MULTIPLICATIVE *K* HYPER-BANHATTI INDICES AND COINDICES OF GRAPHS

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ABSTRACT

The vertices and edges of a graph G are called its elements, If e = uv is an edge of G, then the vertex u and edge e are incident as are v and e. The first multiplicative K hyper-Banhatti index of G is defined as the product of the squares of the sum of the degrees of pairs of incident elements and the second multiplicative K hyper-Banhatti index of G is defined as the product of the squares of the product of the degrees of pairs of incident elements. In this paper, we initiate a study of multiplicative K hyper-Banhatti indices and coindices of graphs.

Keywords: Banhatti indices, multiplicative K hyper-Banhatti indices, multiplicative K hyper-Banhatti coincides.

Mathematics Subject Classification: 05C05, 05C07, 05C12.

1. INTRODUCTION

By a graph, we mean a finite, undirected, without loops, multiple edges and isolated vertices. Let G be a graph with n vertices and m edges with vertex set $V(G)$ and the edge set $E(G)$. Any undefined term in this paper may be found in Kulli [1].

The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v is denoted by uv. If $e = uv$ is an edge of G then the vertex u and edge e are incident as are v and e. Let $d_G(e)$ denote the degree of an edge e in G, which is defined by $d_G(e) = d_G(u) + d_G(v) - 2$ with $e = uv$. The vertices and edges of a graph are called its elements.

The first and second K Banhatti indices are defined as

$$B_1(G) = \sum_{ue} \left[d_G(u) + d_G(e) \right]$$
$$B_2(G) = \sum_{ue} d_G(u) d_G(e)$$

where ue means that the vertex u and edge e are incident in G.

The first and second K Banhatti coindices are defined as

$$\overline{B_1}(G) = \sum_{u*e} \left[d_G(u) + d_G(e) \right]$$
$$\overline{B_2}(G) = \sum_{u*e} d_G(u) d_G(e)$$

where $u*e$ means that the vertex u and edge e are nonincident in G.

The first and second K Banhatti indices and coincides were introduced by Kulli in [2]. Recently many other indices and coincides were studied, for example, in [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13].

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of mathematical chemistry which has an important effect on the development of the Chemical Sciences.

In Chemical Science, the physico-chemical properties of chemical compounds are often modeled by means of molecular graph based structure descriptors, which are also referred to as topological indices, see [3].

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The first and second K hyper-Banhatti indices of a graph G are defined as

$$HB_1(G) = \sum_{ue} \left[d_G(u) + d_G(e) \right]^2$$

$$HB_2(G) = \sum_{ue} \left(d_G(u) d_G(e) \right)^2.$$

The first and second K hyper-Banhatti coincides of a graph G are defined as

$$\overline{HB_1}(G) = \sum_{u*e} \left[d_G(u) + d_G(e) \right]^2$$

$$\overline{HB_2}(G) = \sum_{u*e} \left(d_G(u) d_G(e) \right)^2.$$

The K hyper-Banhatti indices and coincides were introduced by Kulli in [14].

The first multiplicative K Banhatti index and first multiplicative K Banhatti coindex of G are defined as

$$BII_1(G) = \prod_{ue} \left[d_G(u) + d_G(e) \right]$$

$$\overline{BII}_1(G) = \prod_{u*e} \left[d_G(u) + d_G(e) \right].$$

These invariants were introduced by Kulli in [15].

The second multiplicative K Banhatti index and second multiplicative K Banhati coindex of G are defined as

$$BII_2(G) = \prod_{ue} d_G(u) d_G(e)$$

$$\overline{BII}_2(G) = \prod_{u*e} d_G(u) d_G(e).$$

These invariants were introduced by Kulli in [16]. Recently many other multiplicative indices and coindices of graphs were studied, for example, in [17, 18, 19, 20, 21].

In this paper, we consider the multiplicative variants of K hyper-Banhatti indices and coindices of graphs.

2. FIRST MULTIPLICATIVE *K* HYPER-BANHATTI INDEX

We introduce the first multiplicative K hyper-Banhatti index of a graph G in terms of incident vertex-edge degrees.

Definition 1: The first multiplicative K hyper-Banhatti index of a graph G is defined as

$$HBII_1(G) = \prod_{ue} \left[d_G(u) + d_G(e) \right]^2$$

where ue means that the vertex u and edge e are incident in G.

We compute first multiplicative K hyper-Banhatti index of cycles, complete graphs, complete bipartite graphs, r-regular graphs.

Proposition 2: Let C_n be a cycle with $n \geq 3$ vertices. Then

$$HBII_1(C_n) = 4^{4n}.$$

Proof: Let C_n be a cycle with $n \geq 3$ vertices. Then C_n has n edges. Every edge of C_n is incident with exactly two vertices. Consider

$$HBII_1(C_n) = \prod_{ue} \left[d_{C_n}(u) + d_{C_n}(e) \right]^2$$

$$= \prod_{e=uv \in E(C_n)} \left[d_{C_n}(u) + d_{C_n}(e) \right]^2 \times \left[d_{C_n}(v) + d_{C_n}(e) \right]^2$$

$$= \left[(2+2)^2 \right]^n \times \left[(2+2)^2 \right]^n$$

$$= 4^{4n}.$$

Proposition 3: Let K_n be a complete graph with n vertices. Then
$$HBII_1(K_n) = (3n-5)^{2n(n-1)}.$$

Proof: Let K_n be a complete graph with n vertices. Then K_n has $\dfrac{n(n-1)}{2}$ edges. Every edge of K_n is incident with exactly two vertices. Consider

$$HBII_1(K_n) = \prod_{ue} \left[d_{K_n}(u) + d_{K_n}(e) \right]^2$$

$$= \prod_{e=uv \in E(K_n)} \left[d_{K_n}(u) + d_{K_n}(e) \right]^2 \times \left[d_{K_n}(v) + d_{K_n}(e) \right]^2$$

$$= \left[\{(n-1) + (2n-4)\}^2 \right]^{\frac{n(n-1)}{2}} \times \left[\{(n-1) + (2n-4)\}^2 \right]^{\frac{n(n-1)}{2}}$$

$$= (3n-5)^{2n(n-1)}.$$

Proposition 4: Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$. Then
$$HBII_1(K_{m,n}) = (m+2n-2)^{2mn} \times (2m+n-2)^{2mn}.$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m + n$ vertices, mn edges and $|V_1| = m$, $|V_2| = n$, $V(K_{m,n}) = V_1 \cup V_2$.

Every edge of $K_{m,n}$ is incident with exactly two vertices. Every vertex of V_1 is incident with n vertices and every vertex of V_2 is incident with m vertices. Consider

$$HBII_1(K_{m,n}) = \prod_{ue} \left[d_{K_{m,n}}(u) + d_{K_{m,n}}(e) \right]^2$$

$$= \prod_{\substack{e=uv \in E(K_n) \\ u \in V_1, v \in V_2}} \left[d_{K_{m,n}}(u) + d_{K_{m,n}}(e) \right]^2 \times \left[d_{K_{m,n}}(v) + d_{K_{m,n}}(e) \right]^2$$

$$= \left[\{n + (m+n-2)\}^2 \right]^{mn} \times \left[\{m + (m+n-2)\}^2 \right]^{mn}$$

$$= (m+2n-2)^{2mn} \times (2m+n-2)^{2mn}.$$

The following results are immediate from Proposition 4.

Corollary 5: Let $K_{n,n}$ be a complete bipartite graph. Then
$$HBII_1(K_{n,n}) = (3n-2)^{4n^2}.$$

Corollary 6: Let $K_{1,n}$ be a star. Then
$$HBII_1(K_{1,n}) = n^{2n}(2n-1)^{2n}.$$

Theorem 7: Let G be an r-regular graph with n vertices. Then
$$HBII_1(G) = (3r-2)^{2nr}.$$

Proof: Let G be an r-regular graph with n vertices. Then G has $\dfrac{nr}{2}$ edges. Every edge of G is incident with exactly two vertices. Every vertex of G is adjacent with r vertices. Consider

$$HBII_1(G) = \prod_{ue} \left[d_G(u) + d_G(e) \right]^2$$

$$= \prod_{e=uv \in E(G)} \left[d_G(u) + d_G(e) \right]^2 \times \left[d_G(v) + d_G(e) \right]^2$$

$$= \left[\{r + (2r-2)\}^2 \right]^{\frac{nr}{2}} \times \left[\{r + (2r-2)\}^2 \right]^{\frac{nr}{2}}$$

$$= (3r-2)^{2nr}.$$

3. SECOND MULTIPLICATIVE *K* HYPER-BANHATTI INDEX

We define the second multiplicative *K* hyper-Banhatti index of a graph in terms of incident vertex-edge degrees.

Definition 8: The second multiplicative K hyper-Banhatti index of a graph G is defined as

$$HBII_2(G) = \prod_{ue} \left[d_G(u) d_G(e) \right]^2$$

where ue means that the vertex u and edge e are incident in G.

We determine second multiplicative K hyper-Banhatti index of cycles, complete graphs, complete bipartite graphs, r-regular graphs.

Proposition 9: Let C_n be a cycle with $n \geq 3$ vertices. Then
$$HBII_2(C_n) = 4^{4n}.$$

Proof: Let C_n be a cycle with $n \geq 3$ vertices. Then C_n has n edges. Every edge of C_n is incident with exactly two vertices. Consider

$$HBII_2(C_n) = \prod_{ue} \left[d_G(u) d_G(e) \right]^2$$

$$= \prod_{e=uv \in E(C_n)} \left[d_{C_n}(u) d_{C_n}(e) \right]^2 \times \left[d_{C_n}(v) + d_{C_n}(e) \right]^2$$

$$= \left[(2 \times 2)^2 \right]^n \times \left[(2 \times 2)^2 \right]^n$$

$$= 4^{4n}.$$

Proposition 10: Let K_n be a complete graph with n vertices. Then
$$HBII_2(K_n) = \left[2(n-1)(n-2) \right]^{2n(n-1)}.$$

Proof: Let K_n be a complete graph with n vertices. Then K_n has $\dfrac{n(n-1)}{2}$ edges. Every edge of K_n is incident with exactly two vertices. Consider

$$HBII_2(K_n) = \prod_{ue} \left[d_{K_n}(u) d_{K_n}(e) \right]^2$$

$$= \prod_{e=uv \in E(K_n)} \left[d_{K_n}(u) d_{K_n}(e) \right]^2 \times \left[d_{K_n}(v) d_{K_n}(e) \right]^2$$

$$= \left[\left\{ (n-1)(2n-4) \right\}^2 \right]^{\frac{n(n-1)}{2}} \times \left[\left\{ (n-1)(2n-4) \right\}^2 \right]^{\frac{n(n-1)}{2}}$$

$$= \left[2(n-1)(n-2) \right]^{2n(n-1)}.$$

Proposition 11: Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$. Then
$$HBII_2(K_{m,n}) = (mn)^{2mn} \times (m+n-2)^{4mn}.$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m + n$ vertices, mn edges, and $|V_1| = m$, $|V_2| = n$, $V(K_{m,n}) = V_1 \cup V_2$.

Every edge of $K_{m,n}$ is incident with exactly two vertices. Every vertex of V_1 is incident with n vertices and every vertex of V_2 is incident with m vertices. Consider

$$HBII_2(K_{m,n}) = \prod_{ue} \left[d_{K_{m,n}}(u) d_{K_{m,n}}(e) \right]^2$$

$$= \prod_{\substack{e=uv \in E(K_{m,n}) \\ u \in V_1, v \in V_2}} \left[d_{K_{m,n}}(u) d_{K_{m,n}}(e) \right]^2 \times \left[d_{K_{m,n}}(v) d_{K_{m,n}}(e) \right]^2$$

$$= \left[\left\{ n(m+n-2) \right\}^2 \right]^{mn} \times \left[\left\{ m(m+n-2) \right\}^2 \right]^{mn}$$

$$= (mn)^{2mn} \times (m+n-2)^{4mn}.$$

The following results are immediate from Proposition 11.

Corollary 12: Let $K_{n,n}$ be a complete bipartite graph. Then
$$HBII_2(K_{n,n}) = (2)^{4n^2} \times (n)^{4n^2} \times (n-1)^{4n^2}.$$

Corollary 13: Let $K_{1,n}$ be a star. Then

$$HBII_2\left(K_{1,n}\right) = n^{2n}\left(n-1\right)^{4n}.$$

Theorem 14: Let G be an r-regular graph with n vertices. Then

$$HBII_2\left(G\right) = \left[r\left(2r-2\right)\right]^{2nr}.$$

Proof: Let G be an r-regular graph with n vertices. Then G has $\dfrac{nr}{2}$ edges. Every edge of G is incident with exactly two vertices. Every vertex of G is adjacent with r vertices. Consider

$$
\begin{aligned}
HBII_2\left(G\right) &= \prod_{ue}\left[d_G\left(u\right)d_G\left(e\right)\right]^2 \\
&= \prod_{e=uv\in E(G)}\left[d_G\left(u\right)d_G\left(e\right)\right]^2 \times \left[d_G\left(v\right)d_G\left(e\right)\right]^2 \\
&= \left[r\times\left(2r-2\right)^2\right]^{\frac{nr}{2}} \times \left[r\times\left(2r-2\right)^2\right]^{\frac{nr}{2}} \\
&= \left[r\left(2r-2\right)\right]^{2nr}.
\end{aligned}
$$

4. FIRST AND SECOND MULTIPLICATIVE *K* HYPER-BANHATTI COINDICES

We define the first and second multiplicative *K* hyper-Banhatti coindices of a graph in terms of nonincident vertex-edge degrees.

Definition 15: The first and second multiplicative *K* hyper-Banhatti coindices of a graph G are defined as

$$\overline{HBII}_1\left(G\right) = \prod_{u*e}\left[d_G\left(u\right)+d_G\left(e\right)\right]^2$$

$$\overline{HBII}_2\left(G\right) = \prod_{u*e}\left[d_G\left(u\right)d_G\left(e\right)\right]^2$$

where $u*e$ means that the vertex u and edge e are nonincident in G.

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