.4, q=0.6, $E(R_1)$=10, $E(R_2)$ =20, $E(R_3)$=15, E(V)=5, E(S)=10, λ=5, δ=1, 2,3,4,5 and, μ =2, 4, 6,8,10.

Here f is exponential probability distribution function with parameter δ.

The table and graph for $E(\hat{R})$

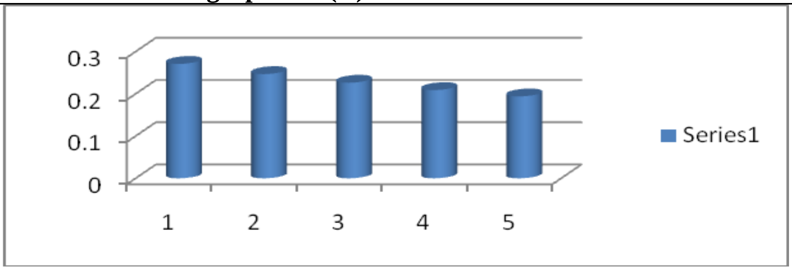
δ	$E(\hat{V})$
1	1.354595
2	2.467301
3	3.394901
4	4.178145
5	4.846939



When δ increases, the expected value of the repair time decreases.

Table and graph of E(X)

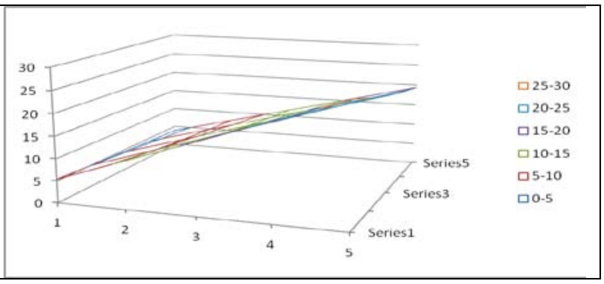
δ	E(X)
1	0.270919
2	0.24673
3	0.226327
4	0.208907
5	0.193878



When δ increases, the expected value of the operation time decreases

The table and graph for $E(\hat{S})$

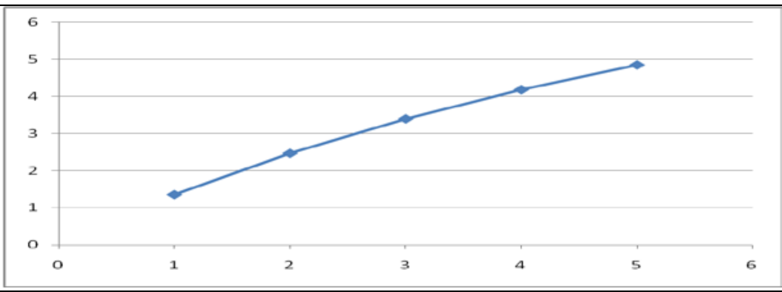
δ / μ	2	4	6	8	10
1	5.418381	10.83676	16.25514	21.67353	27.09191
2	4.934602	9.869209	14.8038	19.73841	24.67301
3	4.526535	9.05307	13.5796	18.10614	22.62267
4	4.178145	8.35629	12.53444	16.71258	20.89073
5	3.877551	7.755102	11.63265	15.5102	19.38776



When δ and μ increase, the expected value of the sales time decrease

The table and graph for $E(\hat{V})$

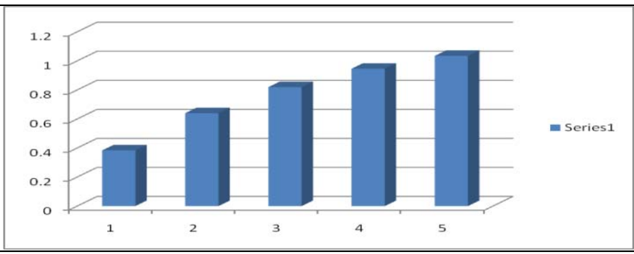
δ	$E(\hat{V})$
1	1.354595
2	2.467301
3	3.394901
4	4.178145
5	4.846939



When δ increases, the expected value of recruitment time increases.

The table and graph for $E(X\hat{V})$

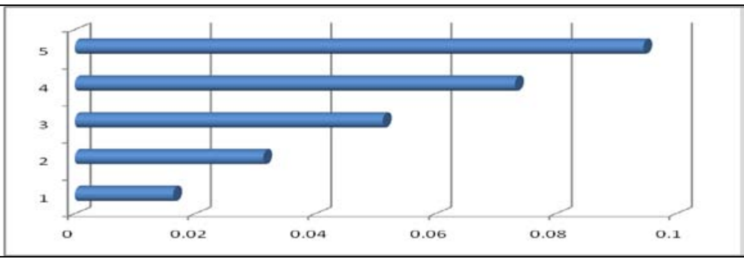
δ	$E(X\hat{V})$
1	0.383347
2	0.640091
3	0.819569
4	0.946043
5	1.034257



When δ increases, the expected value of the product moment, $E(X\hat{V})$ increases

The table and graph for $Cov(X\hat{V})$

δ	$Cov(X\hat{V})$
1	0.01636
2	0.03133
3	0.05121
4	0.0732
5	0.09454



When δ increases, the $Cov(X\hat{V})$ increases.

REFERENCES

1. Barthlomew.D.J, Statistical technique for manpower planning, John Wiley, Chichester (1979)
2. Esary.J.D, A.W. Marshall, F. Proschan, Shock models and wear processes, *Ann. Probability*, 1, No. 4 (1973), 627-649
3. Grinold.R.C, Marshall.K.T, Manpower Planning models, North Holl, Newyork (1977)
4. Hari Kumar.K, P.Sekar and R.Ramanarayanan, Stochastic Analysis of Manpower levels affecting business with varying Recruitment rate, International Journal of Applied Mathematics, Vol.8,2014 no.29, 1421-1428.
5. Hari Kumar.K, Manpower System with Erlang departure and one by one Recruitment, International Journal of Mathematical Archive-5(7), 2014, 89-92.ISSN 2229-5046.
6. Lesson .G.W, Wastage and Promotion in desired Manpower Structures, J. Opl. Res. Soc., 33(1982)433-442.
7. Madhusoodhanan.P., General Production and Sales by Markovian Manpower and Machine System, International Journal of Mathematical Archieve-6(7), July 2015, *ISSN2229-5046, pp. 80-87*.
8. Madhusoodhanan.P., Sekar.P and Ramanarayanan.R, General Production And Sales Time With Two-Units System And Manpower, International Journal of Applied Engineering Research *ISSN 0973-4562 Volume 10, Number 23 (2015) pp 43415-43421*
9. McClean, Semi-Markovian Models in continuous time, J. Appl. Prob., 16(1980), 416-422.
10. Mohan.C and Ramanarayanan.R, An analysis of Manpower, Money and Business with Random Environments, International Journal of Applied Mathematics, 23(5) (2010), 927-940.
11. Setlhare,Modeling of an intermittenly busy manpower system, In Proceedings at the Conference held in sept,2006 at Gabarone, Botswana(2007)
12. Snehalatha.M., Sekar.P and Ramanarayanan.R, General Manpower-SCBZ Machine System with Markovian Production General Sales and General Recruitment, International Journal of Pure and Applied Mathematics[IJPAM], e-ISSN: 1314-3395, p-ISSN: 1311-8080; Volume 106 No. 2 2016, 443-452.
13. Subramanian.V,Optimum promotion rate in manpower models. International Journal of Management and system, 12(2) (1996), 179-184.
14. Vassiliou.P.C.G, A higher order Markovian model for prediction of wastage in manpower system, Operat, Res. Quart., 27 (1976).

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