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A NOTE ON COMMUTATIVITY OF PERIODIC NEAR-FIELD SPACES OVER NEAR-FIELDS

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ABSTRACT

A near-field space N satisfying a polynomial identity of the form ab = ϕ (a, b), where ϕ (A, B) is different word from that AB, must have nil Commutator sub near-field space. First major theorem extends this result to the case where ϕ (A, B) varies with a, b with the restriction that all ϕ (A, B) have length at least three and are not of the form A^nB or AB^n. Further restrictions on the ϕ (A, B) are then shown to yields commutativity of a near-field space N. Among these a semi simple sub near-field space and a near-field space specifically that each ϕ (A, B) begins with B and has at least 2 in A. The final theorem establishes commutativity of near-field spaces N satisfying ab = bas where s = s(a, b) is an element of the center of the sub near-field space generated by a and b. All near-field spaces considered are either periodic by hypothesis or turn out to be periodic near-field spaces in the course of the in depth study and investigation of the near-field spaces.

Key words: *Near-ring, Near-field, periodic Near-field, sub Near-field, sub Near-field space, ideal.*

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SECTION 1: PRELIMINARY RESULT ON PERIODIC NEAR-FIELD SPACES

Let $\phi = \phi$ (A, B) be a mapping or word or monomial in the non-commuting in -determinates A and B i.e., ϕ is a polynomial of form

$$B^{l1} A^{k1} B^{l2} A^{k2} \text{---} B^{ls} A^{ks} \tag{1}$$

Where j_i, k_i $\forall$ i = 1,2,....,s and $\sum_{i=1}^{s}(j_i + k_i) > 0$. By the A-length (respectively B-length) of ϕ, which we denote by $|\phi|_A$ (respectively $|\phi|_B$), we shall mean the non-negative integer $\sum k_i$ (respectively $\sum j_i$). The sum $|\phi|_A + |\phi|_B$ will be called the length of ϕ and denoted by $|\phi|$. It will be convenient to divide the set of all words into nine types as follows:

(a) maps with $|\phi|_A \geq 2$ and $|\phi|_B \geq 2$.
(b) maps of form BA^n, $n \geq 1$
(c) maps of form B^nA, $n \geq 1$
(d) maps with $|\phi|_B = 0$
(e) maps with $|\phi|_A = 0$
(f) maps of form A^nBA^m, n, m $\geq$ 1
(g) maps of form B^nAB^m, n, m $\geq$ 1
(h) maps of form A^nB, $n \geq 1$
(i) maps of form AB^n, $n \geq 1$.

Definition 1.1: A near-field space N is called periodic near-field space if for each $x \in N$, there exist distinct positive integers n, m depending on x, for which $x^n = x^m$.

Example 1.2: Among the periodic near-field spaces in fact finite near-fields which we shall refer to frequently are the cobras (p, k, ϕ) – near-fields which we define as follows.

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Definition 1.3: N^+ is the additive direct sum $GF(p^k) \oplus GF(p^k)$, ϕ is an automorphism of $GF(p^k)$, and near-field multiplication is defined by

$$(a, b)(c, d) = (ac, ad + b\phi(c)) \quad (2)$$

Note 1.4: Near-fields have the property that $D^2 = 0$, where D denoted the set of all zero divisor near-fields including 0 and they have as few zero divisors as non-domain may have – specifically, $|D|^2 = |N|$. They are commutative near-fields only when ϕ is the identity automorphism.

We shall make use of repeated use of two basic theorems on periodic near-field spaces. The second is a special case of an old theorem of I N Herstein. But since deduces it as a corollary of one of his more complicated commutativity theorems, we think it worthwhile to include a simple proof.

Lemma 1.5: If N is any periodic near-field space, then N has each of the following properties:

1. (1) $\forall x \in N$, some power of x is idempotent.
2. (2) $\forall x \in N$, there exists an integer $n(x) > 1$such that $x – x^{n(x)}$ is nilpotent.
3. (3) $\forall x \in N$ can be expressed in the form of y + w, where $y^n = y$ for some $n = n(y) > 1$ and w is nilpotent.
4. (4) If J is an ideal of N and x + J is anon-zero nilpotent element of N /J, then N contains a nilpotent element $u \ni x \equiv u \pmod J$

Proof: To prove (1): If $x^n = x^m$ with n > m, then $x^{j + k (n – m)} = x^j$ for each positive integer k and each $j \geq m$ thus, we may assume $n – m + 1 \geq m$. It follows that $x^{n – m + 1} = (x^{n – m + 1})^{n – m + 1}$ and hence $(x^{n – m + 1})^{n – m}$ is idempotent. Proved (1).

To prove (2): Let $x^n = x^m$, n > m > 1.Then we have,
$x^{m- 1}(x - x^{n – m + 1}) = 0 = x^{m – 2} x (x - x^{n – m + 1}) =x^{m – 2} x^{n – m + 1}(x - x^{n – m + 1})$.

Therefore, $x^{m – 2} (x - x^{n – m + 1}) = 0$ and the result follows by the obvious induction. Proved (2).

To prove (3): If $x^n = x^m$ with $n \geq n – m + 1 > m$, the proofs of (1) and (2) show that we may take $y = x^{n – m + 1}$ and $w = x - x^{n – m + 1}$. Proved (3).

To prove (4): If x + I is a non-zero nilpotent element of N/I, there exists a positive integer k such that $x^q \in I$ for all $q \geq k$. By the proofs of (1) and (2), N contains a nil potent element $v = x – x^q$ with $q \geq k$. Clearly, $v \equiv x$ (mod.1). Proved (4). This completes the proof of the lemma.

Theorem 1.6: If N is a periodic near-field space with all nilpotent elements central, then N is commutative near-field.

Proof: Let N denote the set of nilpotent elements. The usual argument for commutative near-field spaces shows that N is an ideal. Moreover, for $x \in N$ and e is an idempotent in N, both ex - exe and xe – exe are in N, hence commute with e. Thus idempotents in N are central.

By (4) of lemma 1.5, we see that homomorphic images or maps inherit the hypothesis on N. Consequently, we need consider only the case of sub-directly irreducible N. Under this assumption, (1) of lemma 1.5 shows that N is either nil and hence commutative near-field or N has a unique non-zero central idempotent, necessarily a multiplicative identity element 1.

Considering this latter possibility, we see from (1) of lemma 1.5 that each element of N is either nilpotent or invertible. Thus the set D of zero divisor near-fields s is equal to N and hence is a central ideal. Moreover, by (2) of lemma 1.5 $\overline{N} = N / D$ has the $a^n = a$ property of Jacobson. Hence $\overline{N}$ is a commutative near-field and its additive sub near-field is a torsion sub near-field. Thus if $a, b \in N \setminus D$, the sub near-field space of $\overline{N}$ generated by and $\overline{b} = b + D$ is a finite near-field, which has cyclic multiplicative sub near-field. There must therefore exist $g \in N$ and $d_1, d_2 \in D$ such that $a = g' + d_1$ and $b = g' + d_2$ for some positive integers i, j. It follows that a and b must commute and our proof is complete.

SECTION 2: A NIL COMMUTATIVE SUB NEAR-FIELD SPACE AND SOME RELATIVES

Theorem 2.1: Let N be a near-field such that for each $a, b \in N$ there exist a map ϕ (A, B) of one of the types (a) to (g) and with $|\phi| \geq 3$, for which $ab = \phi(a, b)$. Then the set N of nilpotent elements forms an ideal and the Commutator ideal C(N) is contained in N.

Proof: Taking a = b shows that for each $a \in N$, $a^2 = a^k$ for some k > 2. Hence N is periodic near-field space and each nilpotent element squares to zero. We next show that idempotents of N must be central. Let e be a non-zero idempotent.

Let a ∈ N and suppose ϕ (A, B) is a map of the allowed types for which e(ex – exe) = ϕ(e, ex – exe). Clearly, ϕ can not be a type (d) since $(ex – exe)^2 = 0$. Any other types has either two adjacent B's or B preceding an A. Thus e(ex – exe) = ex – exe = 0 and similarly, xe – exe = 0.

A periodic near-field space satisfies the conclusions of the theorem if nilpotent elements commute with each other, so we may complete our proof by showing that ab = 0 for all a, b ∈ N. Accordingly, let a, b ∈ N and ϕ a map such that ab = ϕ (a, b). If ϕ has two adjacent A's and B's then it is immediate that ab = 0. Otherwise, we have one of the following

Cases: (i) $ab = (ab)^k$ for some k > 1. (ii) ab = abab.....a (iii) ab = bab........

In case (i), $(ab)^{k-1}$ is idempotent, hence central and we get $ab = a\,(xb)^{k-1}\,b = 0$. In case (ii) right multiplication by a yields aba = 0 = ab, and in case (iii) left multiplication by b yields bab = 0 = ab. This completed the proof of the theorem.

Note 2.2: The idempotents are central apply (i) of lemma 1.5 to show that some power of each element is central and appeal to a well known theorem of I N Herstein[7].

Note 2.3: In the hypothesis of theorem 2.1, the restriction on the type of ϕ (A, B) is essential, for without it, as the near-field space of 2 x 2 matrices over GF(2) would satisfy the hypothesis.

Note 2.4: In the hypothesis of theorem 2.1 will not yield commutativity of N. The Corbas (2, 2, ϕ)-near-field space is a counter example where ϕ is the non identity automorphism of GF(4) indeed in this near-field space, if u, v ∈ N and a , b ∉∉ N we have $uv = vu^2$, $au = ua^2$, $ua = aua^2$ and $ab = (ba)^3ab$. However, restriction of ϕ (A, B) to words of fixed type (a) to (g) does yield commutativity as we now prove the following theorem.

Theorem 2.5: Let β denote a fixed one of the map-types (a) to (g). Let N be a near-field space such that for each a, b ∈ N, there exists a type - β map ϕ (A, B), depending on a and b and having length at least three, for which ab = ϕ (a, b). Then N is commutative near-field.

Proof: If β is type (a), commutativity follows from a theorem of the present author. Suppose, then that β is type (d) i.e., for each a, b ∈ N, $ab = a^n$ for some n = n(a, b) ≥ 3. Then since nilpotent elements square to 0, they left-annihilate N. Taking a ∈ N and x an element such that $x^k = x$, k > 1 and recalling that idempotents are central, we obtain the result that $xa = xx^{k-1}x = xax^{k-1} = 0$ and by (3) of lemma 1.5 nilpotent elements right annihilate N as well and commutativity follows from theorem 1.6. it is clear that type (e) may be treated similarly.

To complete the proof, we discuss type (f) noting that (g) is similar. Let ∀ x ∈ N, ∀ y ∈ N and $xy = x^n\, yx^m$ with n, m ≥ 1. If either of n, m is greater than 1, then xy = 0. If xy = xyx, right multiplying by x yields xyx = 0 = xy. Also $yx = y^jxy^k$ ∀ k ≥ 1, so yx = 0 as well and again commutativity follows by theorem 1.6. This completes the proof of the theorem.

Theorem 2.6: Suppose that for each x, y∈N there exists an integer n(x, y) > 1 such that $xy = x^{n(x,\, y)}y$. Then the Commutator or ideal C(N) is nil and the nilpotent elements form an ideal. If the idempotents of N are central, then N is commutative.

Proof: Clearly, N is periodic with nilpotent elements squaring to zero and ∀ x ∈ N and v is nilpotent we have $vx = v^nx = 0$. Thus the set N of nilpotent elements is the set of annihilator of near-field space N, hence an ideal. The near-field space N/N has the a^n = a property by lemma 1.5 (2), hence is commutative near-field. Thus C(N) ⊆ N.

Now assume that idempotent are central. If $a^k = a$ ∀ k >1 and v ∈ N, we get $av = a^nv = a^{n-1}aa^{k-1}v = a^nva^{k-1} = 0$. Hence by lemma 1.5 (3) and theorem 1.6 implies N is commutative near-field.

Note 2.7: Centrality of idempotents is not implied by the condition $xy = x^ny$.

Example 2.8: The near-field space N with additive sub near-field space equal to the multiplication given by 0x = cx = 0 and ax = bx = x ∀ x ∈ N. This near-field space satisfies the identity $xy = x^2y$.

Note 2.9: The idempotents are central in theorem 2.6 we can say a bit more about near-field space N specifically it is the direct sum of a zero near-field space and a J-near-field space i.e., one with Jacobson's $a^n = a$ property. For if x, y are arbitrary sub near-field spaces of N, ∃ integers $n_1, n_2 > 1$ ∋ $xy = x^{n1}y$ and $yx = y^{n2}x$.

Note 2.10: A standard computation yields a single n such that $xy = x^n y$ and $yx = y^n x$ and the commutativity now shows that $x^n y = xy^n$. The direct sum decomposition of near-field spaces with the latter type of constraint has essentially been obtained.

SECTION 3: MAIN RESULTS OF TWO COMMUTATIVITY THEOREMS ON PERIODIC NEAR-FIELD SPACES

Theorem 3.1: Let N be a periodic near-field space, the multiplicative semi simple near-field space of which is a semi simple near-field space. Then N is a commutative near-field.

Proof: If a, b $\in$ N and ab = 0 then ba = 0 also. Then the nilpotent elements of N form an ideal N, which since N is periodic near-field space, must coincide with the Jacobson radical J(N).

Again Dr N V Nagendram wish to deduce result from theorem 1.6. Suppose then, that μ is a non-central nilpotent element and b $\in$ N is an element not commuting with μ. Then

$$\mu b = b^j{}_1\, \mu^k{}_1 \text{..........} \mu^k{}_i \quad \forall\, j_1 \geq 1, \forall \sum k_i \geq 2 \tag{3}$$

If $k_1 \geq 2$ we obtain

$$\mu b = b^j{}_1 \mu\, \mu^k{}_1{}^{-1} \text{.......} \mu^k{}_s = \mu^t (b^j{}_1)^q \text{..........} \mu^k{}_1\text{-}^1 \text{.........} \mu^k{}_s \tag{4}$$

If t = 1we make no further substitutions in (4) otherwise we write $\mu b = \mu\mu^{t\,-1}\, b^j{}_1{}^q \ldots \mu^k{}_s{}^{-1} \ldots\ldots \mu^k{}_s = \mu b^j{}_1{}^{qx}\, (\mu^{t\,-1})^n \ldots\ldots \mu^k{}_s$. In either case we have $\mu b = \mu by$ for some y $\in$ J(N) from which it follows that $\mu b = 0 = b\mu$ is a contradiction to our choice of μ. ⊗

If $k_1 = 1$ in equation (3) then some other k_i is positive and a similar computation again yields the same contradiction⊗. Thus nilpotent elements of N are central and this completes the proof of the theorem.

Corollary 3.2: Let N be any near-field space having as multiplicative semi simple near-field space is a semi simple near-field. Then N is a commutative near-field.

Note 3.3: Theorem 3.1 and corollary 3.2 would not be true if the condition $|\phi|_A \geq 2$ were omitted from the definition of maps the Corbas (2, 2, ϕ)-near-field space is the revealing example.

Theorem 3.4: Let N be any near-field space such that for each x, y $\in$ N there exists an element s = s(x, y) in the center of the sub near-field space generated by x and y for which xy = yxs. Then N is commutative near-field.

Proof: Taking x = y shows that $x^2 = x^2 p(x)$, where p(x) is a polynomial with integer coefficients and zero constant term. It follows that N is periodic near-field space. Moreover, the given constraint shows that ab = 0 $\Rightarrow$ ba = 0 = arb $\forall$ r $\in$ N. This result together with the obvious fact that nilpotent elements square to zero shows that uvs =0 $\forall$ nilpotent element u and v and $\forall$ s in the sub near-field space generated by u and v. Thus the nilpotent elements form an ideal N with $N^2 = 0$. Moreover, a standard arguement applied to e, ex – exe and xe – exe shows that all

The hypothesis of the theorem persist under the taking of homeomorphic images, so we need consider only sub-directly irreducible N. Since nil near-field spaces with our condition are zero near-field spaces and since sub directly irreducible near-field spaces can have at most one non-zero central idempotent, lemma 1.5 (1) allows us to assume that N has 1 and that every non nilpotent element is invertible. Hence the set D of zero divisors is an ideal which equal to N.

Since there exist distinct n, m with $(1 + 1)^n = (1 + 1)^m$, N^+ must be a torsion sub near-field space which in view of sub direct irreducibility is a p-sub near-field space for some prime p. Since $D^2 = 0$, we have then $(p \cdot 1)(px) = p^2 x = 0$ for all x $\in$ N.

Now N is clearly a duo near-field space, so we may apply earlier results of near-field spaces on sub directly irreducible duo near-field spaces. Specifically, letting S denote the intersection of the non-zero sub near-field spaces of N and noting that N $\neq$ D, we have S equal to the annihilator of D i.e., S = D. By known lemma 1.5 (2) and the a” = a theorem we know that N/D is commutative near-field and hence that commutator near-fields in a near-field space belongs to D. Suppose now that pN $\neq$ 0 let px $\neq$ 0 and let y be an arbitrary sub near-field space of N. Since pxN is a non-zero sub near-field space , we have xy – yx $\in$ D = S $\subseteq$ pxN and there exists r $\in$ N such that xy – yx = pxr and hence $p(xy - yx) = p^2 xr = 0$. Thus pN = D is central and commutativity of near-fields of N follows from theorem 1.6.

Now suppose that we have a sub directly irreducible counter example with pN = 0. By known lemma 1.5 (3) and the fact that $D^2 = 0$. We can then choose a non-central nilpotent element u and an element $b \in N$ such that $b^{n(b)} = b$ for some n(b) > 1 and b does not commute with u. Since bu = ubs for some s in the sub near-field space generated by u and b, and since uru = 0 for all $r \in N$, we obtain bu = ubp(b), where p(A) is some polynomial with integer coefficients and zero constant term. It follows that the sub near-field space <u, b> of N generated by u and b is finite. Since the hypothesis of the theorem are inherited by sub near-field spaces and by homomorphic images we can conclude that some homomorphic image T of <u, b> is a finite sub directly irreducible counter example with pT = 0.

We can argue that T must be a near-field space for appropriate choices and finite near-field spaces N with 1 and with $D^2 = 0 = pN$ must have additive sub near-field space which is direct sum $K \oplus D$, where K is a finite near- field and D is a left vector space over K. Since one dimensional sub near-field spaces of D are left sub near-field spaces, the fact that our T is sub directly irreducible and a duo near-field space shows that D is one dimensional and $|T| = |D|^2$. We apply an earlier result to show that T is a near-field.

Consider near-field T with ψ a non-identity automorphism of $K = GF(p^k)$. let g be a generator of the multiplicative sub near-field space of K, and let φ be given by $x \to x^{pr}$, $1 \le r < k$. If $(a, b) \in T$ commutes with both (g, 0) and (0, g) then by (2) we have b = 0 and a = φ(a). Then imposing the condition that (g, 0)(0, g) = (0, g)(g, 0)(a, 0) yields g = φ(g)a. Since $\varphi(g) = g^{pr}$ and $g = g^{pk}$ we have $g^{pk} = g^{pr}a$, so that $a = g^{pk - pr} = g^{pr\,(pk\text{-}r}\text{-}^{1})$. Now using fact that φ(a) = a, we get $g^{pr(pk\text{-}r\,-1)(pr\,-1)} = e$, where e denotes the identity element of K. Since g has order p^k-1, which is relatively prime to p′, we conclude that $p^k - 1/\,(p^{k\,-\,r} - 1)(\,p^r - 1)$, which is absurd. The possibility of a counter example is thus demolished. This completes the proof of the theorem.

Note: It is tempting to conjecture that N must be commutative near-field space if it satisfies xy = yxs, where s = s(x, y) is merely assumed to belong to the sub near-field space generated by x and y and not necessarily to its center. However, the near-field space N shows that this is not true.

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