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MHD BLOOD FLOW THROUGH STENOSED INCLINED TUBE OF EXPONENTIAL DIVERGING VESSEL WITH PERIODIC BODY ACCELERATION

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ABSTRACT

The mathematical model for MHD blood flow through stenosed inclined tube of exponential diverging vessel with periodic body acceleration is studied. Using appropriate boundary conditions, analytical expressions for the velocity, volumetric flow rate, fluid acceleration and the wall shear stress have been derived. These expressions are computed numerically and the computational results are presented graphically.

Key Words: *Blood flow, Stenosis, Exponential diverging vessels, Periodic body acceleration, magnetic field, inclined tubes*

INTRODUCTION

Arterial MHD pulsatile flow of blood under the periodic body acceleration is studied by Das and Saha [1]. El-Shahawey *et al.* [2] studied the pulsatile flow of blood through a porous medium under periodic body acceleration. Effect of magnetic field on blood flow under periodic Body acceleration in porous medium is studied by Varun Mohan *et al.* [3]. Bhuvan and Hazarika [4] studied the effect of magnetic field on pulsatile flow of blood in a porous channel. Mishra *et al.*, [5] studied the study of oscillatory blood flow through porous medium in a stenosed artery. Blood flow through an artery having radially non-symmetric mild stenosis is studied by Singh *et al.* [6]. Flow of Casson fluid through an inclined tube of non-uniform cross-section with multiple stenoses is studied by Sreenadh *et al.* [7]. Rathod *et al.*, [8] studied the pulsatile flow of blood under the periodic body acceleration with magnetic field. Rathod and Gopichand [9] studied the pulsatile flow of blood through a stenosed tube under periodic body acceleration with magnetic field. Sharma *et al.*, [10] studied the pulsatile unsteady flow of blood through porous medium in a stenotic artery under the influence of transverse magnetic field. Rathod and Ravi [11] studied the blood flow through stenosed inclined tubes with periodic body accelaration in the presence of magnetic field and it's applications to cardiovascular diseases. Singh and Rathee [12] studied the analysis of non-Newtonian blood flow through stenosed vessel in porous medium under the effect of magnetic field.

Using finite Hankel and Laplace transforms, analytical expressions for velocity profile, volumetric flow rate and wall shear stress have been obtained for an inclined stenosed tube and their natures are portrayed graphically for different parameters such as Hartmann number, phase angle, time etc.

MATHEMATICAL FORMULATION

Let us consider the axially symmetric and fully developed pulsatile flow of blood through a stenosed porous circular artery with body acceleration under the influence of uniform transverse magnetic field. Blood is assumed to be Newtonian and incompressible fluid. Also for mathematical model, we take the artery to be a long cylindrical tube along z-axis. The pressure gradient and body acceleration are respectively given by

$$-\frac{\partial P}{\partial z} = A_0 + A_1 \cos(\omega_P t) \tag{1}$$

$$G = a_0 \cos(\omega_b t + \phi) \tag{2}$$

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where A_0 and A_1 are pressure gradient of steady flow and amplitude of oscillatory part respectively, a_0 is the amplitude of body acceleration, $\omega_p = 2\pi f_p$, $\omega_b = 2\pi f_b$ with f_p is the pulse frequency and f_b is body acceleration frequency, ϕ is the phase angle of body acceleration G with respect to pressure gradient and time, t.

The governing equation of motion for flow in cylindrical polar coordinates is given by

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \rho G + \mu \nabla^2 u - \frac{\mu}{k} u - \sigma B_0{}^2 u + \rho g \sin\theta \tag{3}$$

where u is the axial velocity of blood; $\frac{\partial p}{\partial z}$ is pressure gradient; ρ is density of blood; μ is the viscosity of blood; k is the permeability of the isotropic porous medium; B_0 is the external magnetic field along the radial direction and σ is the conductivity of blood.

The geometry of stenos is is shown in figure-1.

$$R(z) = \begin{cases} a - \delta\left(1 + \cos\frac{\pi z}{2z_0}\right), & -2z_0 \le z \le 2z_0 \\ a & otherwise \end{cases}$$

where $R(z)$ is the radius of the stenosed artery and depends on δ, a is the radius of artery, $4z_0$ is the length of stenosis and 2δ is the maximum protuberance of the stenotic form of the artery wall.

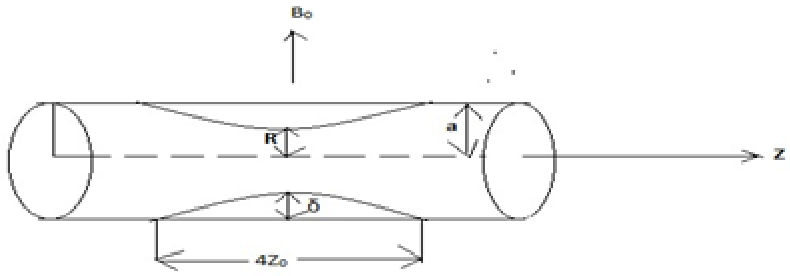

Fig.1.Geometry of artery with stenosis

$$\xi = \frac{r}{R(z)}$$

The equation (3) becomes

$$\rho \frac{\partial u}{\partial t} = A_0 + A_1 \cos(\omega_P t) + \rho a_0 \cos(\omega_b t + \phi) + \frac{\mu}{R^2}\left[\frac{\partial^2 u}{\partial \xi^2} + \frac{1}{\xi}\frac{\delta u}{\delta \xi}\right] - \mu C^2 u + \rho g \sin\theta \tag{4}$$

where

$$C = \sqrt{\frac{1}{k} + \frac{M^2}{R^2}}, \quad M = \sqrt{\frac{\sigma}{\mu}} R B_0 (Hartmann\ number)$$

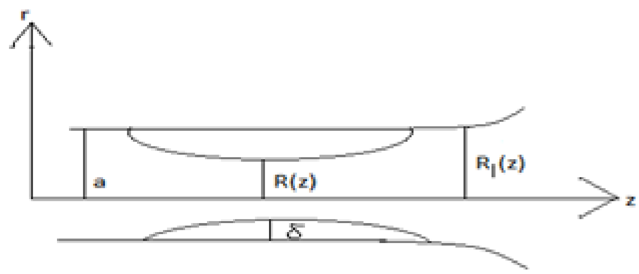

Fig.2.Flow geometry of an arterial stenosis

It is considered that the blood is undergoing periodic pulsatile motion through a rigid circular channel with impermeable walls which diverge exponentially as, $R_1(z) = \frac{a}{2}\left(1 + e^{\frac{\varepsilon z}{z_0}}\right)$ where 'a' is the tube radius, ε the divergence parameter over a characteristic axial distance Z_0 and is considered to be very less than one.

It is assumed that t < 0 only the pumping action of the heart is present and at t = 0, the flow in the artery corresponds to the instantaneous pressure gradient i.e.,

$$-\frac{\partial P}{\partial z} = A_0 + A_1$$

As a result, the flow velocity at t=0 is given by

$$u(\xi, 0) = \frac{A_0 + A_1}{\mu C^2}\left[1 - \frac{I_0(CR\xi)}{I_0(CR\xi_1)}\right] \tag{5}$$

where I_0 is modified Bessel function of first kind of order zero and $\xi_1 = \frac{R_1(z)}{R_0}$

The initial and boundary conditions to the problem are

$$\begin{aligned} &u(\xi, 0) = \frac{A_0 + A_1}{\mu C^2}\left[1 - \frac{I_0(CR\xi)}{I_0(CR\xi_1)}\right] \\ &u = 0 \quad at \quad \xi = \xi_1 \\ &u \;\; is \;\; finite \quad at \; \xi = 0 \end{aligned} \tag{6}$$

Solutions: Applying Laplace transform to equation (4) and first boundary condition of (6), we get

$$\rho s\bar{u} - \frac{\rho(A_0 + A_1)}{\mu C^2}\left[1 - \frac{I_0(CR\xi)}{I_0(CR\xi_1)}\right] = \frac{A_0}{s} + \frac{A_1 s}{(s^2 + \omega_P{}^2)} + \frac{\rho a_0 (s\cos\phi - \omega_b \sin\phi)}{(s^2 + \omega_b{}^2)} + \frac{\mu}{R^2}\left[\frac{\partial^2 \bar{u}}{\partial \xi^2} + \frac{1}{\xi}\frac{\delta \bar{u}}{\delta \xi}\right] - \mu C^2 \bar{u} + \frac{\rho g \sin\theta}{s} \tag{7}$$

where $\bar{u}(\xi, s) = \int_0^\infty e^{-st} u(\xi, t)\, dt \, (s \succ 0)$

Then applying the finite Hankel transform to equation (7), we obtain

$$\bar{u}^*(\lambda_n, s) = \frac{J_1(\xi_1 \lambda_n) R^2}{\lambda_n[\rho s R^2 + \mu(C^2 R^2 + \lambda_n{}^2)]}\left[\frac{A_0}{s} + \frac{A_1 s}{(s^2 + \omega_P{}^2)} + \frac{\rho a_0(s\cos\phi - \omega_b \sin\phi)}{(s^2 + \omega_b{}^2)} + \frac{\rho(A_0 + A_1)R^2}{\mu(C^2 R^2 + \lambda_n{}^2)} + \frac{\rho g \sin\theta}{s}\right] \tag{8}$$

where $\bar{u}^*(\lambda_n, s) = \int_0^1 r u(r, s) J_0(r\lambda_n)\, dr$ and λ_n are zeros of J_0 ,Bessel function of first kind and $\nu = \frac{\mu}{\rho}$

The Laplace and Hankel inversions of equation (8) give the final solution for blood velocity as

$$u(\xi,t)=2\sum_{n=1}^{\infty}\frac{1}{\xi_1\lambda_n}\frac{J_0(\lambda_n\xi)}{J_1(\xi_1\lambda_n)}\left[\begin{array}{l}\left\{\begin{array}{l}\dfrac{(A_0+\rho g\sin\theta)R^2}{\mu(\lambda_n{}^2+C^2R^2)}+\dfrac{A_1R^2[\nu(\lambda_n{}^2+C^2R^2)\cos\omega_P t+\omega_P R^2\sin\omega_P t]}{\rho[R^4\omega_P{}^2+\nu^2(\lambda_n{}^2+C^2R^2)^2]}\\+\dfrac{a_0R^2[\nu(\lambda_n{}^2+C^2R^2)\cos(\omega_b t+\phi)+\omega_b R^2\sin(\omega_b t+\phi)]}{R^4\omega_b{}^2+\nu^2(\lambda_n{}^2+C^2R^2)^2}\end{array}\right\}\\-e^{-\left(\frac{\nu}{R^2}\right)(\lambda_n{}^2+C^2R^2)t}\left\{\begin{array}{l}\dfrac{-A_1\omega_P{}^2R^6}{\mu(\lambda_n{}^2+C^2R^2)[R^4\omega_P{}^2+\nu^2(\lambda_n{}^2+C^2R^2)^2]}\\+\dfrac{a_0R^2[\nu(\lambda_n{}^2+C^2R^2)\cos\phi+\omega_b R^2\sin\phi]}{R^4\omega_b{}^2+\nu^2(\lambda_n{}^2+C^2R^2)^2}\\+\dfrac{\rho g\sin\theta\ R^2}{\nu(\lambda_n{}^2+C^2R^2)}\end{array}\right\}\end{array}\right] \tag{9}$$

which can be written in the form

$$u(\xi,t)=\frac{2A_0R^2}{\mu}\sum_{n=1}^{\infty}\frac{1}{\xi_1\lambda_n}\frac{J_0(\lambda_n\xi)}{J_1(\xi_1\lambda_n)}\left[\begin{array}{l}\left\{\dfrac{A_0+\rho g\sin\theta}{A_0\,(\lambda_n{}^2+C^2R^2)}+\dfrac{\varepsilon[(\lambda_n{}^2+C^2R^2)\cos\omega_P t+\alpha^2\sin\omega_P t]}{(\lambda_n{}^2+C^2R^2)^2+\alpha^4}\right\}\\+\dfrac{\rho a_0}{A_0}\left\{\dfrac{(\lambda_n{}^2+C^2R^2)\cos(\omega_b t+\phi)+\beta^2\sin(\omega_b t+\phi)}{(\lambda_n{}^2+C^2R^2)^2+\beta^4}\right\}\\-e^{-\left(\frac{\nu}{R^2}\right)(\lambda_n{}^2+C^2R^2)t}\left\{\begin{array}{l}\dfrac{-\varepsilon\alpha^4}{(\lambda_n{}^2+C^2R^2)[\alpha^4+(\lambda_n{}^2+C^2R^2)^2]}\\+\dfrac{\frac{\rho a_0}{A_0}\{(\lambda_n{}^2+C^2R^2)\cos\phi+\beta^2\sin\phi\}}{(\lambda_n{}^2+C^2R^2)^2+\beta^4}\\+\dfrac{\rho^2 g\sin\theta}{A_0}\dfrac{1}{(\lambda_n{}^2+C^2R^2)}\end{array}\right\}\end{array}\right] \tag{10}$$

where $\alpha^2=\dfrac{\omega_P R^2}{\nu}=\mathrm{Re}_P$, $\beta^2=\dfrac{\omega_b R^2}{\nu}=\mathrm{Re}_b$, $\varepsilon=\dfrac{A_1}{A_0}$

The analytical expression of u consists of four parts. The first and second parts correspond to steady and oscillatory parts of pressure gradient, the third term indicates body acceleration and the last term is the transient term .As $t\to\infty$, the transient term approaches to zero. Then from equation (10), we get

$$u(\xi,t)=\frac{2A_0R^2}{\mu}\sum_{n=1}^{\infty}\frac{1}{\xi_1\lambda_n}\frac{J_0(\lambda_n\xi)}{J_1(\xi_1\lambda_n)}\left[\begin{array}{l}\left\{\dfrac{A_0+\rho g\sin\theta}{A_0\,(\lambda_n{}^2+C^2R^2)}+\dfrac{\varepsilon[(\lambda_n{}^2+C^2R^2)\cos\omega_P t+\alpha^2\sin\omega_P t]}{(\lambda_n{}^2+C^2R^2)^2+\alpha^4}\right\}\\+\dfrac{\rho a_0}{A_0}\left\{\dfrac{(\lambda_n{}^2+C^2R^2)\cos(\omega_b t+\phi)+\beta^2\sin(\omega_b t+\phi)}{(\lambda_n{}^2+C^2R^2)^2+\beta^4}\right\}\end{array}\right] \tag{11}$$

The volumetric flow rate Q is given by $Q(\xi,t)=2\pi\int_0^R ru\,dr$

$$Q(\xi,t)=\frac{4\pi A_0R^2}{\mu}\sum_{n=1}^{\infty}\frac{1}{\lambda_n^{\,2}}\left[\left\{\frac{A_0+\rho g\sin\theta}{A_0\,(\lambda_n^{\,2}+C^2R^2)}+\frac{\varepsilon[(\lambda_n^{\,2}+C^2R^2)\cos\omega_p t+\alpha^2\sin\omega_p t]}{(\lambda_n^{\,2}+C^2R^2)^2+\alpha^4}\right\}+\frac{\rho a_0}{A_0}\left\{\frac{(\lambda_n^{\,2}+C^2R^2)\cos(\omega_b t+\phi)+\beta^2\sin(\omega_b t+\phi)}{(\lambda_n^{\,2}+C^2R^2)^2+\beta^4}\right\}\right] \tag{12}$$

The fluid acceleration F is given by $F(\xi,t)=\frac{\partial u}{\partial t}$

$$F(\xi,t)=\frac{2a_0}{\rho}\sum_{n=1}^{\infty}\frac{1}{\xi_1\lambda_n}\frac{J_0(\lambda_n\xi)}{J_1(\xi_1\lambda_n)}\left[\left\{\frac{\alpha^2\varepsilon\{-(\lambda_n^{\,2}+C^2R^2)\sin\omega_p t+\alpha^2\cos\omega_p t\}}{(\lambda_n^{\,2}+C^2R^2)^2+\alpha^4}\right\}+\frac{\rho a_0\beta^2}{A_0}\left\{\frac{-(\lambda_n^{\,2}+C^2R^2)\sin(\omega_b t+\phi)+\beta^2\cos(\omega_b t+\phi)}{(\lambda_n^{\,2}+C^2R^2)^2+\beta^4}\right\}\right] \tag{13}$$

The expression for the wall shear stress τ_w can be obtained from $\tau_w=\mu\left(\frac{\partial u}{\partial r}\right)_{r=R}$

$$\tau_w(\xi,t)=-\frac{2A_0R^2}{\xi_1}\sum_{n=1}^{\infty}\left[\left\{\frac{A_0+\rho g\sin\theta}{A_0\,(\lambda_n^{\,2}+C^2R^2)}+\frac{\varepsilon[(\lambda_n^{\,2}+C^2R^2)\cos\omega_p t+\alpha^2\sin\omega_p t]}{(\lambda_n^{\,2}+C^2R^2)^2+\alpha^4}\right\}+\frac{\rho a_0}{A_0}\left\{\frac{(\lambda_n^{\,2}+C^2R^2)\cos(\omega_b t+\phi)+\beta^2\sin(\omega_b t+\phi)}{(\lambda_n^{\,2}+C^2R^2)^2+\beta^4}\right\}\right] \tag{14}$$

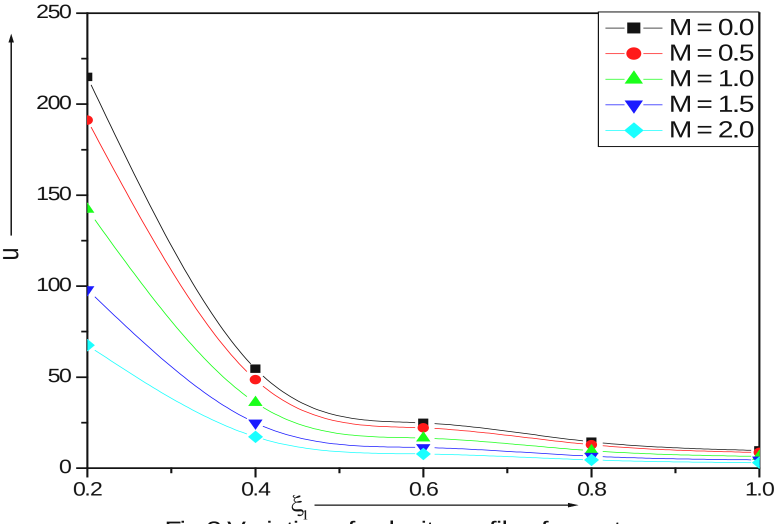

Fig.3.Variation of velocity profiles for aorta artery against ξ_1 with $\phi=90^0$, $\theta=30^0$ and t=2

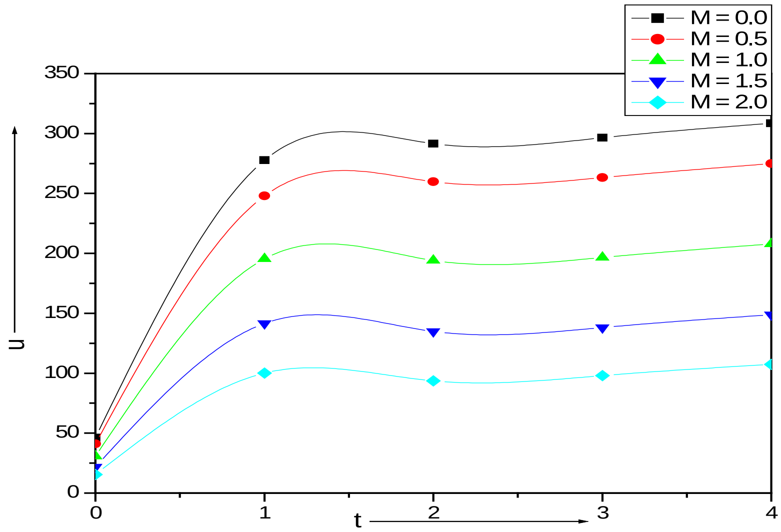

Fig.4.Variation of velocity profilesfor aorta artery against t with $\phi = 90^0$, $\theta = 45^0$ and ξ_1=0.2

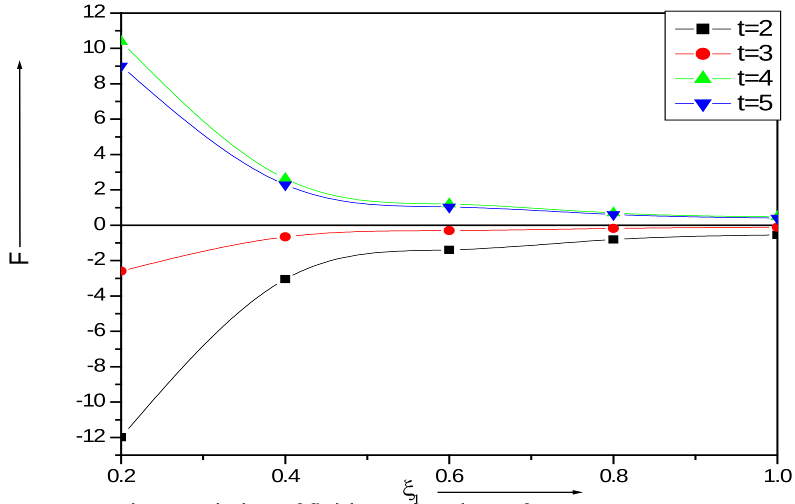

Fig.5.Variation of fluid accelaration F for aorta artery against ξ_1 with $\phi = 90^0$, $\theta = 30^0$ and M=2

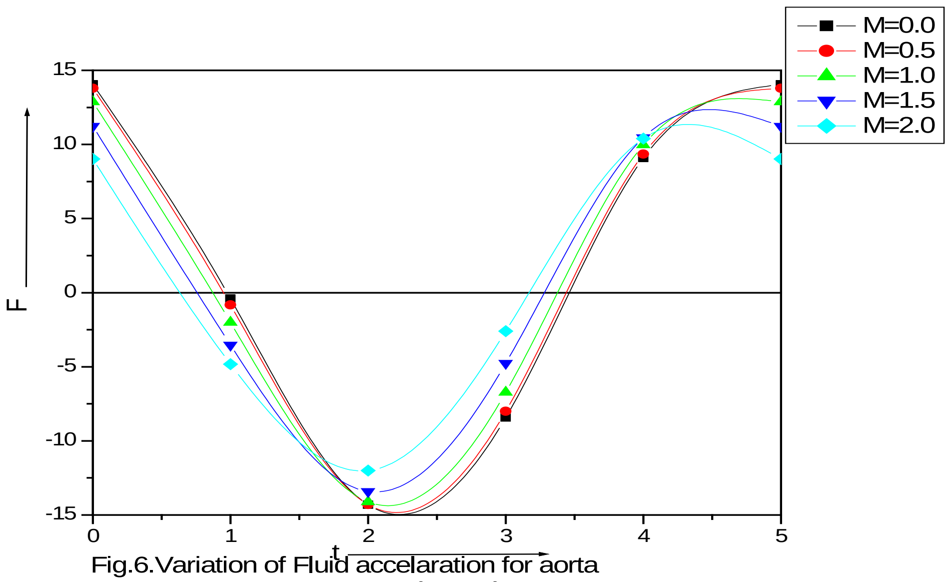

Fig.6.Variation of Fluid accelaration for aorta artery against t with $\phi = 90^0$, $\theta = 30^0$ and ξ_1=0.2

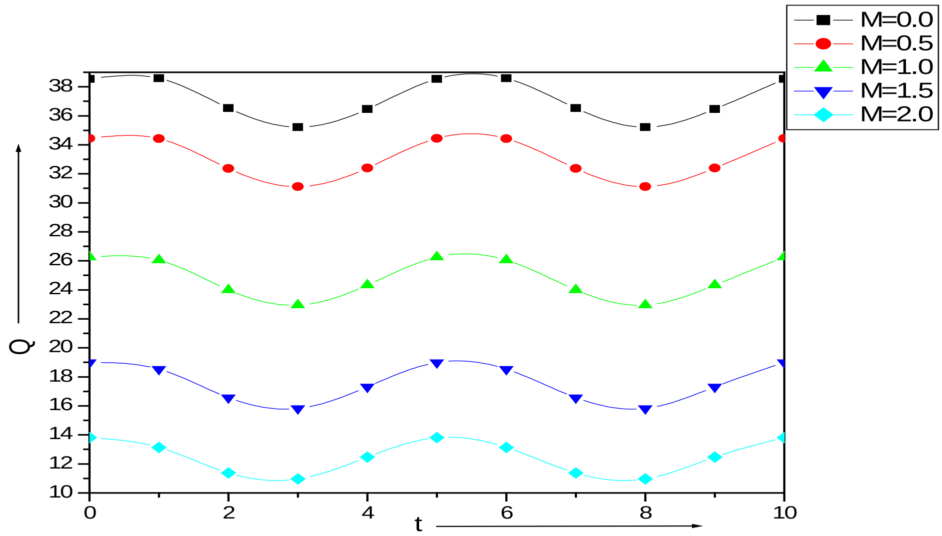

Fig.7.Variation of flow rate for aorta artery against t when $\phi=45^0$, $\theta=30^0$

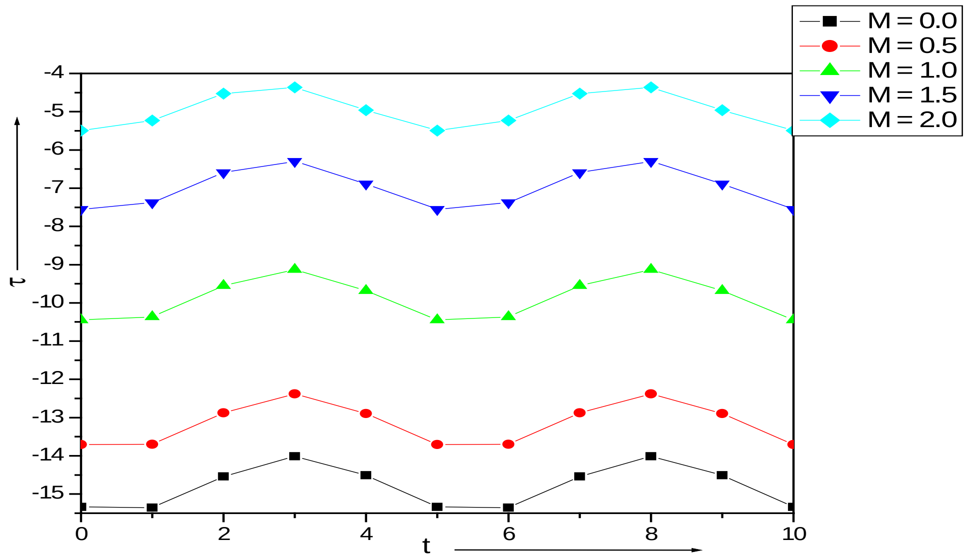

Fig.8.Variation of wall shear stress τ for aorta artery against t when $\phi=45^0$, $\theta=30^0$and ξ_1=0.4

Fig.-3: Velocity decreases with increase in ξ_1 for increase in magnetic field for particular value of $\phi = 90^0, \theta = 30^0$ and $t = 2$

Fig.-4: Velocity increases with increase in t for increase in magnetic field for particular value of $\phi = 90^0, \theta = 30^0$ and $\xi_1 = 0.2$

Fig.-5: If we plot F vs ξ_1 for increasing t for particular value of $\phi = 90^0, \theta = 30^0$ *and* $M = 2$ then the nature of the curve is as shown in the figure.

Fig.-6: If we plot F vs t for increasing M for particular value of $\phi = 90^0, \theta = 30^0$ *and* $\xi_1 = 0.2$ then the nature of the curve is as shown in the figure.

Fig.-7: If we draw Q vs t for different M, as M increases Q decreases for increasing t for particular $\phi = 45^0, \theta = 30^0$

Fig.-8: τ increases with increase in M for different t and the nature of the curve is as shown in fig. for particular $\phi = 45^0, \theta = 30^0$ *and* $\xi_1 = 0.4$

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