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NOTE ON THE BOUNDS FOR THE DEGREE SUM ENERGY OF A GRAPH, DEGREE SUM ENERGY OF A COMMON NEIGHBORHOOD GRAPH AND TERMINAL DISTANCE ENERGY OF A GRAPH

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ABSTRACT

Let G be connected graph with n vertices. The concept of degree sum matrix DS(G) of a simple graph G is introduced by H. S. Ramane et.al. [2]. And the degree sum energy $E_{DS}(G)$ [2] is defined by the sum of the absolute values of eigenvalues of the degree sum matrix DS(G) of G. The degree sum energy of a common neighborhood graph G [4] is defined by the sum of the absolute values of eigenvalues of the degree sum matrix of a common neighborhood graph DS[con(G)]. The terminal distance energy $E_T(G)$ of a graph [3] is defined by the sum of the absolute values of eigenvalues of the terminal distance matrix T(G) of a connected graph G. In this paper we modify upper bounds for the above defined energies.

Keywords: *Degree sum matrix, Eigenvalues, Common neighborhood graph, Congraph, Energy of a graph, Terminal distance energy.*

AMS Subject Classification: *05C50.*

1. INTRODUCTION

Let G be a simple graph with n vertices and m edges. Let the vertices of G be labeled as $\{v_1, v_2, \ldots ,v_n\}$. The degree of a vertex v in a graph G, denoted by $d(v)$ is the number of edges incident to v. Let d_i be the degree of v_i, $i =1, 2, \ldots , n$. Then $DS(G) = [d_{ij}]$ is called the degree sum matrix of G [2] where

$$d_{ij} = \begin{cases} d_i + d_j, & \text{if} \quad i \neq j \\ 0, & \text{otherwise} \end{cases}$$

Since $DS(G)$ is real symmetric matrix, the roots of $\phi(G: \gamma) = 0$ are real and it can be ordered as $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_n$, where γ_1 is largest and γ_n is smallest eigenvalues.

The degree sum energy of a graph G [2] is defined as, $E_{DS}(G) = \sum_{i=1}^{n} |\gamma_i|$

The common neighborhood graph of a graph G with vertex set $\{v_1, v_2, \ldots ,v_n\}$denoted by $con(G)$ and is defined by, two vertices are adjacent in $con(G)$ if and only if they have at least one common neighbor in G.

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The degree sum matrix of a common neighborhood graph [4] $DS[con(G)]$ of a graph G is defined by, $DS[con(G)] = [d_{ij}]$,

where, $$d_{ij} = \begin{cases} d_i + d_j, & \text{if } v_i \text{ and } v_j \text{ shares common vertex} \\ 0, & \text{otherwise} \end{cases}$$

Since $DS[con(G)]$ is real symmetric matrix, the roots of $\phi(DS[con(G)] : \beta) = 0$ are real and they can be ordered as $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_n$.

The degree sum energy of a common neighborhood graph is defined as [4],

$$E_{DS}[con(G)] = \sum_{i=1}^{n} |\beta_i|$$

Terminal distance matrix of a connected graph G [3] is $T(G) = [t_{ij}]$, where t_{ij} is the distance between the terminal vertices v_i and v_j in G.

Since $T(G)$ is real symmetric matrix, the roots of $\phi(G:t) = 0$ denoted by $\{t_1, t_2, \ldots, t_k\}$are called terminal distance eigenvalues of G.

Since $T(G)$ is real and symmetric matrix, it's eigenvalues are real and can be ordered as $t_1 \geq t_2 \geq \cdots \geq t_k$. Where t_1 is largest and t_k is smallest eigenvalues.

The terminal distance energy $E_T(G)$ of a graph G [3] is defined as,

$$E_T(G) = \sum_{i=1}^{k} |t_i| \ .$$

In this paper, we obtain an upper bound for the degree sum energy of any connected graph in terms of the number of vertices and determinant of the degree sum matrix. We obtain an upper bound for the degree sum energy of a common neighborhood graph of any connected graph G in terms of the number of vertices and determinant of the degree sum matrix of a common neighborhood graph and we obtain an upper bound for the terminal distance energy of any connected graph in terms of the number of vertices and determinant of the terminal distance matrix.

In order to obtain bounds we need following lemmas.

Lemma 1.1: [2] Let G be a connected n-vertex graph and let $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_n$, be the eigenvalues of degree sum matrix, where γ_1 is largest and γ_n is smallest eigenvalues. Then,

$$\sum_{i=1}^{n} \gamma_i^2 = 2 \sum_{1 \leq i < j \leq n} (d_i + d_j)^2 = 2M. \tag{1}$$

Lemma 1.2: [4] Let G be a connected n-vertex graph and let $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_n$, be the eigenvalues of degree sum matrix of a common neighborhood graph $DS[con(G)]$, where β_1 is largest and β_n is smallest eigenvalues. Then,

$$\sum_{i=1}^{n} \beta_i^2 = 2 \sum_{1 \leq i < j \leq n} (d_i + d_j)^2 = 2N. \tag{2}$$

Lemma 1.3: [3] Let G be a connected n-vertex graph and let $t_1 \geq t_2 \geq \cdots \geq t_k$ be the eigenvalues of terminal distance matrix of a graph, Where t_1 is largest and t_k is smallest eigenvalues. Then,

$$\sum_{i=1}^{k} t_i^2 = 2 \sum_{1 \leq i < j \leq k} (t_{ij})^2 = 2P. \tag{3}$$

Lemma 1.4: [1] Let $a_1, a_2, \ldots, a_n$ be non negative numbers. Then

$$n\left[\frac{1}{n}\sum_{i=1}^{n} a_i - \left(\prod_{i=1}^{n} a_i\right)^{1/n}\right] \leq n\sum_{i=1}^{n} a_i - \left(\sum_{i=1}^{n}\sqrt{a_i}\right)^2$$

$$\leq n(n-1)\left[\frac{1}{n}\sum_{i=1}^{n} a_i - \left(\prod_{i=1}^{n} a_i\right)^{1/n}\right] \tag{4}$$

2. RESULTS

In [2] for a connected graph G with n vertices the following result for $E_{DS}(G)$ were obtained:

$$\sqrt{2M + n(n-1)\Delta^{2/n}} \leq E_{DS}(G) \leq \sqrt{2Mn} \tag{5}$$

Where Δ is absolute value of the determinant of the degree sum matrix $DS(G)$.

Theorem 2.1: Let G be a connected n-vertex graph and Δ be the absolute value of the determinant of the degree sum matrix $DS(G)$. Then,

$$\sqrt{2M + n(n-1)\Delta^{2/n}} \leq E_{DS}(G) \leq \sqrt{2(n-1)M + n\Delta^{2/n}} \tag{6}$$

Proof: Let $a_i = \gamma_i^2$, $i = 1, 2, \ldots, n$. Then from Lemma (1.1) and Lemma (1.4) we obtain,

$$n\left[\frac{1}{n}\sum_{i=1}^{n}\gamma_i^2 - \left(\prod_{i=1}^{n}\gamma_i^2\right)^{1/n}\right] \leq n\sum_{i=1}^{n}\gamma_i^2 - \left(\sum_{i=1}^{n}\gamma_i\right)^2$$
$$\leq n(n-1)\left[\frac{1}{n}\sum_{i=1}^{n}\gamma_i^2 - \left(\prod_{i=1}^{n}\gamma_i^2\right)^{1/n}\right]$$

That is,

$$2M - n\Delta^{2/n} \leq 2nM - [E_{DS}(G)]^2 \leq 2(n-1)M - n(n-1)\Delta^{2/n}$$

Thus,

$$2M + n(n-1)\Delta^{2/n} \leq [E_{DS}(G)]^2 \leq 2(n-1)M + n\Delta^{2/n}\text{. Where } M = \sum_{1\leq i<j\leq n}\left(d_i + d_j\right)^2 .$$

Hence we get the equality in Eq.(6).

Remark: Since for nonnegative numbers the arithmetic mean is not smaller than the geometric mean,

$$\frac{1}{n}\sum_{i=1}^{n}\gamma_i^2 \geq \left(\prod_{i=1}^{n}\gamma_i^2\right)^{1/n}$$

That is, $\quad n\Delta^{2/n} - 2M \leq 0$.

Hence the upper bound in Eq.(6) improves the upper in Eq.(5).

The lower bound in Eq.(5) coincides the lower bound in Eq.(6). Whereas Eq.(6) has better upper bound than that of in Eq.(5).

In [4] for a connected graph G with n-vertices the following result for $E_{DS}[\text{con}(G)]$ were obtained:

$$\sqrt{2N + n(n-1)\Delta^{2/n}} \leq E_{DS}(G) \leq \sqrt{2Nn} \tag{7}$$

Where Δ is absolute value of the determinant of the degree sum matrix of a common neighborhood graph $DS[con(G)]$.

Theorem 2.2: Let G be a connected n-vertex graph and Δ be the absolute value of the determinant of the matrix of common neighborhood graph $DS[\text{con}(G)]$. Then,

$$\sqrt{2M + n(n-1)\Delta^{2/n}} \leq E_{DS}(G) \leq \sqrt{2(n-1)M + n\Delta^{2/n}} \tag{8}$$

Proof: Proof is similar to the proof given for Theorem (2.1).

Remark: Hence the upper bound in Eq.(8) improves the upper in Eq. (7).

In [3] for a connected graph G with $k \geq 1$ pendent vertices, we have,

$$\sqrt{2P + n(n-1)\left|\det(T(K))\right|\Delta^{2/n}} \leq E_{DS}(G) \leq \sqrt{2Pn} \tag{9}$$

Where $|\det(T(K))|$ is the absolute value of the determinant of the terminal matrix $T(K)$ of any graph G.

Theorem 2.3: Let G be a connected n-vertex graph and $|\det(T(K))|$ be the absolute value of the determinant of the terminal matrix $T(K)$of G. Then,

$$\sqrt{2P + n(n-1)\left|\det(T(K))\right|^{2/n}} \le E_T(G) \le \sqrt{2(n-1)P + n\left|\det(T(K))\right|^{2/n}} \tag{10}$$

Proof: Proof is similar to the proof given for Theorem (2.1).

Remark: Hence the upper bound in Eq. (10) improves the upper in Eq. (9).

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