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MULTIPLICATIVE *K* HYPER-BANHATTI INDICES AND COINDICES OF GRAPHS

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ABSTRACT

The vertices and edges of a graph G are called its elements, If e = uv is an edge of G, then the vertex u and edge e are incident as are v and e. The first multiplicative K hyper-Banhatti index of G is defined as the product of the squares of the sum of the degrees of pairs of incident elements and the second multiplicative K hyper-Banhatti index of G is defined as the product of the squares of the product of the degrees of pairs of incident elements. In this paper, we initiate a study of multiplicative K hyper-Banhatti indices and coindices of graphs.

Keywords: *Banhatti indices, multiplicative K hyper-Banhatti indices, multiplicative K hyper-Banhatti coincides.*

Mathematics Subject Classification: *05C05, 05C07, 05C12.*

1. INTRODUCTION

By a graph, we mean a finite, undirected, without loops, multiple edges and isolated vertices. Let *G* be a graph with *n* vertices and *m* edges with vertex set *V(G)* and the edge set *E(G)*. Any undefined term in this paper may be found in Kulli [1].

The degree $d_G(v)$ of a vertex *v* is the number of vertices adjacent to *v*. The edge connecting the vertices *u* and *v* is denoted by *uv*. If *e* = *uv* is an edge of *G* then the vertex *u* and edge *e* are incident as are *v* and *e*. Let $d_G(e)$ denote the degree of an edge *e* in G, which is defined by $d_G(e) = d_G(u) + d_G(v) – 2$ with *e* = *uv*. The vertices and edges of a graph are called its elements.

The first and second *K* Banhatti indices are defined as

$$B_1(G) = \sum_{ue}\left[d_G(u) + d_G(e)\right]$$

$$B_2(G) = \sum_{ue} d_G(u)\, d_G(e)$$

where *ue* means that the vertex *u* and edge *e* are incident in *G*.

The first and second *K* Banhatti coindices are defined as

$$\overline{B_1}(G) = \sum_{u*e}\left[d_G(u) + d_G(e)\right]$$

$$\overline{B_2}(G) = \sum_{u*e} d_G(u)\, d_G(e)$$

where *u*e* means that the vertex *u* and edge *e* are nonincident in *G*.

The first and second *K* Banhatti indices and coincides were introduced by Kulli in [2]. Recently many other indices and coincides were studied, for example, in [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13].

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of mathematical chemistry which has an important effect on the development of the Chemical Sciences.

In Chemical Science, the physico-chemical properties of chemical compounds are often modeled by means of molecular graph based structure descriptors, which are also referred to as topological indices, see [3].

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The first and second *K* hyper-Banhatti indices of a graph *G* are defined as

$$HB_1(G) = \sum_{ue}\left[d_G(u) + d_G(e)\right]^2$$

$$HB_2(G) = \sum_{ue}\left(d_G(u)d_G(e)\right)^2.$$

The first and second *K* hyper-Banhatti coincides of a graph *G* are defined as

$$\overline{HB_1}(G) = \sum_{u*e}\left[d_G(u) + d_G(e)\right]^2$$

$$\overline{HB_2}(G) = \sum_{u*e}\left(d_G(u)d_G(e)\right)^2.$$

The *K* hyper-Banhatti indices and coincides were introduced by Kulli in [14].

The first multiplicative *K* Banhatti index and first multiplicative *K* Banhatti coindex of *G* are defined as

$$BII_1(G) = \prod_{ue}\left[d_G(u) + d_G(e)\right]$$

$$\overline{BII}_1(G) = \prod_{u*e}\left[d_G(u) + d_G(e)\right].$$

These invariants were introduced by Kulli in [15].

The second multiplicative *K* Banhatti index and second multiplicative *K* Banhati coindex of *G* are defined as

$$BII_2(G) = \prod_{ue} d_G(u)d_G(e)$$

$$\overline{BII}_2(G) = \prod_{u*e} d_G(u)d_G(e).$$

These invariants were introduced by Kulli in [16]. Recently many other multiplicative indices and coindices of graphs were studied, for example, in [17, 18, 19, 20, 21].

In this paper, we consider the multiplicative variants of *K* hyper-Banhatti indices and coindices of graphs.

2. FIRST MULTIPLICATIVE *K* HYPER-BANHATTI INDEX

We introduce the first multiplicative *K* hyper-Banhatti index of a graph *G* in terms of incident vertex-edge degrees.

Definition 1: The first multiplicative *K* hyper-Banhatti index of a graph *G* is defined as

$$HBII_1(G) = \prod_{ue}\left[d_G(u) + d_G(e)\right]^2$$

where *ue* means that the vertex *u* and edge *e* are incident in *G*.

We compute first multiplicative *K* hyper-Banhatti index of cycles, complete graphs, complete bipartite graphs, *r*-regular graphs.

Proposition 2: Let C_n be a cycle with $n \geq 3$ vertices. Then

$$HBII_1(C_n) = 4^{4n}.$$

Proof: Let C_n be a cycle with $n \geq 3$ vertices. Then C_n has *n* edges. Every edge of C_n is incident with exactly two vertices. Consider

$$HBII_1(C_n) = \prod_{ue}\left[d_{C_n}(u) + d_{C_n}(e)\right]^2$$

$$= \prod_{e=uv\in E(C_n)}\left[d_{C_n}(u) + d_{C_n}(e)\right]^2 \times \left[d_{C_n}(v) + d_{C_n}(e)\right]^2$$

$$= \left[(2+2)^2\right]^n \times \left[(2+2)^2\right]^n$$

$$= 4^{4n}.$$

Proposition 3: Let K_n be a complete graph with n vertices. Then

$$HBII_1(K_n) = (3n-5)^{2n(n-1)}.$$

Proof: Let K_n be a complete graph with n vertices. Then K_n has $\frac{n(n-1)}{2}$ edges. Every edge of K_n is incident with exactly two vertices. Consider

$$HBII_1(K_n) = \prod_{ue}\left[d_{K_n}(u) + d_{K_n}(e)\right]^2$$

$$= \prod_{e=uv \in E(K_n)} \left[d_{K_n}(u) + d_{K_n}(e)\right]^2 \times \left[d_{K_n}(v) + d_{K_n}(e)\right]^2$$

$$= \left[\left\{(n-1)+(2n-4)\right\}^2\right]^{\frac{n(n-1)}{2}} \times \left[\left\{(n-1)+(2n-4)\right\}^2\right]^{\frac{n(n-1)}{2}}$$

$$= (3n-5)^{2n(n-1)}.$$

Proposition 4: Let $K_{m,n}$ be a complete bipartite graph with $1 \le m \le n$. Then

$$HBII_1(K_{m,n}) = (m+2n-2)^{2mn} \times (2m+n-2)^{2mn}.$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m + n$ vertices, mn edges and $|V_1| = m$, $|V_2| = n$, $V(K_{m,n}) = V_1 \cup V_2$.

Every edge of $K_{m,n}$ is incident with exactly two vertices. Every vertex of V_1 is incident with n vertices and every vertex of V_2 is incident with m vertices. Consider

$$HBII_1(K_{m,n}) = \prod_{ue}\left[d_{K_{m,n}}(u) + d_{K_{m,n}}(e)\right]^2$$

$$= \prod_{\substack{e=uv \in E(K_n) \\ u \in V_1, v \in V_2}} \left[d_{K_{m,n}}(u) + d_{K_{m,n}}(e)\right]^2 \times \left[d_{K_{m,n}}(v) + d_{K_{m,n}}(e)\right]^2$$

$$= \left[\left\{n+(m+n-2)\right\}^2\right]^{mn} \times \left[\left\{m+(m+n-2)\right\}^2\right]^{mn}$$

$$= (m+2n-2)^{2mn} \times (2m+n-2)^{2mn}.$$

The following results are immediate from Proposition 4.

Corollary 5: Let $K_{n,n}$ be a complete bipartite graph. Then

$$HBII_1(K_{n,n}) = (3n-2)^{4n^2}.$$

Corollary 6: Let $K_{1,n}$ be a star. Then

$$HBII_1(K_{1,n}) = n^{2n}(2n-1)^{2n}.$$

Theorem 7: Let G be an r-regular graph with n vertices. Then

$$HBII_1(G) = (3r-2)^{2nr}.$$

Proof: Let G be an r-regular graph with n vertices. Then G has $\frac{nr}{2}$ edges. Every edge of G is incident with exactly two vertices. Every vertex of G is adjacent with r vertices. Consider

$$HBII_1(G) = \prod_{ue}\left[d_G(u) + d_G(e)\right]^2$$

$$= \prod_{e=uv \in E(G)} \left[d_G(u) + d_G(e)\right]^2 \times \left[d_G(v) + d_G(e)\right]^2$$

$$= \left[\left\{r+(2r-2)\right\}^2\right]^{nr/2} \times \left[\left\{r+(2r-2)\right\}^2\right]^{nr/2}$$

$$= (3r-2)^{2nr}.$$

3. SECOND MULTIPLICATIVE *K* HYPER-BANHATTI INDEX

We define the second multiplicative K hyper-Banhatti index of a graph in terms of incident vertex-edge degrees.

Definition 8: The second multiplicative K hyper-Banhatti index of a graph G is defined as

$$HBII_2(G)=\prod_{ue}\left[d_G(u)d_G(e)\right]^2$$

where ue means that the vertex u and edge e are incident in G.

We determine second multiplicative K hyper-Banhatti index of cycles, complete graphs, complete bipartite graphs, r-regular graphs.

Proposition 9: Let C_n be a cycle with $n \geq 3$ vertices. Then

$$HBII_2(C_n)=4^{4n}.$$

Proof: Let C_n be a cycle with $n \geq 3$ vertices. Then C_n has n edges. Every edge of C_n is incident with exactly two vertices. Consider

$$\begin{aligned} HBII_2(C_n) &= \prod_{ue}\left[d_G(u)d_G(e)\right]^2 \\ &= \prod_{e=uv\in E(C_n)}\left[d_{C_n}(u)d_{C_n}(e)\right]^2\times\left[d_{C_n}(v)+d_{C_n}(e)\right]^2 \\ &= \left[(2\times 2)^2\right]^n\times\left[(2\times 2)^2\right]^n \\ &= 4^{4n}. \end{aligned}$$

Proposition 10: Let K_n be a complete graph with n vertices. Then

$$HBII_2(K_n)=\left[2(n-1)(n-2)\right]^{2n(n-1)}.$$

Proof: Let K_n be a complete graph with n vertices. Then K_n has $\frac{n(n-1)}{2}$ edges. Every edge of K_n is incident with exactly two vertices. Consider

$$\begin{aligned} HBII_2(K_n) &= \prod_{ue}\left[d_{K_n}(u)d_{K_n}(e)\right]^2 \\ &= \prod_{e=uv\in E(K_n)}\left[d_{K_n}(u)d_{K_n}(e)\right]^2\times\left[d_{K_n}(v)d_{K_n}(e)\right]^2 \\ &= \left[\left\{(n-1)(2n-4)\right\}^2\right]^{\frac{n(n-1)}{2}}\times\left[\left\{(n-1)(2n-4)\right\}^2\right]^{\frac{n(n-1)}{2}} \\ &= \left[2(n-1)(n-2)\right]^{2n(n-1)}. \end{aligned}$$

Proposition 11: Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$. Then

$$HBII_2(K_{m,n})=(mn)^{2mn}\times(m+n-2)^{4mn}.$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m + n$ vertices, mn edges, and $|V_1|= m$, $|V_2|=n$, $V(K_{m,n}) = V_1 \cup V_2$.

Every edge of $K_{m,n}$ is incident with exactly two vertices. Every vertex of V_1 is incident with n vertices and every vertex of V_2 is incident with m vertices. Consider

$$\begin{aligned} HBII_2(K_{m,n}) &= \prod_{ue}\left[d_{K_{m,n}}(u)d_{K_{m,n}}(e)\right]^2 \\ &= \prod_{\substack{e=uv\in E(K_{m,n}) \\ u\in V_1, v\in V_2}}\left[d_{K_{m,n}}(u)d_{K_{m,n}}(e)\right]^2\times\left[d_{K_{m,n}}(v)d_{K_{m,n}}(e)\right]^2 \\ &= \left[\left\{n(m+n-2)\right\}^2\right]^{mn}\times\left[\left\{m(m+n-2)\right\}^2\right]^{mn} \\ &= (mn)^{2mn}\times(m+n-2)^{4mn}. \end{aligned}$$

The following results are immediate from Proposition 11.

Corollary 12: Let $K_{n,n}$ be a complete bipartite graph. Then

$$HBII_2(K_{n,n})=(2)^{4n^2}\times(n)^{4n^2}\times(n-1)^{4n^2}.$$

Corollary 13: Let $K_{1,\,n}$ be a star. Then

$$HBII_2\left(K_{1,n}\right)=n^{2n}\left(n-1\right)^{4n}.$$

Theorem 14: Let G be an r-regular graph with n vertices. Then

$$HBII_2\left(G\right)=\left[r\left(2r-2\right)\right]^{2nr}.$$

Proof: Let G be an r-regular graph with n vertices. Then G has $\frac{nr}{2}$ edges. Every edge of G is incident with exactly two vertices. Every vertex of G is adjacent with r vertices. Consider

$$HBII_2\left(G\right)=\prod_{ue}\left[d_G\left(u\right)d_G\left(e\right)\right]^2$$

$$=\prod_{e=uv\in E(G)}\left[d_G\left(u\right)d_G\left(e\right)\right]^2\times\left[d_G\left(v\right)d_G\left(e\right)\right]^2$$

$$=\left[r\times\left(2r-2\right)^2\right]^{\frac{nr}{2}}\times\left[r\times\left(2r-2\right)^2\right]^{\frac{nr}{2}}$$

$$=\left[r\left(2r-2\right)\right]^{2nr}.$$

4. FIRST AND SECOND MULTIPLICATIVE *K* HYPER-BANHATTI COINDICES

We define the first and second multiplicative K hyper-Banhatti coindices of a graph in terms of nonincident vertex-edge degrees.

Definition 15: The first and second multiplicative K hyper-Banhatti coindices of a graph G are defined as

$$\overline{HBII_1}\left(G\right)=\prod_{u*e}\left[d_G\left(u\right)+d_G\left(e\right)\right]^2$$

$$\overline{HBII_2}\left(G\right)=\prod_{u*e}\left[d_G\left(u\right)d_G\left(e\right)\right]^2$$

where $u*e$ means that the vertex u and edge e are nonincident in G.

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