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ON SOFT FUZZY ALMOST P-SPACES

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ABSTRACT

In this paper the concepts of soft fuzzy almost P-spaces soft weak fuzzy P-spaces and soft fuzzy P-spaces are introduced and studied.

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Keywords: *Fuzzy sets, soft fuzzy topological spaces, soft fuzzy P-space, soft fuzzy almost P-space.*

1. INTRODUCTION

Zadeh introduced the concept of fuzzy sets and fuzzy set operations in [9]. Chang in [3] introduced and developed the concept of fuzzy topological spaces. The concept of P-Spaces in fuzzy setting was introduced by G. Balasubramanian in [2]. The concept of almost GP-spaces in classical topology was introduced by M.R. Ahmadi Zand [1]. The concept of almost P-spaces in fuzzy setting was introduced by the authors in [4, 5, 6]. The concept of soft fuzzy topological space is introduced by I.U. Tiryaki [7]. In this paper, the concepts of soft fuzzy almost P-spaces and soft fuzzy P-spaces are introduced and studied.

2. PRELIMINARIES

We introduce some basic notions and results that are used in the sequel.

Definition 2.1: [2] Let (X,τ) be a fuzzy topological space. Let λ be any fuzzy set. Then λ is said to be fuzzy G_δ set if $\lambda = \wedge_{i=1}^{\infty} \mu_i$ where each μ_i is fuzzy open set. The complement of a fuzzy G_δ set is fuzzy F_σ.

Definition 2.2: [7] Let X be a set, μ be a fuzzy subset of X and $M \subseteq X$. Then the pair (μ, M) will be called a soft fuzzy subset of X. The set of all soft fuzzy subsets of X will be denoted by $SF(X)$.

Proposition 2.3: [7] If $(\mu_j, M_j)_{j\in J} \in SF(X)$, then the family $\{(\mu_j, M_j) | j \in J\}$ has a meet, that is greatest lower bound, in $(SF(X), \sqsubseteq$, denoted by $\prod_{j\in J}(\mu_j, M_j)$ such that $\prod_{j\in J}(\mu_j, M_j) = (\mu, M)$ where $\mu(x) = \wedge_{j\in J} \mu_j(x)$, $\forall x$, $M = \cap_{j\in J} M_j$.

Definition 2.4: [7] Let X be a non-empty set and the soft fuzzy sets A and B in the form,

$$A = \{(\mu, M) | \mu(x) \epsilon I^X, \forall x \in X, M \subseteq X\}$$
$$B = \{(\lambda, N) | \lambda(x) \epsilon I^X, \forall x \in X, N \subseteq X\}$$

Then,

(i) $A \subseteq B \Leftrightarrow \mu(x) \leq \lambda(x), \forall x \in X, M \subseteq N$.
(ii) $A = B \Leftrightarrow \mu(x) = \lambda(x), \forall x \in X, M = N$.
(iii) $A' \Leftrightarrow 1 - \mu(x), \forall x \in X, X \setminus M$.
(iv) $A \sqcap B \Leftrightarrow \mu(x) \wedge \lambda(x), \forall x \in X$ and $M \cap N$, for all $(\mu, M), (\lambda, N) \epsilon SF(X)$.
(v) $A \sqcup B \Leftrightarrow \mu(x) \vee \lambda(x), \forall x \in X$ and $M \cup N$, for all $(\mu, M), (\lambda, N) \epsilon SF(X)$.

Definition 2.5: [7]

$$(0, \emptyset) = \{(\lambda, N) | \lambda = 0, N = \emptyset\}$$
$$(1, X) = \{(\lambda, N) | \lambda = 1, N = X\}$$

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Definition 2.6: [7] For $(\boldsymbol{\mu}, \boldsymbol{M}) \in \boldsymbol{SF}(\boldsymbol{X})$ the soft fuzzy set
$(\boldsymbol{\mu}, \boldsymbol{M})' = (\mathbf{1} - \boldsymbol{\mu}, \boldsymbol{X} \setminus \boldsymbol{M})$ is called the complement of $(\boldsymbol{\mu}, \boldsymbol{M})$.

Definition 2.7: [7] A subset $\boldsymbol{\tau} \subseteq \boldsymbol{SF}(\boldsymbol{X})$ is called an $\boldsymbol{SF}$-topology on $\boldsymbol{X}$ if
(i) $(\mathbf{0}, \emptyset)$ and $(\mathbf{1}, \boldsymbol{X}) \in \boldsymbol{\tau}$
(ii) $(\boldsymbol{\mu}_j, \boldsymbol{M}_j) \in \boldsymbol{\tau}, \boldsymbol{j} = \mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{n} \Rightarrow \prod_{j=1}^{n} (\boldsymbol{\mu}_j, \boldsymbol{M}_j) \in \boldsymbol{\tau}$
(iii) $(\boldsymbol{\mu}_j, \boldsymbol{M}_j), \boldsymbol{j} \in \boldsymbol{J} \Rightarrow \coprod_{j \in J} (\boldsymbol{\mu}_j, \boldsymbol{M}_j) \in \boldsymbol{\tau}$. The elements of $\boldsymbol{\tau}$ are called soft fuzzy open, and those of
$\boldsymbol{\tau}' = \{(\boldsymbol{\mu}, \boldsymbol{M}) | (\boldsymbol{\mu}, \boldsymbol{M})' \in \boldsymbol{\tau}\}$ soft fuzzy closed.

If $\boldsymbol{\tau}$ is $\boldsymbol{SF}$-topology on $\boldsymbol{X}$ we call the pair $(\boldsymbol{X}, \boldsymbol{\tau})$ $\boldsymbol{SF}$-topological space (in short SFTS).

Definition 2.8: [7] The closure of a soft fuzzy set $(\boldsymbol{\mu}, \boldsymbol{M})$ will be denoted by $\overline{(\boldsymbol{\mu}, \boldsymbol{M})}$. It is given by
$\overline{(\boldsymbol{\mu}, \boldsymbol{M})} = \sqcap \{(\boldsymbol{\gamma}, \boldsymbol{N}) | (\boldsymbol{\mu}, \boldsymbol{M}) \sqsubseteq (\boldsymbol{\gamma}, \boldsymbol{N}), (\boldsymbol{\gamma}, \boldsymbol{N}) \in \boldsymbol{\tau}'\}$.

Likewise the interior is given by
$(\boldsymbol{\mu}, \boldsymbol{M})^\circ = \sqcup \{(\boldsymbol{\gamma}, \boldsymbol{N}) | (\boldsymbol{\gamma}, \boldsymbol{N}) \in \boldsymbol{\tau}, (\boldsymbol{\gamma}, \boldsymbol{N}) \sqsubseteq (\boldsymbol{\mu}, \boldsymbol{M})\}$.

Note: $\overline{(\boldsymbol{\mu}, \boldsymbol{M})} = \boldsymbol{cl}(\boldsymbol{\mu}, \boldsymbol{M})$ and $(\boldsymbol{\mu}, \boldsymbol{M})^\circ = \boldsymbol{int}(\boldsymbol{\mu}, \boldsymbol{M})$.

Definition 2.9: [8] Let $(\boldsymbol{X}, \boldsymbol{\tau})$be a soft fuzzy topological space. Let $(\boldsymbol{\lambda}, \boldsymbol{N})$ be a soft fuzzy set in $(\boldsymbol{X}, \boldsymbol{\tau})$. Then
(i) $(\boldsymbol{\lambda}, \boldsymbol{N})$ is said to be soft fuzzy regular open if $(\boldsymbol{\lambda}, \boldsymbol{N}) = \boldsymbol{int}(\boldsymbol{cl}(\boldsymbol{\lambda}, \boldsymbol{N}))$.
(ii) $(\boldsymbol{\lambda}, \boldsymbol{N})$ is said to be soft fuzzy regular closed if $(\boldsymbol{\lambda}, \boldsymbol{N}) = \boldsymbol{cl}(\boldsymbol{int}(\boldsymbol{\lambda}, \boldsymbol{N}))$.

Definition 2.10: [5] A fuzzy topological space $(\boldsymbol{X}, \boldsymbol{\tau})$ is called a fuzzy P-space if countable intersection of fuzzy open sets in $(\boldsymbol{X}, \boldsymbol{\tau})$ is fuzzy open. That is, every non-zero fuzzy $\boldsymbol{G_\delta}$ set in $(\boldsymbol{X}, \boldsymbol{\tau})$, is fuzzy open in $(\boldsymbol{X}, \boldsymbol{\tau})$.

Definition 2.11: [5] A fuzzy topological space $(\boldsymbol{X}, \boldsymbol{\tau})$ is called a fuzzy almost P-space if for every non-zero fuzzy $\boldsymbol{G_\delta}$ set $\boldsymbol{\lambda}$ in $(\boldsymbol{X}, \boldsymbol{\tau})$, $\boldsymbol{int}(\boldsymbol{\lambda}) \neq \mathbf{0}$ in $(\boldsymbol{X}, \boldsymbol{\tau})$.

Definition 2.12: [5] A fuzzy topological space $(\boldsymbol{X}, \boldsymbol{\tau})$ is called a weak fuzzy P-space if the countable intersection fuzzy regular open sets in $(\boldsymbol{X}, \boldsymbol{\tau})$is a fuzzy regular open set in $(\boldsymbol{X}, \boldsymbol{\tau})$.

3. ON SOFT FUZZY ALMOST P-SPACES

Definition 3.1: A soft fuzzy topological space $(\boldsymbol{X}, \boldsymbol{\tau})$ is called a soft fuzzy P-space if countable intersection of soft fuzzy open sets in $(\boldsymbol{X}, \boldsymbol{\tau})$ is soft fuzzy open. That is, every non-zero soft fuzzy $\boldsymbol{G_\delta}$ set in $(\boldsymbol{X}, \boldsymbol{\tau})$ is soft fuzzy open in $(\boldsymbol{X}, \boldsymbol{\tau})$.

Definition 3.2: A soft fuzzy topological space $(\boldsymbol{X}, \boldsymbol{\tau})$ is called a soft fuzzy almost P-space if for every non-zero soft fuzzy $\boldsymbol{G_\delta}$ set (λ, M) in $(\boldsymbol{X}, \boldsymbol{\tau})$, $int(\lambda, M) \neq (0, \emptyset)$in $(\boldsymbol{X}, \boldsymbol{\tau})$.

It is clear that in soft fuzzy topological spaces, we have the following implication:
Soft fuzzy P-space ⇒ Soft fuzzy almost P-space.

Proposition 3.3: If the soft fuzzy topological space $(\boldsymbol{X}, \boldsymbol{\tau})$ is a soft P-space, then
$\boldsymbol{int}(\sqcap_{i=1}^{\infty} (\boldsymbol{\mu}_i, \boldsymbol{M}_i)) = \sqcap_{i=1}^{\infty} (\boldsymbol{\mu}_i, \boldsymbol{M}_i)$,
where $(\boldsymbol{\mu}_i, \boldsymbol{M}_i)$'s are non-zero soft fuzzy open sets in $(\boldsymbol{X}, \boldsymbol{\tau})$.

Proof: Let $(\boldsymbol{\mu}_i, \boldsymbol{M}_i)$'s be non-zero soft fuzzy open sets in a soft fuzzy P-space $(\boldsymbol{X}, \boldsymbol{\tau})$. Then $(\boldsymbol{\mu}, \boldsymbol{M}) = \sqcap_{i=1}^{\infty} (\boldsymbol{\mu}_i, \boldsymbol{M}_i)$ is a soft fuzzy $\boldsymbol{G_\delta}$ set in $(\boldsymbol{X}, \boldsymbol{\tau})$. Since $(\boldsymbol{X}, \boldsymbol{\tau})$ is a soft fuzzy P-space, the soft fuzzy $\boldsymbol{G_\delta}$ set $(\boldsymbol{\mu}, \boldsymbol{M})$ is soft fuzzy open in $(\boldsymbol{X}, \boldsymbol{\tau})$.

Hence, we have $\boldsymbol{int}(\boldsymbol{\mu}, \boldsymbol{M}) = (\boldsymbol{\mu}, \boldsymbol{M})$. This implies that
$\boldsymbol{int}(\sqcap_{i=1}^{\infty} (\boldsymbol{\mu}_i, \boldsymbol{M}_i)) = \sqcap_{i=1}^{\infty} (\boldsymbol{\mu}_i, \boldsymbol{M}_i) = \sqcap_{i=1}^{\infty} \boldsymbol{int}(\boldsymbol{\mu}_i, \boldsymbol{M}_i)$, and hence
$\boldsymbol{int}(\sqcap_{i=1}^{\infty} (\boldsymbol{\mu}_i, \boldsymbol{M}_i)) = \sqcap_{i=1}^{\infty} (\boldsymbol{\mu}_i, \boldsymbol{M}_i)$,
where $(\boldsymbol{\mu}_i, \boldsymbol{M}_i)$'s are non-zero soft fuzzy open sets in $(\boldsymbol{X}, \boldsymbol{\tau})$.

Proposition 3.4: If $(\boldsymbol{\lambda}_i, \boldsymbol{M}_i)$'s are soft fuzzy regular closed sets in a soft fuzzy P-space $(\boldsymbol{X}, \boldsymbol{\tau})$, then
$\boldsymbol{cl}(\sqcup_{i=1}^{\infty} (\boldsymbol{\lambda}_i, \boldsymbol{M}_i)) = \sqcup_{i=1}^{\infty} (\boldsymbol{\lambda}_i, \boldsymbol{M}_i)$.

Proof: Let $(\boldsymbol{\lambda}_i, \boldsymbol{M}_i)$'s be soft fuzzy regular closed sets in a soft fuzzy P-space $(\boldsymbol{X}, \boldsymbol{\tau})$. Then $(\boldsymbol{\lambda}_i, \boldsymbol{M}_i)$'s are soft fuzzy closed sets in $(\boldsymbol{X}, \boldsymbol{\tau})$, which implies that $(\mathbf{1}, \boldsymbol{X}) - (\boldsymbol{\lambda}_i, \boldsymbol{M}_i)$'s are soft fuzzy open sets in $(\boldsymbol{X}, \boldsymbol{\tau})$. Then $\sqcap_{i=1}^{\infty} [(\mathbf{1}, \boldsymbol{X}) - (\boldsymbol{\lambda}_i, \boldsymbol{M}_i)]$ is a non-zero soft fuzzy $\boldsymbol{G_\delta}$ set in $(\boldsymbol{X}, \boldsymbol{\tau})$.

Hence $\boldsymbol{int}(\sqcap_{i=1}^{\infty}[(\mathbf{1},\boldsymbol{X})-(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)])=\sqcap_{i=1}^{\infty}[(\mathbf{1},\boldsymbol{X})-(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)]$.

Therefore $(\mathbf{1},\boldsymbol{X})-\boldsymbol{cl}(\sqcup_{i=1}^{\infty}(\boldsymbol{\lambda}_i,\boldsymbol{M}_i))=(\mathbf{1},\boldsymbol{X})-\sqcup_{i=1}^{\infty}(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)$. Hence

we have $\boldsymbol{cl}(\sqcup_{i=1}^{\infty}(\boldsymbol{\lambda}_i,\boldsymbol{M}_i))=\sqcup_{i=1}^{\infty}(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)$.

Definition 3.5: A soft fuzzy set $(\boldsymbol{\lambda},\boldsymbol{M})$ in a soft fuzzy topological space $(\boldsymbol{X},\boldsymbol{\tau})$is called a soft fuzzy nowhere dense if there exists no non-zero soft fuzzy open set $(\boldsymbol{\mu},\boldsymbol{N})$ in $(\boldsymbol{X},\boldsymbol{\tau})$such that $(\boldsymbol{\mu},\boldsymbol{N})\sqsubseteq\boldsymbol{cl}(\boldsymbol{\lambda},\boldsymbol{M})$.

That is, $\boldsymbol{int}(\boldsymbol{cl}(\boldsymbol{\lambda},\boldsymbol{M}))=(\mathbf{0},\emptyset)$.

Definition 3.6: A soft fuzzy set $(\boldsymbol{\lambda},\boldsymbol{M})$ in a soft fuzzy topological space $(\boldsymbol{X},\boldsymbol{\tau})$is called a soft fuzzy dense if there exists no soft fuzzy closed set $(\boldsymbol{\mu},\boldsymbol{N})$ in $(\boldsymbol{X},\boldsymbol{\tau})$such that $(\boldsymbol{\lambda},\boldsymbol{M})\sqsubseteq(\boldsymbol{\mu},\boldsymbol{N})\sqsubseteq(\mathbf{1},\boldsymbol{X})$. That is, $\boldsymbol{cl}(\boldsymbol{\lambda},\boldsymbol{M})=(\mathbf{1},\boldsymbol{X})$.

Definition 3.7: A soft fuzzy topological space $(\boldsymbol{X},\boldsymbol{\tau})$is called a soft fuzzy submaximal space if for each soft fuzzy set $(\boldsymbol{\lambda},\boldsymbol{M})$in $(\boldsymbol{X},\boldsymbol{\tau})$such that $\boldsymbol{cl}(\boldsymbol{\lambda},\boldsymbol{M})=(\mathbf{1},\boldsymbol{X})$, then $(\boldsymbol{\lambda},\boldsymbol{M})$ in $(\boldsymbol{X},\boldsymbol{\tau})$.

Proposition 3.8: If each soft fuzzy $\boldsymbol{G}_{\boldsymbol{\delta}}$ set is a soft fuzzy dense set in a soft fuzzy submaximal space $(\boldsymbol{X},\boldsymbol{\tau})$, then $(\boldsymbol{X},\boldsymbol{\tau})$is a soft fuzzy P-space.

Proof: Let $(\boldsymbol{\lambda},\boldsymbol{M})$ be a soft fuzzy $\boldsymbol{G}_{\boldsymbol{\delta}}$ set is in a soft fuzzy submaximal space $(\boldsymbol{X},\boldsymbol{\tau})$. By hypothesis, $(\boldsymbol{\lambda},\boldsymbol{M})$ is a soft fuzzy dense set in $(\boldsymbol{X},\boldsymbol{\tau})$. Then $(\boldsymbol{\lambda},\boldsymbol{M})$ is a soft fuzzy open set in $(\boldsymbol{X},\boldsymbol{\tau})$. That is, every soft fuzzy $\boldsymbol{G}_{\boldsymbol{\delta}}$ set in $(\boldsymbol{X},\boldsymbol{\tau})$ is a soft fuzzy open set in $(\boldsymbol{X},\boldsymbol{\tau})$. Hence $(\boldsymbol{X},\boldsymbol{\tau})$ is a soft fuzzy P-space.

Proposition 3.9: If $\boldsymbol{cl}(\boldsymbol{int}(\boldsymbol{\lambda},\boldsymbol{M}))=(\mathbf{1},\boldsymbol{X})$, for each soft fuzzy $\boldsymbol{G}_{\boldsymbol{\delta}}$ set $(\boldsymbol{\lambda},\boldsymbol{M})$ in a soft fuzzy submaximal space $(\boldsymbol{X},\boldsymbol{\tau})$, then $(\boldsymbol{X},\boldsymbol{\tau})$ is a soft fuzzy P-space.

Proof: Let $(\boldsymbol{\lambda},\boldsymbol{M})$ be a soft fuzzy $\boldsymbol{F}_{\boldsymbol{\sigma}}$ set in a soft fuzzy submaximal space $(\boldsymbol{X},\boldsymbol{\tau})$. Then $(\boldsymbol{\lambda},\boldsymbol{M})'$ is a soft fuzzy $\boldsymbol{G}_{\boldsymbol{\delta}}$ set in $(\boldsymbol{X},\boldsymbol{\tau})$. By hypothesis,

$\boldsymbol{cl}(\boldsymbol{int}(\boldsymbol{\lambda},\boldsymbol{M})')=(\mathbf{1},\boldsymbol{X})$. Then $(\mathbf{1},\boldsymbol{X})-\boldsymbol{cl}(\boldsymbol{int}(\boldsymbol{\lambda},\boldsymbol{M})')=(\mathbf{0},\emptyset)$.

This implies that $(\mathbf{1},\boldsymbol{X})-[(\mathbf{1},\boldsymbol{X})-\boldsymbol{int}(\boldsymbol{cl}(\boldsymbol{\lambda},\boldsymbol{M})]=(\mathbf{0},\emptyset)$.

That is, $\boldsymbol{int}(\boldsymbol{cl}(\boldsymbol{\lambda},\boldsymbol{M}))=(\mathbf{0},\emptyset)$ and hence $(\boldsymbol{\lambda},\boldsymbol{M})$ is a soft fuzzy nowhere dense set in $(\boldsymbol{X},\boldsymbol{\tau})$. Thus the soft fuzzy $\boldsymbol{F}_{\boldsymbol{\sigma}}$ set $(\boldsymbol{\lambda},\boldsymbol{M})$ is a soft fuzzy nowhere dense set in a soft fuzzy submaximal space $(\boldsymbol{X},\boldsymbol{\tau})$. Since each soft fuzzy $\boldsymbol{F}_{\boldsymbol{\sigma}}$ set is a soft fuzzy nowhere dense set in a soft fuzzy submaximal space $(\boldsymbol{X},\boldsymbol{\tau})$, then $(\boldsymbol{X},\boldsymbol{\tau})$ is a soft fuzzy P-space.

Definition 3.10: A soft fuzzy topological space $(\boldsymbol{X},\boldsymbol{\tau})$ is called a soft fuzzy weak P-space if the countable intersection of soft fuzzy regular open sets in $(\boldsymbol{X},\boldsymbol{\tau})$ is a soft fuzzy regular open sets in $(\boldsymbol{X},\boldsymbol{\tau})$. That is, $\prod_{i=1}^{\infty}(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)$ is a soft fuzzy regular open in $(\boldsymbol{X},\boldsymbol{\tau})$, where $(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)$'s are soft fuzzy regular open sets in $(\boldsymbol{X},\boldsymbol{\tau})$. It is clear that in soft fuzzy topological spaces, we have the following implication:

Soft fuzzy P-space ⇒ Soft fuzzy weak P-space.

Proposition 3.11: A soft fuzzy topological space $(\boldsymbol{X},\boldsymbol{\tau})$ is a soft fuzzy weak P-space iff $\coprod_{i=1}^{\infty}(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)$, where $(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)$'s are soft fuzzy regular closed sets in $(\boldsymbol{X},\boldsymbol{\tau})$ is a soft fuzzy regular closed in $(\boldsymbol{X},\boldsymbol{\tau})$.

Proof: Let $(\boldsymbol{X},\boldsymbol{\tau})$ be a soft fuzzy weak P-space. Then $\boldsymbol{int}(\boldsymbol{cl}(\prod_{i=1}^{\infty}(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)))=\prod_{i=1}^{\infty}(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)$, where $(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)$'s are soft fuzzy regular open sets in $(\boldsymbol{X},\boldsymbol{\tau})$. Now

$$(\mathbf{1},\boldsymbol{X})-[\boldsymbol{int}(\boldsymbol{cl}(\prod_{i=1}^{\infty}(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)))]=(\mathbf{1},\boldsymbol{X})-\prod_{i=1}^{\infty}(\boldsymbol{\lambda}_i,\boldsymbol{M}_i),$$

implies that

$$\boldsymbol{cl}(\boldsymbol{int}[(\coprod_{i=1}^{\infty}(\mathbf{1},\boldsymbol{X})-(\boldsymbol{\lambda}_i,\boldsymbol{M}_i))]=\coprod_{i=1}^{\infty}[(\mathbf{1},\boldsymbol{X})-(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)].$$

Since $[(\mathbf{1},\boldsymbol{X})-(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)]$ is a soft fuzzy regular closed set in $(\boldsymbol{X},\boldsymbol{\tau})$. Then we have

$$\boldsymbol{cl}(\boldsymbol{int}(\coprod_{i=1}^{\infty}[(\mathbf{1},\boldsymbol{X})-(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)]))=\coprod_{i=1}^{\infty}[(\mathbf{1},\boldsymbol{X})-(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)].$$

Hence $\coprod_{i=1}^{\infty}[(\mathbf{1},\boldsymbol{X})-(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)]$ is a soft fuzzy regular closed in $(\boldsymbol{X},\boldsymbol{\tau})$.

Conversely, suppose that $\boldsymbol{cl}(\boldsymbol{int}(\coprod_{i=1}^{\infty}[(\mathbf{1},\boldsymbol{X})-(\boldsymbol{\lambda}_i,\boldsymbol{M}_i]))=\coprod_{i=1}^{\infty}[(\mathbf{1},\boldsymbol{X})-(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)]$,

where $[(\mathbf{1},\boldsymbol{X})-(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)]$ are soft fuzzy regular closed sets in $(\boldsymbol{X},\boldsymbol{\tau})$. Then

$$(\mathbf{1},\boldsymbol{X})-\boldsymbol{cl}(\boldsymbol{int}(\coprod_{i=1}^{\infty}[(\mathbf{1},\boldsymbol{X})-(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)]))=(\mathbf{1},\boldsymbol{X})-\coprod_{i=1}^{\infty}[(\mathbf{1},\boldsymbol{X})-(\boldsymbol{\lambda}_i,\boldsymbol{M}_i)],$$

which implies that

$$int(cl(\prod_{i=1}^{\infty}[(1,X)-((1,X)-(\lambda_i,M_i))]))= \prod_{i=1}^{\infty}\left[(1,X)-\left((1,X)-(\lambda_i,M_i)\right)\right]=\prod_{i=1}^{\infty}(\lambda_i,M_i).$$

Hence (X,τ) is a soft fuzzy weak P-space.

Proposition 3.12: If a soft fuzzy topological space (X,τ) is a soft fuzzy weak P-space, then

$$cl(\coprod_{i=1}^{\infty}(\lambda_i,M_i))=\coprod_{i=1}^{\infty}cl(\lambda_i,M_i),$$

where (λ_i,M_i)'s are non-zero soft fuzzy open sets in (X,τ).

Proof: Proof is similar to the Proposition 3.4.

Definition 3.13: A soft fuzzy topological space (X,τ) is called a soft fuzzy almost Lindelöf space if every soft fuzzy open cover $(\lambda_\alpha,M_\alpha)_{\alpha\in\wedge}$ of (X,τ) there exists a countable subcover $(\lambda_n,M_n)_{n\in\mathbb{N}}$ such that $\sqcup_{n\in\mathbb{N}}\, cl(\lambda_n,M_n)=(1,X)$.

Definition 3.14: A soft fuzzy topological space (X,τ) is said to be soft fuzzy weakly Lindelöf space if every soft fuzzy open cover $(\lambda_\alpha,M_\alpha)_{\alpha\in\wedge}$ of (X,τ) there exists a countable subcover $(\lambda_n,M_n)_{n\in\mathbb{N}}$ such that

$$cl(\sqcup_{n\in\mathbb{N}}(\lambda_n,M_n))=(1,X).$$

Obviously every soft fuzzy almost Lindelöf space is a soft fuzzy weakly Lindelöf space.

Proposition 3.15: If the soft fuzzy topological space (X,τ) is a soft fuzzy weak P-space, then every soft fuzzy weakly Lindelöf space is a soft fuzzy almost Lindelöf space.

Proof: Let (X,τ) be a soft fuzzy weakly Lindelöf space and $(\lambda_\alpha,M_\alpha)_{\alpha\in\wedge}$ be a soft fuzzy open cover of (X,τ). Then there exists a countable subcover$(\lambda_n,M_n)_{n\in\mathbb{N}}$ such that $cl(\sqcup_{n\in\mathbb{N}}(\lambda_n,M_n))=(1,X)$.

Since (X,τ) is a soft fuzzy weak P-space,

$$cl(\sqcup_{n\in\mathbb{N}}(\lambda_n,M_n))=\sqcup_{n\in\mathbb{N}}cl(\lambda_n,M_n)$$

where (λ_n,M_n)'s are non-zero soft fuzzy open sets in (X,τ). Hence for the soft fuzzy open cover $(\lambda_\alpha,M_\alpha)_{\alpha\in\wedge}$ of (X,τ), there exists a countable subcover

$(\lambda_n,M_n)_{n\in\mathbb{N}}$such that $\sqcup_{n\in\mathbb{N}}cl(\lambda_n,M_n)=(1,X)$.

Hence (X,τ) is a soft fuzzy almost Lindelöf space.

Proposition 3.16: If a soft fuzzy topological space (X,τ) is a soft fuzzy P-space, then (X,τ) is a soft fuzzy weak P-space.

Proof: Let (λ_i,M_i)'s be soft fuzzy regular closed sets in (X,τ). Since (X,τ) is a soft fuzzy P-space, we have

$$cl(\coprod_{i=1}^{\infty}(\lambda_i,M_i))=\coprod_{i=1}^{\infty}(\lambda_i,M_i).$$

Now $cl(int(\coprod_{i=1}^{\infty}(\lambda_i,M_i)))\sqsubseteq cl(\coprod_{i=1}^{\infty}(\lambda_i,M_i))=\coprod_{i=1}^{\infty}(\lambda_i,M_i)$.

Since $cl\left(int(\lambda_i,M_i)\right)=(\lambda_i,M_i)$, then

$\coprod_{i=1}^{\infty}cl\left(int(\lambda_i,M_i)\right)=\coprod_{i=1}^{\infty}(\lambda_i,M_i)$, which implies that

$\coprod_{i=1}^{\infty}(\lambda_i,M_i)\sqsubseteq cl(int(\coprod_{i=1}^{\infty}(\lambda_i,M_i)))$.

Hence $cl(int(\coprod_{i=1}^{\infty}(\lambda_i,M_i)))=\coprod_{i=1}^{\infty}(\lambda_i,M_i)$. From the Proposition 3.11, (X,τ) is a soft fuzzy weak P-space.

Proposition 3.17: If (λ,M) is a non-zero soft fuzzy nowhere dense and soft fuzzy G_δ set in a soft fuzzy topological space (X,τ), then (X,τ) is not a soft fuzzy almost P-space.

Proof: Let (λ,M) be a non-zero soft fuzzy nowhere dense soft fuzzy G_δ set (λ,M)in (X,τ). Then $int(\lambda,M)\sqsubseteq int(cl(\lambda,M))$ and $int\left(cl(\lambda,M)\right)=(0,\emptyset)$, implies that $int(\lambda,M)=(0,\emptyset)$. Hence for the non-zero soft fuzzy G_δ set (λ,M)in (X,τ), $int(\lambda,M)=(0,\emptyset)$ in (X,τ). Therefore (X,τ) is not a soft fuzzy almost P-space.

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