

INTERVAL VALUED INTUITIONISTIC ANTI FUZZY PRIMARY IDEAL

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ABSTRACT

The concept of interval valued intuitionistic anti fuzzy primary ideals and interval valued intuitionistic anti fuzzy semiprimary ideals is defined. Various properties of interval valued intuitionistic anti fuzzy primary ideals and interval valued intuitionistic anti fuzzy semiprimary ideals are discussed. Finally, interval valued intuitionistic anti fuzzy primary ideals and interval valued intuitionistic anti fuzzy semiprimary ideals is defined, some properties are established.

Keywords: Intuitionistic fuzzy set, Intuitionistic anti fuzzy primary ideal, Intuitionistic anti fuzzy semi-primary ideal, Interval valued intuitionistic anti fuzzy primary ideals, Interval valued intuitionistic anti fuzzy semi primary ideals.

1. INTRODUCTION

After on introduction of fuzzy sets by A.Zadeh[8], the fuzzy concept has introduced almost all branches of mathematics. The notion of intuitionistic fuzzy set and its operations were introduced by Atanassov [1], as a generalization of the notion of fuzzy set. Atanassov [2, 3] discussed the operators over interval valued intuitionistic fuzzy sets. Palanivelrajan and Nandakumar [6] introduced the definition and some properties of intuitionistic anti fuzzy primary and semiprimary ideals and interval valued intuitionistic fuzzy primary ideals. In this paper interval valued intuitionistic anti fuzzy primary ideals and intuitionistic anti fuzzy semiprimary ideal are established.

2. PRELIMINARIES

In this section, some basic definitions that are essential for this paper are assembled.

Definition 2.1: Let S be any nonempty set. A mapping $\mu : S \rightarrow [0,1]$ is called a fuzzy subset of S .

Definition 2.2: A fuzzy subset μ of a ring R is called anti fuzzy ideal if for all $x, y \in R$ the following conditions are satisfied

$$(i) \mu(x - y) \leq \max(\mu(x), \mu(y))$$

$$(ii) \mu(xy) \leq \min(\mu(x), \mu(y))$$

Definition 2.3: A fuzzy ideal μ of a ring R is called anti fuzzy primary ideal, if for all $a, b \in R$ either $\mu(a) = \mu(b)$ or else $\mu(ab) \geq \mu(b^m)$ for some $m \in \mathbb{Z}^+$.

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Definition 2.4: A fuzzy ideal μ of a ring R is called anti fuzzy semiprimary ideal, if for all $a, b \in R$ either $\mu(ab) \geq \mu(a^n)$, for some $n \in \mathbb{Z}^+$, or else $\mu(ab) \geq \mu(b^m)$ for some $m \in \mathbb{Z}^+$.

Definition 2.5: An intuitionistic fuzzy set (IFS) A in X is defined as an object of the form $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in X \}$, where $\mu_A : X \rightarrow [0,1]$ and $\gamma_A : X \rightarrow [0,1]$ are the degree of membership and the degree of non-membership of the element $x \in X$, respectively, for every $x \in X$, satisfying $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.

Definition 2.6: A fuzzy subset μ of a ring R is called intuitionistic anti fuzzy ideal if for all $x, y \in R$, the following conditions are satisfied,

- (i) $\mu_A(x-y) \leq \min(\mu_A(x), \mu_A(y))$
- (ii) $\mu_A(xy) \leq \min(\mu_A(x), \mu_A(y))$
- (iii) $\gamma_A(x-y) \geq \min(\gamma_A(x), \gamma_A(y))$ (iv) $\gamma_A(xy) \geq \max(\gamma_A(x), \gamma_A(y))$

Definition 2.7: An intuitionistic anti fuzzy ideal R of a ring R is called Intuitionistic anti fuzzy primary ideal if for all $a, b \in R$ either $\mu_A(ab) \geq \mu_A(a)$ and $\gamma_A(ab) = \gamma_A(a)$, or $\mu_A(ab) \geq \mu_A(b^m)$ and $\gamma_A(ab) \leq \gamma_A(b^m)$, for some $m \in \mathbb{Z}^+$.

Example 2.1:

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \in \langle 4 \rangle \\ 0.3 & \text{if } x \in \langle 2 \rangle \sim \langle 4 \rangle \end{cases}$$

$$\gamma_A(x) = \begin{cases} 1 & \text{if } x \in \langle 8 \rangle \\ 0.6 & \text{if } x \in \langle 2 \rangle \sim \langle 8 \rangle \end{cases}$$

Definition 2.8: An intuitionistic anti fuzzy ideal A of a ring R is called intuitionistic anti fuzzy semiprimary ideal if for all $a, b \in R$ either $\mu_A(ab) \geq \mu_A(a^n)$ and $\gamma_A(ab) \leq \gamma_A(a^n)$, for some $n \in \mathbb{Z}^+$ or else $\mu_A(ab) \geq \mu_A(b^m)$ and $\gamma_A(ab) \leq \gamma_A(b^m)$ for some $m \in \mathbb{Z}^+$.

Definition 2.9: An interval valued fuzzy set A is specified by a function $M_A : E \rightarrow D[0,1]$, where $D[0,1]$ is the set of all intervals with in $[0, 1]$, for all $x \in E$, $M_A(x)$ is an interval $[a, b]$, where $0 \leq a \leq b \leq 1$.

Remark 2.1: An interval valued intuitionistic fuzzy set. A over E is defined as an object of the form $A = \{ \langle x, M_A(x), N_A(x) \rangle \mid x \in E \}$ where $M_A(x) \subseteq [0,1]$ and $N_A(x) \subseteq [0,1]$ are interval and for all $x \in E$, $rmaxM_A(x) + rmaxN_A(x) \leq 1$.

Definition 2.10: Let A be an interval valued intuitionistic fuzzy sets. A fuzzy ideal A of a ring R is said to be interval valued intuitionistic anti fuzzy primary ideal of R if for all $a, b \in R$ then either

$$\mu_A(ab) = rmaxM_A(ab) = rmaxM_A(a) = \mu_A(a) \text{ and } \gamma_A(ab) = rmaxN_A(ab) = rmaxN_A(a) = \gamma_A(a) \text{ or}$$

$$\gamma_A(ab) = rmaxN_A(ab) \geq rmaxN_A(b^n) = \gamma_A(b^n) \text{ and } \mu_A(ab) = rmaxM_A(ab) \leq rmaxM_A(b^n) = \mu_A(b^n),$$

for some $n \in \mathbb{Z}^+$.

Definition 2.11: Let A be an interval valued intuitionistic fuzzy sets. A fuzzy ideal A of a ring R is said to be interval valued intuitionistic anti fuzzy primary ideal of R if for all $a, b \in R$ then either

$$\begin{aligned} \mu_A(ab) &= rmaxM_A(ab) \geq rmaxM_A(a^n) = \mu_A(a^n) \text{ and} \\ \gamma_A(ab) &= rmax N_A(ab) \leq rmax N_A(a^n) = \gamma_A(a^n), \text{ for some } n \in \mathbb{Z}^+ \text{ or} \\ \mu_A(ab) &= rmax M_A(ab) \geq rmax \mu_A(b^m) = \mu_A(b^m) \text{ and} \\ \gamma_A(ab) &= rmax N_A(ab) \leq rmaxN_A(b^m) = \gamma_A(b^m) \text{ for some } m \in \mathbb{Z}^+. \end{aligned}$$

Example 2.2:

$$\mu_A(x) = rmaxM_A(x) = \begin{cases} [0,0] & \text{if } x = 0 \\ [0.2, 0.3] & \text{otherwise} \end{cases}$$

$$\gamma_A(x) = rmax N_A(x) = \begin{cases} [1,1] & \text{if } x = 0 \\ [0.4, 0.5] & \text{otherwise} \end{cases}$$

Definition 2.12: Let A and B are interval valued intuitionistic fuzzy set then $A \cap B$ is an interval valued intuitionistic fuzzy set E . If the following conditions are satisfied

$$A \cap B = \{ \langle x, [\min(rmin M_A(x), rmin M_B(x)), \max(rmin N_A(x), rmin N_B(x))] \rangle \mid x \in E \},$$

Definition 2.13: Let A and B are interval valued intuitionistic fuzzy set then $A \cup B$ is an interval valued intuitionistic fuzzy set E . If the following conditions are satisfied

$$A \cup B = \{ \langle x, [\max(rmin M_A(x), rmin M_B(x)), \min(rmin N_A(x), rmin N_B(x))] \rangle \mid x \in E \},$$

Definition 2.14: Let A and B are interval valued intuitionistic fuzzy set then $A + B$ is an interval valued intuitionistic fuzzy set E . If the following conditions are satisfied

$$A + B = \{ \langle x, [rmin M_A(x) + rmin M_B(x) - rmin M_A(x), rmin M_B(x), rmin N_A(x), rmin N_B(x)] \rangle \mid x \in E \},$$

Definition 2.15: Let A and B are interval valued intuitionistic fuzzy set then $A.B$ is an interval valued intuitionistic fuzzy set E . If the following conditions are satisfied

$$A.B = \{ \langle x, [rmin M_A(x).rmin M_B(x), rmin N_A(x) + rmin N_B(x) - rmin M_A(x).rmin N_B(x)] \rangle \mid x \in E \},$$

Definition 2.16: Let A and B are interval valued intuitionistic fuzzy set then $A @ B$ is an interval valued intuitionistic fuzzy set E . If the following conditions are satisfied

$$A @ B = \left\{ \langle x, \left[\frac{rmin M_A(x) + rmin M_B(x)}{2}, \frac{rmin N_A(x) + rmin N_B(x)}{2} \right] \right\} \mid x \in E \}.$$

Definition 2.17: Let A and B are interval valued intuitionistic fuzzy set then $A \$ B$ is an interval valued intuitionistic fuzzy set E . If the following conditions are satisfied

$$A \$ B = \left\{ \langle x, \left[\sqrt{rmin M_A(x).rmin M_B(x)}, \sqrt{rmin N_A(x).rmin N_B(x)} \right] \right\} \mid x \in E \}.$$

Definition 2.18: Let A and B are interval valued intuitionistic fuzzy set then $A \# B$ is an interval valued intuitionistic fuzzy set E . If the following conditions are satisfied

$$A \# B = \left\{ \langle x, \left[\frac{2.rmin M_A(x).rmin M_B(x)}{(rmin M_A(x) + rmin M_B(x))}, \frac{2.rmin N_A(x).rmin N_B(x)}{(rmin N_A(x) + rmin N_B(x))} \right] \right\} \mid x \in E \}$$

Definition 2.19: Let A and B are interval valued intuitionistic fuzzy set then $A * B$ is an interval valued intuitionistic fuzzy set E . If the following conditions are satisfied

$$A * B = \left\{ \langle x, \left[\frac{rmin M_A(x) + rmin M_B(x)}{2.(rmin M_A(x).rmin M_B(x) + 1)}, \frac{rmin N_A(x) + rmin N_B(x)}{2.(rmin N_A(x).rmin N_B(x) + 1)} \right] \right\} \mid x \in E.$$

3. INTERVAL VALUED INTUITIONISTIC ANTI FUZZY PRIMARY IDEALS AND INTERVAL VALUED INTUITIONISTIC ANTI FUZZY SEMIPRIMARY IDEALS

Theorem 3.1: If A and B are interval valued intuitionistic anti fuzzy semiprimary ideal of R, then $A \cap B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

Proof: Let A be an interval valued intuitionistic anti fuzzy semiprimary ideal of a ring R, then

$$\begin{aligned} \mu_A(xy) &= rmaxM_A(xy) \geq rmaxM_A(x^n) = \mu_A(x^n) \text{ and} \\ \gamma_A(xy) &= rmaxN_A(xy) \leq rmaxN_A(x^n) = \gamma_A(x^n) \text{ for some } n \in Z^+ \text{ and } x, y \in R. \end{aligned}$$

Let B be an interval valued intuitionistic anti fuzzy semiprimary ideal of the ring R, then

$$\mu_B(xy) = rmax\mu_B(xy) \geq rmaxM_B(x^n) = \mu_B(x^n) \text{ and } \gamma_B(xy) = rmaxN_B(xy) \leq rmaxN_B(x^n) = \gamma_B(x^n),$$

for some $n \in Z^+$ and $x, y \in R$.

Consider $x, y \in R$, then $x, y \in A \cap B$ implies $x, y \in A$ and $x, y \in B$

Consider

$$\begin{aligned} \mu_{A \cap B}(xy) &= r \max M_{A \cap B}(xy) \\ &= \max(r \max M_A(xy), r \max M_B(xy)) \\ &\geq \max(r \max M_A(x^n), r \max M_B(x^n)) \\ &= r \max M_{A \cap B}(x^n) \\ &= \mu_{A \cap B}(x^n) \end{aligned}$$

Therefore, $\mu_{A \cap B}(xy) \geq \mu_{A \cap B}(x^n)$.

Consider

$$\begin{aligned} \gamma_{A \cap B}(xy) &= r \max N_{A \cap B}(xy) \\ &= \min(r \max N_A(xy), r \max N_B(xy)) \\ &\leq \min(r \max N_A(x^n), r \max N_B(x^n)) \\ &= r \max N_{A \cap B}(x^n) \\ &= \gamma_{A \cap B}(x^n) \end{aligned}$$

Therefore, $\gamma_{A \cap B}(xy) \leq \gamma_{A \cap B}(x^n)$.

Therefore $A \cap B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

Theorem 3.2: If A and B are interval valued intuitionistic anti fuzzy semiprimary ideal of R, then $A \cup B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

Proof: Let A be an interval valued intuitionistic anti fuzzy semiprimary ideal of a ring R, then

$$\begin{aligned} \mu_A(xy) &= rmaxM_A(xy) \geq rmaxM_A(x^n) = \mu_A(x^n) \text{ and} \\ \gamma_A(xy) &= rmaxN_A(xy) \leq rmaxN_A(x^n) = \gamma_A(x^n) \text{ for some } n \in Z^+ \text{ and } x, y \in R. \end{aligned}$$

Let B be an interval valued intuitionistic anti fuzzy semiprimary ideal of the ring R, then

$$\begin{aligned} \mu_B(xy) &= rmaxM_B(xy) \geq rmaxM_B(x^n) = \mu_B(x^n) \text{ and} \\ \gamma_B(xy) &= rmaxN_B(xy) \leq rmaxN_B(x^n) = \gamma_B(x^n), \text{ for some } n \in Z^+ \text{ and } x, y \in R. \end{aligned}$$

Consider $x, y \in R$, then $x, y \in A \cup B$ implies $x, y \in A$ and $x, y \in B$

Consider

$$\begin{aligned} \mu_{A \cup B}(xy) &= r \max M_{A \cup B}(xy) \\ &= \min(r \max M_A(xy), r \max M_B(xy)) \\ &\geq \min(r \max M_A(x^n), r \max M_B(x^n)) \\ &= r \max M_{A \cap B}(x^n) \\ &= \mu_{A \cap B}(x^n) \end{aligned}$$

Therefore, $\mu_{A \cup B}(xy) \geq \mu_{A \cap B}(x^n)$.

Consider

$$\begin{aligned} \gamma_{A \cup B}(xy) &= r \max N_{A \cup B}(xy) \\ &= \max(r \max N_A(xy), r \max N_B(xy)) \\ &\leq \max(r \max N_A(x^n), r \max N_B(x^n)) \\ &= r \max N_{A \cap B}(x^n) \\ &= \gamma_{A \cap B}(x^n) \end{aligned}$$

Therefore, $\gamma_{A \cup B}(xy) \leq \gamma_{A \cap B}(x^n)$.

Therefore $A \cup B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

Theorem 3.3: If A and B are interval valued intuitionistic anti fuzzy semiprimary ideal of R, then $A + B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

Proof: Let A be an interval valued intuitionistic anti fuzzy semiprimary ideal of a ring R, then

$$\begin{aligned} \mu_A(xy) &= r \max M_A(xy) \geq r \max M_A(x^n) = \mu_A(x^n) \text{ and} \\ \gamma_A(xy) &= r \max N_A(xy) \leq r \max N_A(x^n) = \gamma_A(x^n) \text{ for some } n \in \mathbb{Z}^+ \text{ and } x, y \in R. \end{aligned}$$

Let B be an interval valued intuitionistic anti fuzzy semiprimary ideal of the ring R, then

$$\begin{aligned} \mu_B(xy) &= r \max M_B(xy) \geq r \max M_B(x^n) = \mu_B(x^n) \text{ and} \\ \gamma_B(xy) &= r \max N_B(xy) \leq r \max N_B(x^n) = \gamma_B(x^n), \text{ for some } n \in \mathbb{Z}^+ \text{ and } x, y \in R. \end{aligned}$$

Consider $x, y \in R$, then $x, y \in A + B$ implies $x, y \in A$ and $x, y \in B$

consider

$$\begin{aligned} \mu_{A+B}(xy) &= r \max M_{A+B}(xy) \\ &= (r \max M_A(xy) + r \max M_B(xy) - r \max M_A(xy).r \max M_B(xy)) \\ &\geq (r \max M_A(x^n) + r \max M_B(x^n) - r \max M_A(x^n).r \max M_B(x^n)) \\ &= \mu_A(x^n) + \mu_B(x^n) - \mu_A(x^n).\mu_B(x^n) \\ &= \mu_{A+B}(x^n). \end{aligned}$$

Therefore, $\mu_{A+B}(xy) \geq \mu_{A+B}(x^n)$

;

Consider

$$\begin{aligned} \gamma_{A+B}(xy) &= r \max N_{A+B}(xy) \\ &= r \max N_A(xy).r \max N_B(xy) \\ &\geq r \max N_A(x^n).r \max N_B(x^n) \\ &= \gamma_A(x^n).\gamma_B(x^n) \\ &= \gamma_{A+B}(x^n). \end{aligned}$$

Therefore, $\gamma_{A+B}(xy) \leq \gamma_{A+B}(x^n)$

Therefore, $A + B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R .

Theorem 3.4: If A and B are interval valued intuitionistic anti fuzzy semiprimary ideal of R , then $A.B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R .

Proof: Let A be an interval valued intuitionistic anti fuzzy semiprimary ideal of a ring R , then

$$\begin{aligned} \mu_A(xy) &= r \max M_A(xy) \geq r \max M_A(x^n) = \mu_A(x^n) \text{ and} \\ \gamma_A(xy) &= r \max N_A(xy) \leq r \max N_A(x^n) = \gamma_A(x^n) \text{ for some } n \in \mathbb{Z}^+ \text{ and } x, y \in R. \end{aligned}$$

Let B be an interval valued intuitionistic anti fuzzy semiprimary ideal of the ring R , then

$$\begin{aligned} \mu_B(xy) &= r \max M_B(xy) \geq r \max M_B(x^n) = \mu_B(x^n) \text{ and} \\ \gamma_B(xy) &= r \max N_B(xy) \leq r \max N_B(x^n) = \gamma_B(x^n), \text{ for some } n \in \mathbb{Z}^+ \text{ and } x, y \in R. \end{aligned}$$

Consider $x, y \in R$, then $x, y \in A.B$ implies $x, y \in A$ and $x, y \in B$

Consider

$$\begin{aligned} \mu_{A.B}(xy) &= r \max M_{A.B}(xy) \\ &= r \max M_A(xy).r \max M_B(xy) \\ &\geq r \max M_A(x^n).r \max M_B(x^n) \\ &= \mu_A(x^n).\mu_B(x^n) \\ &= \mu_{A.B}(x^n). \end{aligned}$$

Therefore, $\mu_{A.B}(xy) \geq \mu_{A.B}(x^n)$

Consider

$$\begin{aligned} \gamma_{A.B}(xy) &= r \max N_{A.B}(xy) \\ &= (r \max N_A(xy) + r \max N_B(xy) - r \max N_A(xy).r \max N_B(xy)) \\ &\leq (r \max N_A(x^n) + r \max N_B(x^n) - r \max N_A(x^n).r \max N_B(x^n)) \\ &= \gamma_A(x^n).\gamma_B(x^n) - \gamma_A(x^n).\gamma_B(x^n) \\ &= \gamma_{A.B}(x^n). \end{aligned}$$

Therefore, $\gamma_{A.B}(xy) \leq \gamma_{A.B}(x^n)$

Therefore, $A.B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R .

Theorem 3.5: If A and B are interval valued intuitionistic anti fuzzy semiprimary ideal of R , then $A@B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R .

Proof: Let A be an interval valued intuitionistic anti fuzzy semiprimary ideal of a ring R, then

$$\begin{aligned} \mu_A(xy) &= rmaxM_A(xy) \geq rmaxM_A(x^n) = \mu_A(x^n) \text{ and} \\ \gamma_A(xy) &= rmaxN_A(xy) \leq rmaxN_A(x^n) = \gamma_A(x^n) \text{ for some } n \in Z^+ \text{ and } x, y \in R. \end{aligned}$$

Let B be an interval valued intuitionistic anti fuzzy semiprimary ideal of the ring R, then

$$\begin{aligned} \mu_B(xy) &= rmaxM_B(xy) \geq rmaxM_B(x^n) = \mu_B(x^n) \text{ and} \\ \gamma_B(xy) &= rmaxN_B(xy) \leq rmaxN_B(x^n) = \gamma_B(x^n), \text{ for some } n \in Z^+ \text{ and } x, y \in R. \end{aligned}$$

Consider $x, y \in R$, then $x, y \in A @ B$ implies $x, y \in A$ and $x, y \in B$

Consider

$$\begin{aligned} \mu_{A@B}(xy) &= r \max M_{A@B}(xy) \\ &= \frac{r \max M_A(xy) + r \max M_B(xy)}{2} \\ &\geq \frac{r \max M_A(x^n) + r \max M_B(x^n)}{2} \\ &= \frac{\mu_A(x^n) + \mu_B(x^n)}{2} \\ &= \mu_{A@B}(x^n) \end{aligned}$$

Therefore, $\mu_{A@B}(xy) \geq \mu_{A@B}(x^n)$

Consider

$$\begin{aligned} \gamma_{A@B}(xy) &= r \max N_{A@B}(xy) \\ &= \frac{r \max N_A(xy) + r \max N_B(xy)}{2} \\ &\leq \frac{r \max N_A(x^n) + r \max N_B(x^n)}{2} \\ &= \frac{\gamma_A(x^n) + \gamma_B(x^n)}{2} \\ &= \gamma_{A@B}(x^n) \end{aligned}$$

Therefore, $\gamma_{A@B}(xy) \leq \gamma_{A@B}(x^n)$

Therefore, $A @ B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

Theorem 3.6: If A and B are interval valued intuitionistic anti fuzzy semiprimary ideal of R, then $A\$B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

Proof: Let A be an interval valued intuitionistic anti fuzzy semiprimary ideal of a ring R, then

$$\begin{aligned} \mu_A(xy) &= rmaxM_A(xy) \geq rmaxM_A(x^n) = \mu_A(x^n) \text{ and} \\ \gamma_A(xy) &= rmaxN_A(xy) \leq rmaxN_A(x^n) = \gamma_A(x^n) \text{ for some } n \in Z^+ \text{ and } x, y \in R. \end{aligned}$$

Let B be an interval valued intuitionistic anti fuzzy semiprimary ideal of the ring R, then

$$\begin{aligned} \mu_B(xy) &= rmaxM_B(xy) \geq rmaxM_B(x^n) = \mu_B(x^n) \text{ and} \\ \gamma_B(xy) &= rmaxN_B(xy) \leq rmaxN_B(x^n) = \gamma_B(x^n), \text{ for some } n \in Z^+ \text{ and } x, y \in R. \end{aligned}$$

Consider $x, y \in R$, then $x, y \in A\$B$ implies $x, y \in A$ and $x, y \in B$

Consider

$$\begin{aligned} \mu_{A\$B}(xy) &= r \max M_{A\$B}(xy) \\ &= \sqrt{r \max M_A(xy) r \max M_B(xy)} \\ &\geq \sqrt{r \max M_A(x^n) r \max M_B(x^n)} \\ &= \sqrt{\mu_A(x^n) \mu_B(x^n)} \\ &= \mu_{A\$B}(x^n) \end{aligned}$$

Therefore, $\mu_{A\$B}(xy) \geq \mu_{A\$B}(x^n)$.

Consider

$$\begin{aligned} \gamma_{A\$B}(xy) &= r \max N_{A\$B}(xy) \\ &= \sqrt{r \max N_A(xy) r \max N_B(xy)} \\ &\leq \sqrt{r \max N_A(x^n) r \max N_B(x^n)} \\ &= \sqrt{\gamma_A(x^n) \gamma_B(x^n)} \\ &= \gamma_{A\$B}(x^n) \end{aligned}$$

Therefore, $\gamma_{A\$B}(xy) \leq \gamma_{A\$B}(x^n)$

Therefore, $A\$B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R .

Theorem 3.7: If A and B are interval valued intuitionistic anti fuzzy semiprimary ideal of R , then $A\#B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R .

Proof: Let A be an interval valued intuitionistic anti fuzzy semiprimary ideal of a ring R , then

$$\begin{aligned} \mu_A(xy) &= r \max M_A(xy) \geq r \max M_A(x^n) = \mu_A(x^n) \text{ and} \\ \gamma_A(xy) &= r \max N_A(xy) \leq r \max N_A(x^n) = \gamma_A(x^n) \text{ for some } n \in \mathbb{Z}^+ \text{ and } x, y \in R. \end{aligned}$$

Let B be an interval valued intuitionistic anti fuzzy semiprimary ideal of the ring R , then

$$\begin{aligned} \mu_B(xy) &= r \max M_B(xy) \geq r \max M_B(x^n) = \mu_B(x^n) \text{ and} \\ \gamma_B(xy) &= r \max N_B(xy) \leq r \max N_B(x^n) = \gamma_B(x^n), \text{ for some } n \in \mathbb{Z}^+ \text{ and } x, y \in R. \end{aligned}$$

Consider $x, y \in R$, then $x, y \in A\#B$ implies $x, y \in A$ and $x, y \in B$

$$\begin{aligned} \mu_{A\#B}(xy) &= r \max M_{A\#B}(xy) \\ &= \frac{2r \max M_A(xy) r \max M_B(xy)}{r \max M_A(xy) + r \max M_B(xy)} \\ &\geq \frac{2r \max M_A(x^n) r \max M_B(x^n)}{r \max M_A(x^n) + r \max M_B(x^n)} \\ &= \frac{2\mu_A(x^n) \mu_B(x^n)}{\mu_A(x^n) + \mu_B(x^n)} \\ &= \mu_{A\#B}(x^n). \end{aligned}$$

Therefore, $\mu_{A\#B}(xy) \geq \mu_{A\#B}(x^n)$

Consider

$$\begin{aligned} \gamma_{A\#B}(xy) &= r \max N_{A\#B}(xy) \\ &= \frac{2r \max N_A(xy) r \max N_B(xy)}{r \max N_A(xy) + r \max N_B(xy)} \\ &\leq \frac{2r \max N_A(x^n) r \max N_B(x^n)}{r \max N_A(x^n) + r \max N_B(x^n)} \\ &= \frac{2\gamma_A(x^n)\gamma_B(x^n)}{\gamma_A(x^n) + \gamma_B(x^n)} \\ &= \gamma_{A\#B}(x^n). \end{aligned}$$

Therefore, $\gamma_{A\#B}(xy) \leq \gamma_{A\#B}(x^n)$

Therefore, $A\#B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R .

Theorem 3.8: If A and B are interval valued intuitionistic anti fuzzy semiprimary ideal of R , then $A*B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R .

Proof: Let A be an interval valued intuitionistic anti fuzzy semiprimary ideal of a ring R , then

$$\begin{aligned} \mu_A(xy) &= r \max M_A(xy) \geq r \max M_A(x^n) = \mu_A(x^n) \text{ and} \\ \gamma_A(xy) &= r \max N_A(xy) \leq r \max N_A(x^n) = \gamma_A(x^n) \text{ for some } n \in Z^+ \text{ and } x, y \in R. \end{aligned}$$

Let B be an interval valued intuitionistic anti fuzzy semiprimary ideal of the ring R , then

$$\begin{aligned} \mu_B(xy) &= r \max M_B(xy) \geq r \max M_B(x^n) = \mu_B(x^n) \text{ and} \\ \gamma_B(xy) &= r \max N_B(xy) \leq r \max N_B(x^n) = \gamma_B(x^n), \text{ for some } n \in Z^+ \text{ and } x, y \in R. \end{aligned}$$

Consider $x, y \in R$, then $x, y \in A*B$ implies $x, y \in A$ and $x, y \in B$

Consider

$$\begin{aligned} \mu_{A*B}(xy) &= r \max M_{A*B}(xy) \\ &= \frac{r \max M_A(xy) + r \max M_B(xy)}{2(r \max M_A(xy) r \max M_B(xy) + 1)} \\ &\geq \frac{r \max M_A(x^n) + r \max M_B(x^n)}{2(r \max M_A(x^n) r \max M_B(x^n) + 1)} \\ &= \frac{\mu_A(x^n) + \mu_B(x^n)}{2(\mu_A(x^n)\mu_B(x^n) + 1)} \\ &= \mu_{A*B}(x^n) \end{aligned}$$

Therefore, $\mu_{A*B}(xy) \geq \mu_{A*B}(x^n)$

Consider

$$\begin{aligned} \gamma_{A*B}(xy) &\geq r \max N_{A*B}(xy) \\ &= \frac{r \max N_A(xy) + r \max N_B(xy)}{2(r \max N_A(x^n) r \max N_B(x^n) + 1)} \end{aligned}$$

$$\begin{aligned} &\leq \frac{r \max N_A(x^n) + r \max N_B(x^n)}{2(r \max N_A(x^n) r \max N_B(x^n) + 1)} \\ &= \frac{\gamma_A(x^n) + \gamma_B(x^n)}{2 \gamma_A(x^n) \gamma_B(x^n) + 1} \\ &= \gamma_{A*B}(x^n). \end{aligned}$$

Therefore, $\gamma_{A*B}(xy) \leq \gamma_{A*B}(x^n)$

Therefore, $A*B$ is an interval valued intuitionistic anti fuzzy semiprimary ideal of R.

Theorem 3.9: If A and B are interval valued intuitionistic anti fuzzy primary ideal of R then $A \cap B$ is an interval valued intuitionistic anti fuzzy primary ideal of R.

Theorem 3.10: If A and B are interval valued intuitionistic anti fuzzy primary ideal of R then $A \cup B$ is an interval valued intuitionistic anti fuzzy primary ideal of R.

Theorem 3.11: If A and B are interval valued intuitionistic anti fuzzy primary ideal of R then $A + B$ is an interval valued intuitionistic anti fuzzy primary ideal of R.

Theorem 3.12: If A and B are interval valued intuitionistic anti fuzzy primary ideal of R then $A.B$ is an interval valued intuitionistic anti fuzzy primary ideal of R.

Theorem 3.13: If A and B are interval valued intuitionistic anti fuzzy primary ideal of R then $A @ B$ is an interval valued intuitionistic anti fuzzy primary ideal of R.

Theorem 3.14: If A and B are interval valued intuitionistic anti fuzzy primary ideal of R then $A\$B$ is an interval valued intuitionistic anti fuzzy primary ideal of R.

Theorem 3.15: If A and B are interval valued intuitionistic anti fuzzy primary ideal of R then $A\#B$ is an interval valued intuitionistic anti fuzzy primary ideal of R.

Theorem 3.16: If A and B are interval valued intuitionistic anti fuzzy primary ideal of R then $A*B$ is an interval valued intuitionistic anti fuzzy primary ideal of R.

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