ON SOFT FUZZY ALMOST P-SPACES

A. HAYDAR EŞ*

Department of Mathematics Education,
Baskent University, Baglica, 06490, Ankara, Turkey.

ABSTRACT

***I**n this paper the concepts of soft fuzzy almost P-spaces soft weak fuzzy P-spaces and soft fuzzy P-spaces are introduced and studied.*

Mathematics Subject Classification: 54A40.

Keywords: Fuzzy sets, soft fuzzy topological spaces, soft fuzzy P-space, soft fuzzy almost P-space.

1. INTRODUCTION

Zadeh introduced the concept of fuzzy sets and fuzzy set operations in [9]. Chang in [3] introduced and developed the concept of fuzzy topological spaces. The concept of P-Spaces in fuzzy setting was introduced by G. Balasubramanian in [2]. The concept of almost GP-spaces in classical topology was introduced by M.R. Ahmadi Zand [1]. The concept of almost P-spaces in fuzzy setting was introduced by the authors in [4, 5, 6]. The concept of soft fuzzy topological space is introduced by I.U. Tiryaki [7]. In this paper, the concepts of soft fuzzy almost P-spaces and soft fuzzy P-spaces are introduced and studied.

2. PRELIMINARIES

We introduce some basic notions and results that are used in the sequel.

Definition 2.1: [2] Let (X, τ) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be fuzzy G_δ set if $\lambda = \bigwedge_{i=1}^{\infty} \mu_i$ where each μ_i is fuzzy open set. The complement of a fuzzy G_δ set is fuzzy F_σ.

Definition 2.2: [7] Let X be a set, μ be a fuzzy subset of X and $M \subseteq X$. Then the pair (μ, M) will be called a soft fuzzy subset of X. The set of all soft fuzzy subsets of X will be denoted by $SF(X)$.

Proposition 2.3: [7] If $(\mu_j, M_j)_{j \in J} \in SF(X)$, then the family $\{(\mu_j, M_j) | j \in J\}$ has a meet, that is greatest lower bound, in $(SF(X), \sqsubseteq$, denoted by $\prod_{j \in J}(\mu_j, M_j)$ such that $\prod_{j \in J}(\mu_j, M_j) = (\mu, M)$ where $\mu(x) = \bigwedge_{j \in J} \mu_j(x), \forall x, M = \bigcap_{j \in J} M_j$.

Definition 2.4: [7] Let X be a non-empty set and the soft fuzzy sets A and B in the form,
$$A = \{(\mu, M) | \mu(x) \epsilon I^X, \forall x \in X, M \subseteq X\}$$
$$B = \{(\lambda, N) | \lambda(x) \epsilon I^X, \forall x \in X, N \subseteq X\}$$
Then,
   (i) $A \subseteq B \Leftrightarrow \mu(x) \leq \lambda(x), \forall x \in X, M \subseteq N$.
   (ii) $A = B \Leftrightarrow \mu(x) = \lambda(x), \forall x \in X, M = N$.
   (iii) $A^{'} \Leftrightarrow 1 - \mu(x), \forall x \in X, X \setminus M$.
   (iv) $A \sqcap B \Leftrightarrow \mu(x) \wedge \lambda(x), \forall x \in X$ and $M \cap N$, for all $(\mu, M), (\lambda, N) \epsilon SF(X)$.
   (v) $A \sqcup B \Leftrightarrow \mu(x) \vee \lambda(x), \forall x \in X$ and $M \cup N$, for all $(\mu, M), (\lambda, N) \epsilon SF(X)$.

Definition 2.5: [7]
$$(0, \emptyset) = \{(\lambda, N) | \lambda = 0, N = \emptyset\}$$
$$(1, X) = \{(\lambda, N) | \lambda = 1, N = X\}$$

*Corresponding Author: A. Haydar Eş**
Department of Mathematics Education,
Baskent University, Baglica, 06490, Ankara, Turkey.

Definition 2.6: [7] For $(\mu, M) \in SF(X)$ the soft fuzzy set
$$(\mu, M)' = (1 - \mu, X \setminus M)$$ is called the complement of (μ, M).

Definition 2.7: [7] A subset $\tau \subseteq SF(X)$ is called an SF-topology on X if
(i) $(0, \emptyset)$ and $(1, X) \in \tau$
(ii) $(\mu_j, M_j) \in \tau$, $j = 1, 2, \ldots, n \Rightarrow \prod_{j=1}^{n}(\mu_j, M_j) \in \tau$
(iii) $(\mu_j, M_j), j \in J \Rightarrow \bigsqcup_{j \in J}(\mu_j, M_j) \in \tau$. The elements of τ are called soft fuzzy open, and those of
   $\tau' = \{(\mu, M) | (\mu, M)' \in \tau\}$ soft fuzzy closed.

If τ is SF-topology on X we call the pair (X, τ) SF-topological space (in short SFTS).

Definition 2.8: [7] The closure of a soft fuzzy set (μ, M) will be denoted by $\overline{(\mu, M)}$. It is given by
$$\overline{(\mu, M)} = \sqcap \{(\gamma, N) | (\mu, M) \sqsubseteq (\gamma, N), (\gamma, N) \in \tau'\}.$$

Likewise the interior is given by
$$(\mu, M)^\circ = \sqcup \{(\gamma, N) | (\gamma, N) \in \tau, (\gamma, N) \sqsubseteq (\mu, M)\}.$$

Note: $\overline{(\mu, M)} = cl(\mu, M)$ and $(\mu, M)^\circ = int(\mu, M)$.

Definition 2.9: [8] Let (X, τ) be a soft fuzzy topological space. Let (λ, N) be a soft fuzzy set in (X, τ). Then
   (i) (λ, N) is said to be soft fuzzy regular open if $(\lambda, N) = int(cl(\lambda, N))$.
   (ii) (λ, N) is said to be soft fuzzy regular closed if $(\lambda, N) = cl(int(\lambda, N))$.

Definition 2.10: [5] A fuzzy topological space (X, τ) is called a fuzzy P-space if countable intersection of fuzzy open sets in (X, τ) is fuzzy open. That is, every non-zero fuzzy G_δ set in (X, τ), is fuzzy open in (X, τ).

Definition 2.11: [5] A fuzzy topological space (X, τ) is called a fuzzy almost P-space if for every non-zero fuzzy G_δ set λ in (X, τ), $int(\lambda) \neq 0$ in (X, τ).

Definition 2.12: [5] A fuzzy topological space (X, τ) is called a weak fuzzy P-space if the countable intersection fuzzy regular open sets in (X, τ) is a fuzzy regular open set in (X, τ).

3. ON SOFT FUZZY ALMOST P-SPACES

Definition 3.1: A soft fuzzy topological space (X, τ) is called a soft fuzzy P-space if countable intersection of soft fuzzy open sets in (X, τ) is soft fuzzy open. That is, every non-zero soft fuzzy G_δ set in (X, τ) is soft fuzzy open in (X, τ).

Definition 3.2: A soft fuzzy topological space (X, τ) is called a soft fuzzy almost P-space if for every non-zero soft fuzzy G_δ set (λ, M) in (X, τ), $int(\lambda, M) \neq (0, \emptyset)$ in (X, τ).

It is clear that in soft fuzzy topological spaces, we have the following implication:
   Soft fuzzy P-space $\Rightarrow$ Soft fuzzy almost P-space.

Proposition 3.3: If the soft fuzzy topological space (X, τ) is a soft P-space, then
$$int(\sqcap_{i=1}^{\infty} (\mu_i, M_i)) = \sqcap_{i=1}^{\infty} (\mu_i, M_i),$$
where (μ_i, M_i)'s are non-zero soft fuzzy open sets in (X, τ).

Proof: Let (μ_i, M_i)'s be non-zero soft fuzzy open sets in a soft fuzzy P-space (X, τ). Then $(\mu, M) = \sqcap_{i=1}^{\infty} (\mu_i, M_i)$ is a soft fuzzy G_δ set in (X, τ). Since (X, τ) is a soft fuzzy P-space, the soft fuzzy G_δ set (μ, M) is soft fuzzy open in (X, τ).

Hence, we have $int(\mu, M) = (\mu, M)$. This implies that
$$int(\sqcap_{i=1}^{\infty} (\mu_i, M_i)) = \sqcap_{i=1}^{\infty} (\mu_i, M_i) = \sqcap_{i=1}^{\infty} int(\mu_i, M_i), \text{ and hence}$$
$$int(\sqcap_{i=1}^{\infty} (\mu_i, M_i)) = \sqcap_{i=1}^{\infty} (\mu_i, M_i),$$
where (μ_i, M_i)'s are non-zero soft fuzzy open sets in (X, τ).

Proposition 3.4: If (λ_i, M_i)'s are soft fuzzy regular closed sets in a soft fuzzy P-space (X, τ), then
$$cl(\sqcup_{i=1}^{\infty} (\lambda_i, M_i)) = \sqcup_{i=1}^{\infty} (\lambda_i, M_i).$$

Proof: Let (λ_i, M_i)'s be soft fuzzy regular closed sets in a soft fuzzy P-space (X, τ). Then (λ_i, M_i)'s are soft fuzzy closed sets in (X, τ), which implies that $(1, X) - (\lambda_i, M_i)$'s are soft fuzzy open sets in (X, τ). Then $\sqcap_{i=1}^{\infty} [(1, X) - (\lambda_i, M_i)]$ is a non-zero soft fuzzy G_δ set in (X, τ).

Hence $int(\sqcap_{i=1}^{\infty} [(1, X) - (\lambda_i, M_i)]) = \sqcap_{i=1}^{\infty} [(1, X) - (\lambda_i, M_i)]$.

Therefore $(1, X) - cl(\sqcup_{i=1}^{\infty} (\lambda_i, M_i)) = (1, X) - \sqcup_{i=1}^{\infty} (\lambda_i, M_i)$. Hence

we have $cl(\sqcup_{i=1}^{\infty} (\lambda_i, M_i)) = \sqcup_{i=1}^{\infty} (\lambda_i, M_i)$.

Definition 3.5: A soft fuzzy set (λ, M) in a soft fuzzy topological space (X, τ) is called a soft fuzzy nowhere dense if there exists no non-zero soft fuzzy open set (μ, N) in (X, τ) such that $(\mu, N) \sqsubseteq cl(\lambda, M)$.

That is, $int(cl(\lambda, M)) = (0, \emptyset)$.

Definition 3.6: A soft fuzzy set (λ, M) in a soft fuzzy topological space (X, τ) is called a soft fuzzy dense if there exists no soft fuzzy closed set (μ, N) in (X, τ) such that $(\lambda, M) \sqsubseteq (\mu, N) \sqsubseteq (1, X)$. That is, $cl(\lambda, M) = (1, X)$.

Definition 3.7: A soft fuzzy topological space (X, τ) is called a soft fuzzy submaximal space if for each soft fuzzy set (λ, M) in (X, τ) such that $cl(\lambda, M) = (1, X)$, then (λ, M) in (X, τ).

Proposition 3.8: If each soft fuzzy G_δ set is a soft fuzzy dense set in a soft fuzzy submaximal space (X, τ), then (X, τ) is a soft fuzzy P-space.

Proof: Let (λ, M) be a soft fuzzy G_δ set is in a soft fuzzy submaximal space (X, τ). By hypothesis, (λ, M) is a soft fuzzy dense set in (X, τ). Then (λ, M) is a soft fuzzy open set in (X, τ). That is, every soft fuzzy G_δ set in (X, τ) is a soft fuzzy open set in (X, τ). Hence (X, τ) is a soft fuzzy P-space.

Proposition 3.9: If $cl(int(\lambda, M)) = (1, X)$, for each soft fuzzy G_δ set (λ, M) in a soft fuzzy submaximal space (X, τ), then (X, τ) is a soft fuzzy P-space.

Proof: Let (λ, M) be a soft fuzzy F_σ set in a soft fuzzy submaximal space (X, τ). Then $(\lambda, M)'$ is a soft fuzzy G_δ set in (X, τ). By hypothesis,
$$cl(int(\lambda, M)') = (1, X). \text{ Then } (1, X) - cl(int(\lambda, M)') = (0, \emptyset).$$

This implies that $(1, X) - [(1, X) - int(cl(\lambda, M)] = (0, \emptyset)$.

That is, $int(cl(\lambda, M)) = (0, \emptyset)$ and hence (λ, M) is a soft fuzzy nowhere dense set in (X, τ). Thus the soft fuzzy F_σ set (λ, M) is a soft fuzzy nowhere dense set in a soft fuzzy submaximal space (X, τ). Since each soft fuzzy F_σ set is a soft fuzzy nowhere dense set in a soft fuzzy submaximal space (X, τ), then (X, τ) is a soft fuzzy P-space.

Definition 3.10: A soft fuzzy topological space (X, τ) is called a soft fuzzy weak P-space if the countable intersection of soft fuzzy regular open sets in (X, τ) is a soft fuzzy regular open sets in (X, τ). That is, $\prod_{i=1}^{\infty}(\lambda_i, M_i)$ is a soft fuzzy regular open in (X, τ), where (λ_i, M_i)'s are soft fuzzy regular open sets in (X, τ). It is clear that in soft fuzzy topological spaces, we have the following implication:
$$\text{Soft fuzzy P-space} \Rightarrow \text{Soft fuzzy weak P-space}.$$

Proposition 3.11: A soft fuzzy topological space (X, τ) is a soft fuzzy weak P-space iff $\coprod_{i=1}^{\infty}(\lambda_i, M_i)$, where (λ_i, M_i)'s are soft fuzzy regular closed sets in (X, τ) is a soft fuzzy regular closed in (X, τ).

Proof: Let (X, τ) be a soft fuzzy weak P-space. Then $int(cl(\prod_{i=1}^{\infty}(\lambda_i, M_i))) = \prod_{i=1}^{\infty}(\lambda_i, M_i)$, where (λ_i, M_i)'s are soft fuzzy regular open sets in (X, τ). Now
$$(1, X) - [int(cl(\prod_{i=1}^{\infty}(\lambda_i, M_i)))] = (1, X) - \prod_{i=1}^{\infty}(\lambda_i, M_i),$$
implies that
$$cl(int[(\coprod_{i=1}^{\infty}(1, X) - (\lambda_i, M_i))] = \coprod_{i=1}^{\infty}[(1, X) - (\lambda_i, M_i)].$$

Since $[(1, X) - (\lambda_i, M_i)]$ is a soft fuzzy regular closed set in (X, τ). Then we have
$$cl(int(\coprod_{i=1}^{\infty}[(1, X) - (\lambda_i, M_i)])) = \coprod_{i=1}^{\infty}[(1, X) - (\lambda_i, M_i)].$$

Hence $\coprod_{i=1}^{\infty}[(1, X) - (\lambda_i, M_i)]$ is a soft fuzzy regular closed in (X, τ).

Conversely, suppose that $cl(int(\coprod_{i=1}^{\infty}[(1, X) - (\lambda_i, M_i])) = \coprod_{i=1}^{\infty}[(1, X) - (\lambda_i, M_i)]$,

where $[(1, X) - (\lambda_i, M_i)]$ are soft fuzzy regular closed sets in (X, τ). Then
$$(1, X) - cl(int(\coprod_{i=1}^{\infty}[(1, X) - (\lambda_i, M_i)])) = (1, X) - \coprod_{i=1}^{\infty}[(1, X) - (\lambda_i, M_i)],$$

which implies that
$$int(cl(\prod_{i=1}^{\infty}[(1,X)-((1,X)-(\lambda_i,M_i))])) = \prod_{i=1}^{\infty}[(1,X)-((1,X)-(\lambda_i,M_i))] = \prod_{i=1}^{\infty}(\lambda_i,M_i).$$

Hence (X,τ) is a soft fuzzy weak P-space.

Proposition 3.12: If a soft fuzzy topological space (X,τ) is a soft fuzzy weak P-space, then
$$cl(\coprod_{i=1}^{\infty}(\lambda_i,M_i)) = \coprod_{i=1}^{\infty} cl(\lambda_i,M_i),$$
where (λ_i,M_i)'s are non-zero soft fuzzy open sets in (X,τ).

Proof: Proof is similar to the Proposition 3.4.

Definition 3.13: A soft fuzzy topological space (X,τ) is called a soft fuzzy almost Lindelöf space if every soft fuzzy open cover $(\lambda_\alpha,M_\alpha)_{\alpha\in\wedge}$ of (X,τ) there exists a countable subcover $(\lambda_n,M_n)_{n\in\mathbb{N}}$ such that $\sqcup_{n\in\mathbb{N}} cl(\lambda_n,M_n) = (1,X)$.

Definition 3.14: A soft fuzzy topological space (X,τ) is said to be soft fuzzy weakly Lindelöf space if every soft fuzzy open cover $(\lambda_\alpha,M_\alpha)_{\alpha\in\wedge}$ of (X,τ) there exists a countable subcover $(\lambda_n,M_n)_{n\in\mathbb{N}}$ such that
$$cl(\sqcup_{n\in\mathbb{N}}(\lambda_n,M_n)) = (1,X).$$

Obviously every soft fuzzy almost Lindelöf space is a soft fuzzy weakly Lindelöf space.

Proposition 3.15: If the soft fuzzy topological space (X,τ) is a soft fuzzy weak P-space, then every soft fuzzy weakly Lindelöf space is a soft fuzzy almost Lindelöf space.

Proof: Let (X,τ) be a soft fuzzy weakly Lindelöf space and $(\lambda_\alpha,M_\alpha)_{\alpha\in\wedge}$ be a soft fuzzy open cover of (X,τ). Then there exists a countable subcover $(\lambda_n,M_n)_{n\in\mathbb{N}}$ such that $cl(\sqcup_{n\in\mathbb{N}}(\lambda_n,M_n)) = (1,X)$.

Since (X,τ) is a soft fuzzy weak P-space,
$$cl(\sqcup_{n\in\mathbb{N}}(\lambda_n,M_n)) = \sqcup_{n\in\mathbb{N}} cl(\lambda_n,M_n)$$
where (λ_n,M_n)'s are non-zero soft fuzzy open sets in (X,τ). Hence for the soft fuzzy open cover $(\lambda_\alpha,M_\alpha)_{\alpha\in\wedge}$ of (X,τ), there exists a countable subcover
$$(\lambda_n,M_n)_{n\in\mathbb{N}} \text{such that } \sqcup_{n\in\mathbb{N}} cl(\lambda_n,M_n) = (1,X).$$

Hence (X,τ) is a soft fuzzy almost Lindelöf space.

Proposition 3.16: If a soft fuzzy topological space (X,τ) is a soft fuzzy P-space, then (X,τ) is a soft fuzzy weak P-space.

Proof: Let (λ_i,M_i)'s be soft fuzzy regular closed sets in (X,τ). Since (X,τ) is a soft fuzzy P-space, we have
$$cl(\coprod_{i=1}^{\infty}(\lambda_i,M_i)) = \coprod_{i=1}^{\infty}(\lambda_i,M_i).$$

Now $$cl(int(\coprod_{i=1}^{\infty}(\lambda_i,M_i))) \sqsubseteq cl(\coprod_{i=1}^{\infty}(\lambda_i,M_i)) = \coprod_{i=1}^{\infty}(\lambda_i,M_i).$$

Since $cl(int(\lambda_i,M_i)) = (\lambda_i,M_i)$, then
$$\coprod_{i=1}^{\infty} cl(int(\lambda_i,M_i)) = \coprod_{i=1}^{\infty}(\lambda_i,M_i), \text{ which implies that}$$
$$\coprod_{i=1}^{\infty}(\lambda_i,M_i) \sqsubseteq cl(int(\coprod_{i=1}^{\infty}(\lambda_i,M_i))).$$

Hence $cl(int(\coprod_{i=1}^{\infty}(\lambda_i,M_i))) = \coprod_{i=1}^{\infty}(\lambda_i,M_i)$. From the Proposition 3.11, (X,τ) is a soft fuzzy weak P-space.

Proposition 3.17: If (λ,M) is a non-zero soft fuzzy nowhere dense and soft fuzzy G_δ set in a soft fuzzy topological space (X,τ), then (X,τ) is not a soft fuzzy almost P-space.

Proof: Let (λ,M) be a non-zero soft fuzzy nowhere dense soft fuzzy G_δ set (λ,M) in (X,τ). Then $int(\lambda,M) \sqsubseteq int(cl(\lambda,M))$ and $int(cl(\lambda,M)) = (0,\emptyset)$, implies that $int(\lambda,M) = (0,\emptyset)$. Hence for the non-zero soft fuzzy G_δ set (λ,M) in (X,τ), $int(\lambda,M) = (0,\emptyset)$ in (X,τ). Therefore (X,τ) is not a soft fuzzy almost P-space.

ACKNOWLEDGEMENT

REFERENCES

1. Ahmadi, M.R.Zand, Almost GP-spaces, J.KoreanMath.Soc., 47(1) (2010), 215-222.
2. Balasubramanian, G., Maximal fuzzy topologies, Kybernetika, 31(5) (1995), 459-464.
3. Chang, C.L., Fuzzy topological spaces, J.Math.Anal.Appl., 24 (1968), 182-190.
4. Thangaraj, G., Anbazhagan, C., Some remarks on fuzzy P-spaces, Gen.Math.Notes, 26(1) (2015), 8-16.
5. Thangaraj, G., Anbazhagan, C., Vivakanandan, P., On fuzzy P-spaces, weak fuzzy P-spaces and fuzzy almost P-spaces, Gen.Math.Notes, 18(2) (2013), 128-139.
6. Thangaraj, G., Anbazhagan, C., On fuzzy almost GP-spaces, Annals of Fuzzy Mathematics and Informatics, 10(5) (2015), 727-736.
7. Tiryaki, I.U., Fuzzy sets over the posets I, Hacettepe Journal of Mathematics and Statistics, 37(2) (2008), 143-166.
8. Visalakshi, V., Uma, M.K., Roja, E., On soft fuzzy G_δ pre continuity in soft fuzzy topological space, Annals of Fuzzy Mathematics and Informatics, 8(6) (2014), 921-939.
9. Zadeh, L.A., Fuzzy sets, Information and Control, 8 (1965), 338-353.

Source of support: Nil, Conflict of interest: None Declared