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COMPARISON OF DIFFERENT WAVELET-BASED STATISTICAL METHODS IN BANKING SECTOR

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ABSTRACT

The paper proposes different wavelet-based forecasting techniques for time series. We investigate the stock forecast of two leading banks from the Indian banking sector through these wavelet-based approaches and make a comparison of the techniques for the data. The proposed prediction approach consists of the combination of the wavelet transform and various statistical methods. This approach is applied on the two types of real banking data series: SBI and ICICI

Keywords: *stock prices, wavelet transform, Exponential smoothing, SMA, trigonometric fit.*

MSC 2010 NO.: *91B84, 91G70.*

1. INTRODUCTION

Banking sector plays a dynamic role in the economic development of a country. Banks are considered not merely as dealers in money but also the leaders in economic development. A well-developed banking sector provides a firm and durable foundation for the development of the country. For a developing country like India, banking sector is considered to be the backbone of the economy. It contributes to a country's development in the following ways:

- Banks promote the capital formation by encouraging the habit of saving among people. Economic development depends upon the diversion of the economic resources from consumption to capital formation. Banks help in this direction by encouraging saving and mobilising them for productive uses.
- Banks are very important source of finance and credit for industry and trade. They are the instruments for developing internal as well as external trade.
- Facilities of bank loans enable the entrepreneurs to step up their investment on innovational activities, adopt new methods of production and increase productive capital of the economy.
- Banks influence the nature and volume of industrial production by providing financial resources to the industries. Economic development of the developing countries like India, where most of the population live in the rural areas, requires the development of agriculture and small scale industries. Banks play an important role by providing loans for the growth and modernisation of agriculture.

In recent years, banking sector has experienced the extent of competition. Volatile markets, government intervention and changing customer habits have created a truly dynamic environment. As this environment transforms, banking sector needs to formulate and implement strategic and operational changes to meet the market's demand while simultaneously updating forecasts. Forecasting, the process to predict future situations based on past and present data, is helpful in planning and future growth. Being the key to smart business, it is an important prelude to effective and efficient planning.

Stock market forecasting is required for the investors as it is an important issue in investment decision making. Mostly, the financial time series data is non stationary as it contains extreme variations and these fluctuations occur with high frequency. Standard time series econometric tools such as Fourier transform usually consider only time or frequency component separately. Whereas, wavelets allow us to study the frequency components of the time series with time information simultaneously. So the wavelet transform is very useful tool in time series analysis. Many eminent scientists and mathematicians e.g. I. Daubechies, A. Grossmann, S. Mallat, Y. Meyer, J. Morlet, Coifman, V. Wickerhauser made a remarkable contribution to the wavelet theory. The power of wavelets has been proven in many applications such as wave propagation, data compression, signal processing, image processing, pattern recognition, self similarity or discontinuity detection. Siddiqi [11] discusses the wavelet methods to solve partial differential equation and integral equations. As the trends can be classified with wavelets so they also offer a strong tool for time series analysis.

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Applications of wavelet analysis in financial market analysis have been discussed by various researchers. Ramsey [10] discusses the role of wavelet in various economics fields such as forecasting, business cycles, interest rates etc. R. Gencey [4] investigates the foreign exchange rate scaling properties by wavelet techniques. He has shown that foreign exchange rate volatilities follow different scaling levels at different horizons. In [6] M. Gallegati studies the relationship between stock market returns and economic activity by applying the Maximum Overlap Discrete wavelet transform to the Dow Jone Industrial Average and the USA Industrial production Indices. With the help of wavelet method and Fisher hypothesis S. Kim [7] discusses that there is positive relationship between stock returns and inflation. The result shown by R. Deora [3] reveals the existence of time-scale-dependent comovement between Indian and world stock markets with wavelet-based GARCH approach. In [9] Pablo M. examines the linkage between interest rate fluctuations and Spanish stock market by using wavelet-based tools namely wavelet variance, correlation and cross-correlation. Abiyev R. [1] uses Fuzzy Wavelet Neural Networks (FWNN) for modelling and prediction of stock prices. In [2] A. J. Conejo discusses the applications of wavelet with ARIMA in electricity market forecasting. Gencey [5] provides some important properties of the wavelets and discusses its applications in both economics and finance. Kumar, J. [8] discusses the concept of neuro-fuzzy with wavelet decomposition for stock market. Some statistical methods based on wavelet for prediction are discussed by Weinreich [12]. Our proposed methods are also partially motivated by Weinreich's wavelet-based methods.

The object of this paper is to conduct the stock market analysis of some financial institutions from Indian banking sector based on the decomposition of time series of stock prices in order to forecast stock prices by using the wavelet decompositions and some statistical methods and then make comparison of the results. The data used in the study are the daily closing prices of the two banks namely SBI and ICICI. State bank of India (SBI) is one of the biggest Indian multinational public sectors bank whereas ICICI is one of the largest private sector bank. SBI has nearly 16000 branches, 14 regional hubs and 57 Zonal offices that are located in important cities throughout India. Having a market capital of Rs. 181,804.24 Cr., it is large capital company operating in the banking sector. On the other side ICICI has a network of 4,050 branches. It offers a wide range of banking products and financial services for corporate and retail customers.

The rest of the paper structures as follows: the next section describes the wavelet methodology. The forecasting framework is presented in the third section. The fourth section discusses the empirical results and is followed by the conclusion in the last section.

2. WAVELET METHODOLOGY

Wavelet analysis is a powerful mathematical tool for analyzing time series. It uses a similar strategy like Fourier analysis as it employs some basic functions (wavelets instead of sinusoidal) and uses them to decompose the series. The main difference between two tools is that in the contrast to Fourier analysis, wavelet analysis does not need any stationary assumption in order to decompose the series. Also Fourier methods perform a global analysis whereas the Wavelet methods act locally in time and frequency. This feature makes wavelets ideal tool for analysing non-stationary signals and those with transients or singularities. In this section we first describe a short overview of the Fourier transform and its revised version i.e. the short–term Fourier transform and then provide a brief discussion on the discrete wavelet transform.

2.1 Fourier Transform vs. Wavelet Transform

The Fourier transform breaks a signal into a sum of harmonic components of different frequencies as a linear combination of Fourier basic functions (sines and cosines). It is a frequency domain representation of a signal, containing the same information of the original signal but summarized as a function of frequency. The main drawback of the Fourier transform is that it allows analysis of signals under the main assumption that the observed signal is stationary over the time period of the analysis. This assumption is not valid for many practical signals as mostly economic and financial time series data (financial indices, census data, and spatially distributed econometrics measures) are non stationary, exhibit high complexity and involve both random processes and intermittent deterministic processes. Also Fourier transform gives only frequency information of the signal, not regarding time. To overcome these drawbacks, Dennis Gabor in 1946, first introduced a modified time dependent version of it, namely, Short-Time Fourier Transform (STFT) known later as Gabor transform. The short- time Fourier transform uses a fixed window function with respect to frequency and applies the Fourier transform to the windowed signal. The original signal is partitioned into small enough segments such that these portions of the non stationary signal can be assumed to be stationary over the duration of the window function. Once the window function is chosen both the time as well as frequency resolutions become fixed for all frequencies and times respectively. As a consequence, the short- time Fourier transform does not allow any change in time or frequency resolutions.

Wavelet transform is an alternative approach to the short-time Fourier transform to overcome the resolution problem. The Wavelet transform combines the information from time and frequency domains and therefore, preserves time information. Moreover, it does not require the stationarity of the signal. In contrast to the fixed time frequency partition of the short-time Fourier transform, the Wavelet transform analyzes the signal at different resolutions using multiresloution analysis. The multiresolution analysis approach may overcome the resolution problem as it adaptively partitions the time frequency plane, using short windows at high frequencies and long windows at low frequencies and thus letting both time and frequency resolutions to vary in the time –frequency plane.

As having finite length and oscillatory behaviour, wavelets literally mean small waves. Basic wavelets are characterized into two special functions: the father wavelet (or scaling function) $\phi(t)$ and the mother wavelet $\psi(t)$. Based on the mother wavelet, a family of wavelets $\psi_{a,b}(t)$ can be obtained by simply scaling and translating ψ:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right); a \in \mathbb{R}\setminus\{0\}, b \in \mathbb{R}$$

where a is a scaling or dilation parameter that controls the length of the wavelet (window), while location parameter b determines its position in the time domain. The father wavelet integrates to one and is good at representing the smooth and low frequency part of a signal, whereas the mother wavelet integrates to zero and is good in capturing the detail and high frequency components. To capture the volatile behaviour in time series wavelet analysis has become an increasing popular tool in many fields.

2.2 Discrete Wavelet Transform

There are two types of wavelet transform- the continuous wavelet transform (CWT) and its discretized version, discrete wavelet transform (DWT). The CWT is popular among physicists, whereas the DWT is more common in numerical analysis, signal and image processing. For a long time, wavelet applications in economics have concentrated on the DWT due to its greater simplicity and more parsimonious nature. The DWT produces only the minimal number of coefficients necessary to reconstruct the original signal. The reduction is achieved by discretizing the parameters a and b, so that $a = 2^{-j}$ and $b = k2^{-j}$ where j and k are integers. In the DWT the number of observations has to be dyadic i.e. an integer power of 2.

The aim of the DWT is to decompose the discrete time signal to basic functions, wavelets which provides us to a good analytic view of the analyzed signal When the DWT is applied, the time series signal can be built up as a sequence of projections onto father and mother wavelet generated from ϕ and ψ through scaling and translating as follows:

$$\phi_{j,k} = 2^{\frac{j}{2}}\phi(2^j t - k)$$
$$\psi_{j,k} = 2^{\frac{j}{2}}\psi(2^j t - k)$$

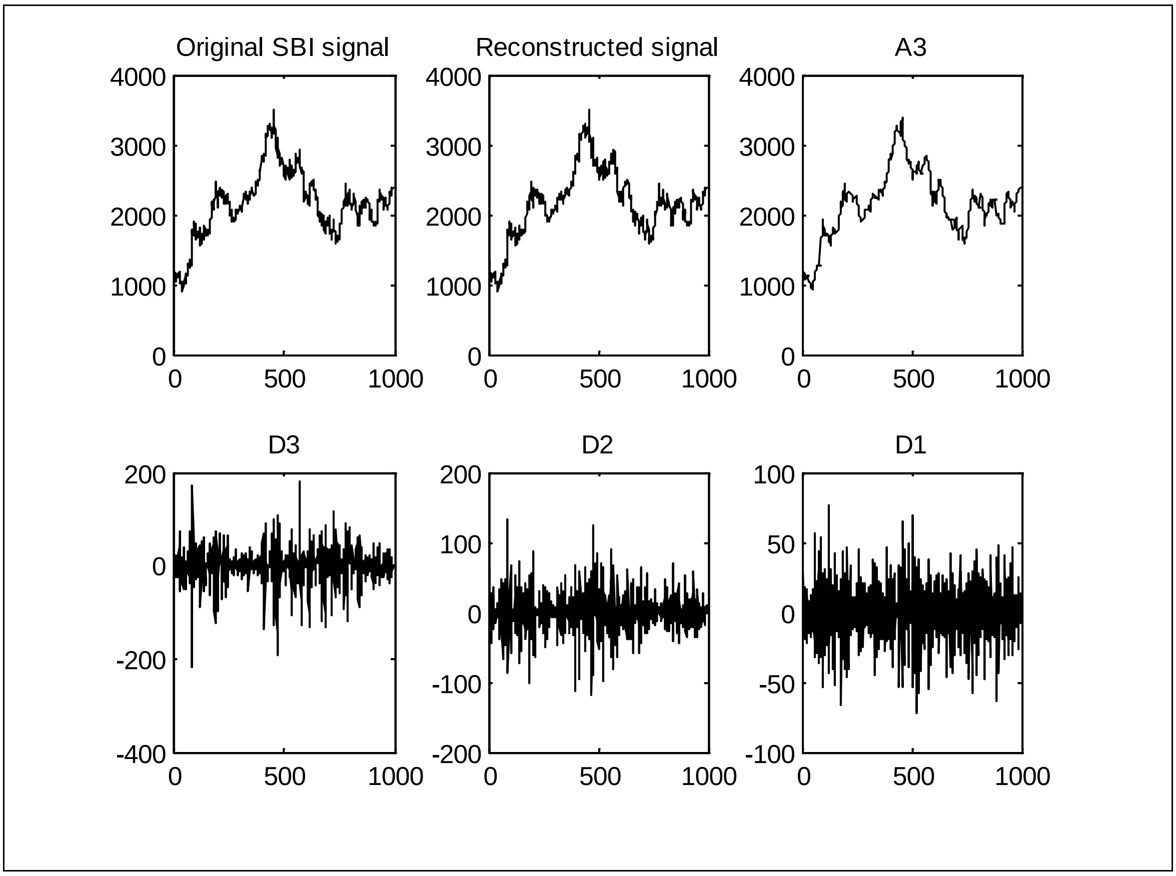
They form a basis for $L^2(\mathbb{R})$. The wavelet representation of the signal $y(t) \in L^2(\mathbb{R})$ can be written as:

$$\mathrm{y}(t) = \sum_k s_{J,k}\,\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \cdots + \sum_k d_{1,k}\psi_{1,k}(t)$$

Where J is the number of multiresolution levels and k ranges from 1 to the number of coefficients in each level. Here $s_{J,k}$ are scaling or smooth coefficients that represent the smooth behaviour of the series and $d_{J,k}$, wavelet coefficients capture the high frequency content of the time series. They are defined as:

$$s_{J,k} = \int_{-\infty}^{\infty}\phi_{J,k}(t)\, y(t)\, dt$$
$$d_{j,k} = \int_{-\infty}^{\infty}\psi_{j,k}(t)\, y(t)dt \quad j = 1,2,\ldots J$$

Figure 1



Forecasting Framework

The main idea of the prediction technique using wavelet approach is to decompose the original time series into a range of frequency scales and then apply the forecasting methods on these individual parts. Here for the prediction of one month values in the time series, the following steps are performed. At first, we shorten the given time series to the size of 974 values and neglect the data from the one month for prediction. Secondly, a three level daubechies wavelet of order two (db2) is performed on the time series which results the following decomposition

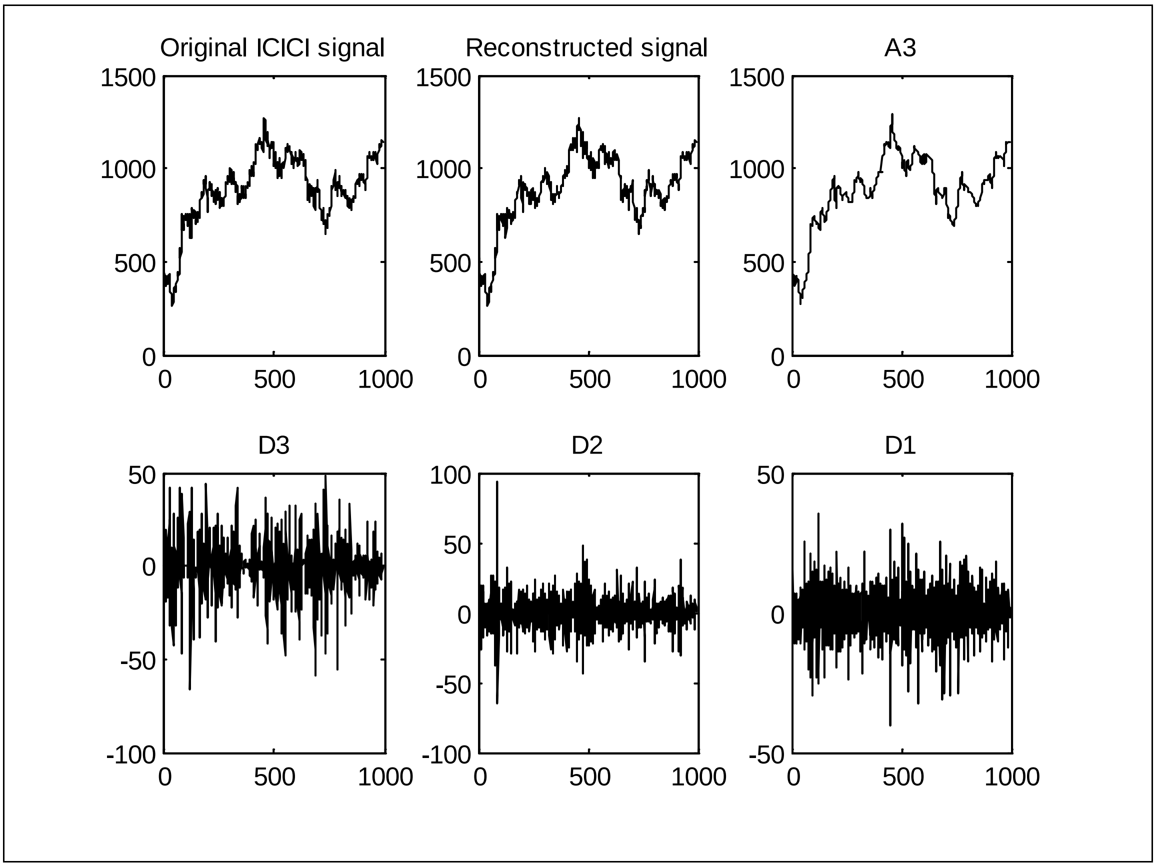
$$s = a_3 + d_3 + d_2 + d_1$$

This is shown in figure 1 and 2. The third step is the extension of the time series s, that is extension of each scale of s, namely a_3, d_3, d_2, d_1. The various techniques how thes scales are extended yield various methods. Ten prediction methods are introduced and denoted as M_1, M_2, M_3 ... M_{10}. These approaches are based on some statistical procedures which are explained below:

<u>SMA(simple moving average)</u>

Moving average provides a simple method for smoothing the past values to estimate trend-cycle component. Taking an average of the points near observation provide a reasonable estimate of the trend-cycle at that observation. The average eliminates some randomness in the data. It is needed to decide how many data points to include in each average. In our application we take a moving average of order 3 centered at time t.

Figure 2



Exponential Smoothing(ES)

In moving averages the past observations are weighted equally but exponential smoothing assigns exponentially decreasing weights as the observation get older. So exponential smoothing method gives relatively more weights to recent observations in forecasting than the older observations. This framework generates reliable forecasts quickly for a wide spectrum of time series which is of great importance and added advantage to applications in industry.

Trigonometric fitting

With this approach we approximate a function by series of trigonometric functions. The approximation (a_3) or detail $(d_i\ , i = 1,2,3)$, components of the given series are interpolated with trigonometric functions which results in a low frequency function fitting the data for approximation and a higher frequency for the detail levels.

Table: 1

	a_3	d_3	d_2	d_1
M_1	SMA	SMA	SMA	SMA
M_2	SMA	SINFIT	SINFIT	0
M_3	SMA	SINFIT	SINFIT	SINFIT
M_4	SMA	SMA	0	0
M_5	SMA	SMA	SINFIT	SINFIT
M_6	ES	ES	ES	ES
M_7	ES	SINFT	SINFIT	0
M_8	ES	SINFIT	SINFIT	SINFIT
M_9	ES	ES	0	0
M_{10}	ES	ES	SINFIT	SINFIT

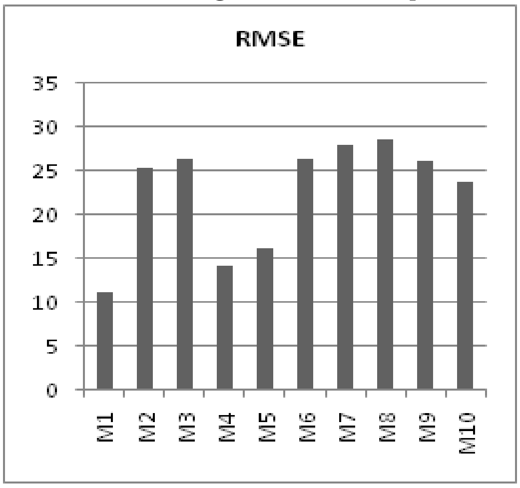
We also consider the case when the finest detail levels are neglected, i.e. either only d_1 or the two finest details levels d_1 and d_2 are treated by hard threshold which means putting the corresponding wavelet coefficients to zero. These ten different prediction methods are described in the table 1.

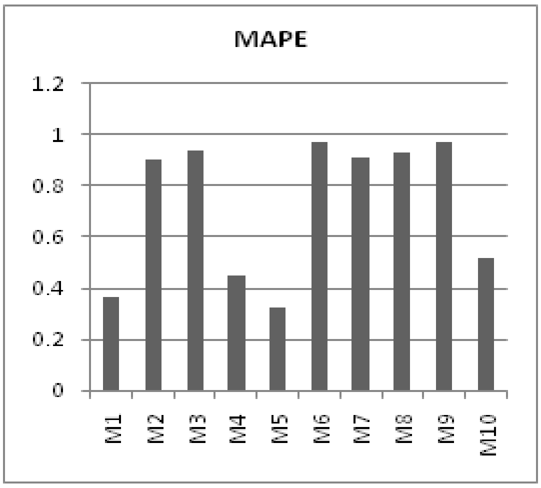
We consider the prediction in method 1 (M_1) by Simple Moving Average (SMA) and method 6 (M_6) by Exponential smoothing (ES) on each level. Methods M_2 -M_5 extend approximation level by SMA and M_7 -M_{10} apply ES. The methods differ mainly with respect to their analysis of the detail levels. In M_2, M_3, M_5, M_7, M_8 and M_{10} trigonometric fit extend the different detail levels.

4. EMPIRICAL RESULTS

For the application with the proposed methodology, we take a large amount of data of SBI and ICICI daily closing prices. The data is collected from BSE site over a period of 1 January 2009 to 31 December 2012. Both data series are divided in two phase: training phase and testing phase. In the training phase, we design predictive models for each of the decomposed component of the original series. The developed forecasting models are used to predict future values and then we compare forecasted values with exact values in the testing phase.

Figure 3: Error comparisons of SBI data for 1 month prediction





For the comparison of the proposed methods we consider the two standard error measures: Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE). Let $y(t)$ be the actual value and $f(t)$ the forecasted value. Then these measures are defined as

$$RMSE = \sqrt{\sum_{t=1}^{n} \frac{(y(t) - f(t))^2}{n}}$$

$$MAPE = \frac{1}{n} \sum_{t=1}^{n} \frac{|y(t) - f(t)|}{n} \times 100$$

These error measures are applied to the time series data. Based on these measures we compare the all proposed wavelet based methods. First we predict one month values using proposed methods. Then it is extended to three month and one year. For one month we calculate RMSE and MAPE which are shown in figures 3 and figure 4.

Figure 4: Error comparisons of ICICI data for 1 month prediction

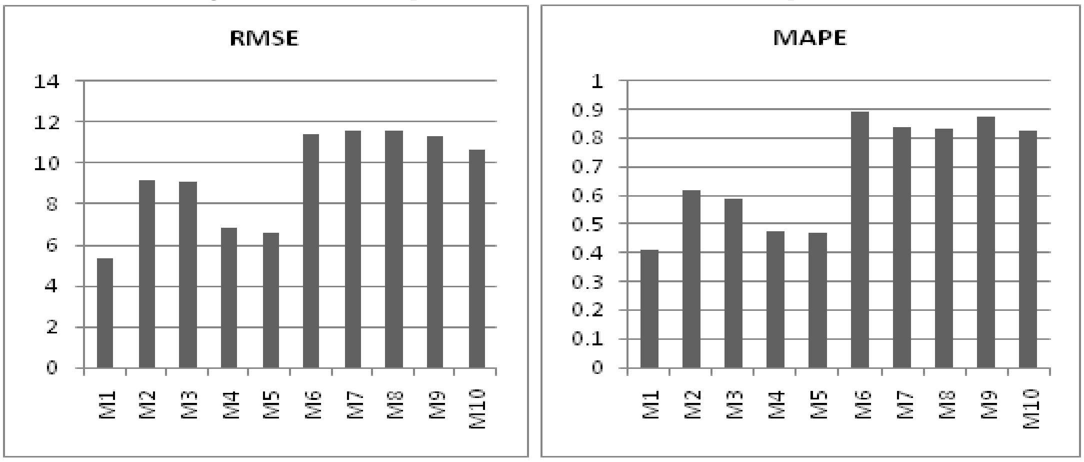
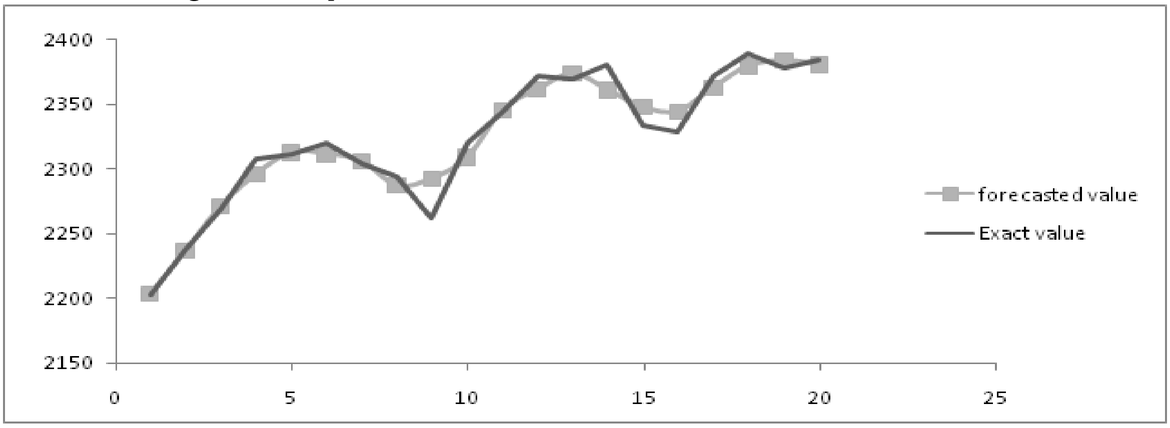
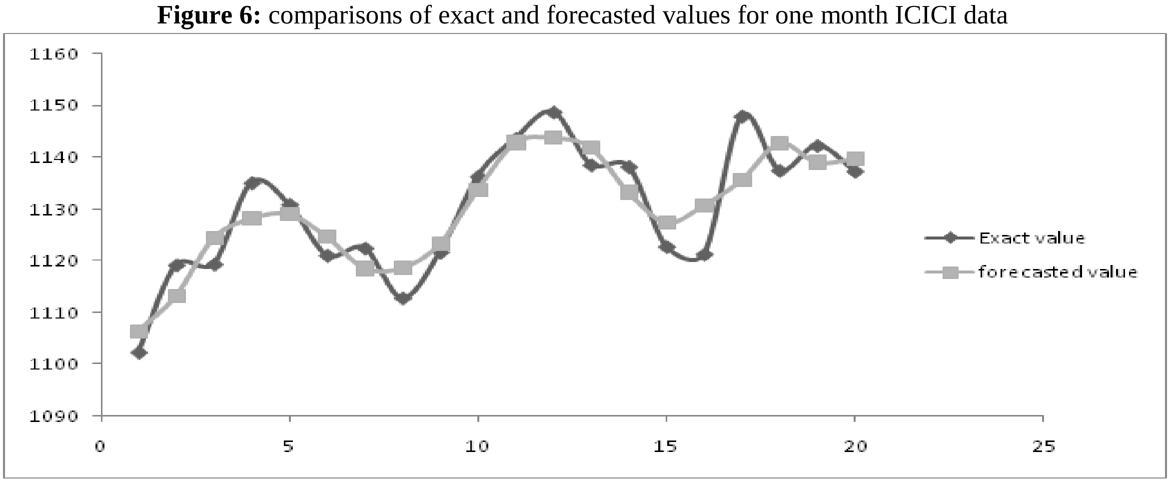


Figure 5: comparisons of exact and forecasted values for one month SBI data



We compare all these wavelet-based prediction results. From both graphs (Figure3 and Figure 4) it is clear that the methods where SMA is considered are superior to those with variants Exponential smoothing. All methods except M_1 and M_6 are based on different extensions of different approximations and details levels. The detail parts d_1 and d_2 are neglected in M_2, M_4, M_7 and M_9 respectively. These scales could be noise as they have no great impact on the quality. Figures 3 and 4 show that with respect to the root mean square error M_1 and M_4 are the best methods and to mean absolute percentage error M_1 and M_5 performs best. Also methods M_2 -M_5 are better than M_6 -M_9 . It means that for the analysis of the approximation level a_3 moving average is better than for the exponential smoothing. As a summary we find that M_1 shows best performance with respect to both measures. Figure 6 and 7 presents the comparisons of exact values and forecasted values of SBI and ICICI respectively by method by M_1. It can be seen that in both type of data predicted values are close to exact values.

Figure 6: comparisons of exact and forecasted values for one month ICICI data



Also we apply same ten methods for three months and one year stock price prediction. Figure 7 and 8 show RMSE and MAPE for three month prediction and figure 9 and 10 for one year. It is clear that same result hold for three months and one year as for one month i.e. M_1 is superior to all other methods.

Figure 7: Error comparisons of SBI data for 3 months prediction

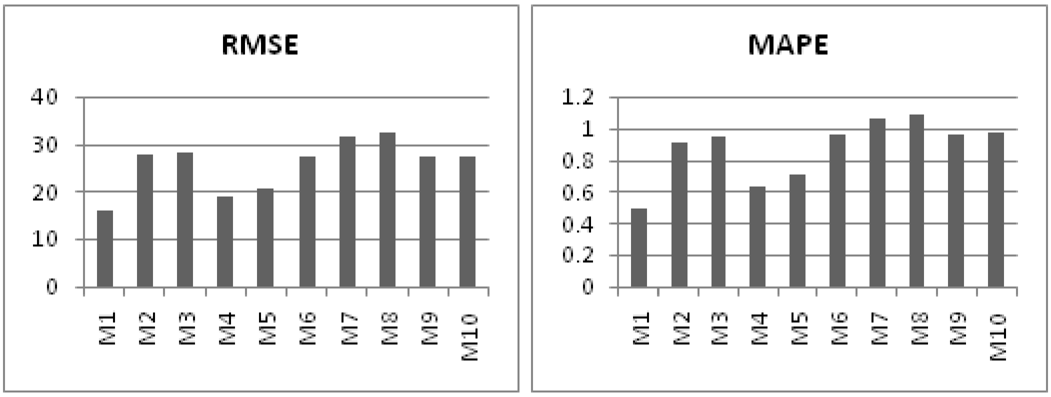


Figure 8: Error comparisons of ICIC data for 3 months prediction

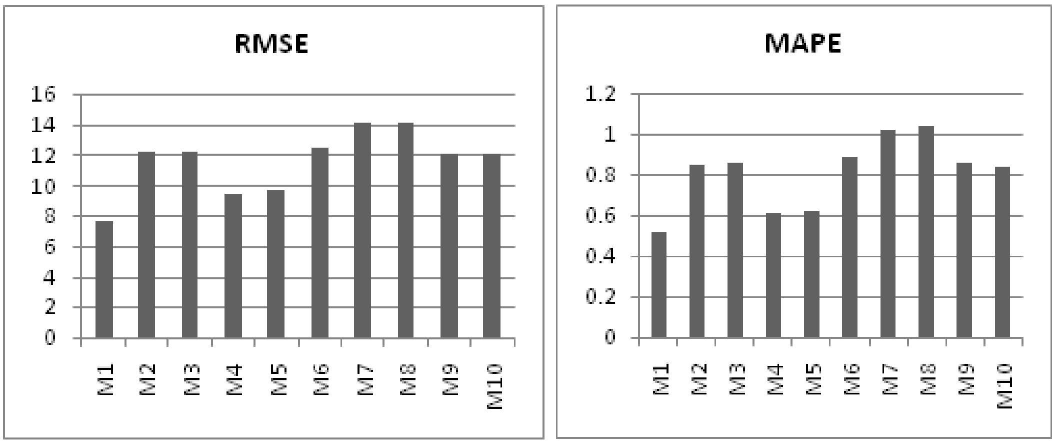


Figure 9: Error comparisons of SBI data for 1 year prediction

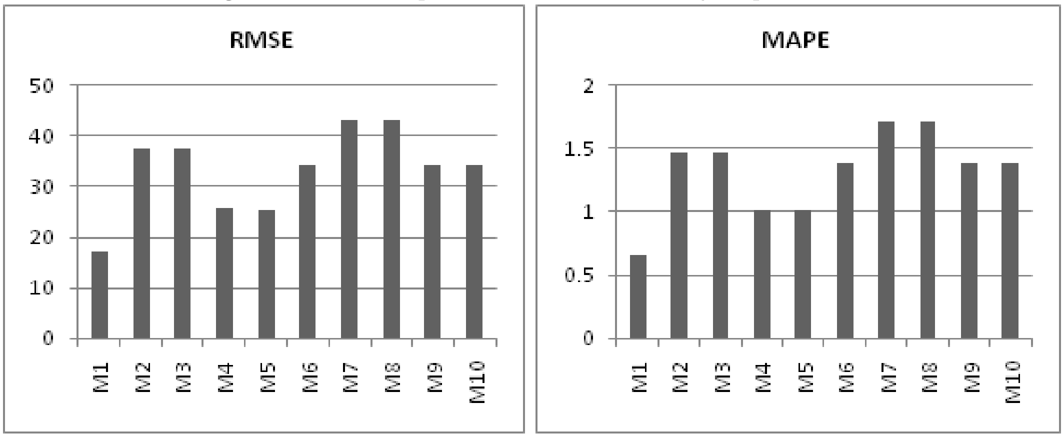
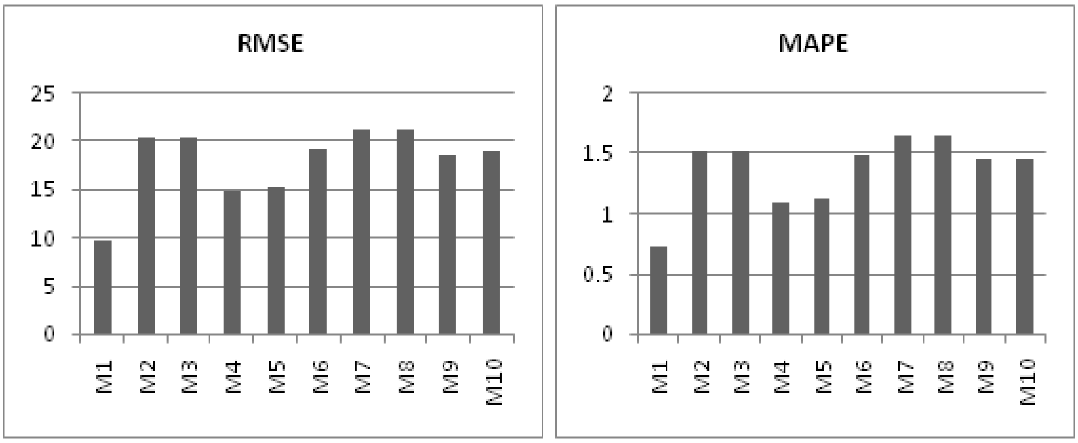


Figure 10: Error comparisons of ICICI data for 1 year prediction



5. CONCLUSION

In the present work, different prediction methods based on wavelet have been discussed. Wavelet transform decomposes the original time series data in a hierarchy of new time series that behave better than the original time series. The decomposed time series can be predicted more accurately. Here we have used the Indian banking stock prices as an example to show the benefits of using wavelet transform in time series analysis. In this paper we use ten various statistical methods (shown in table 1) on the decomposed parts to extend the data for required period. The computational results show that method M_1 which is based on moving average gives best result as compared to other methods.

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