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The Transmuted Weighted Exponential Distribution: Theory and Application

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ABSTRACT

A generalization of the weighted exponential distribution so-called transmuted weighted exponential distribution is proposed and studied. We will use the quadratic rank transmutation map (QRTM) in order to generate a flexible family of probability distributions taking weighted exponential distribution as the base value distribution by introducing a new parameter that would offer more distributional flexibility. Various structural properties including explicit expressions for the moments, quantiles, moment generating function and order statistics of the new distribution are derived. Its parameters were estimated using the method of maximum likelihood for estimating the model parameters and obtain the observed information matrix. Three real data sets are used to compare the flexibility of the transmuted version versus the weighted exponential distribution. Results obtained by Oguntunde et al. (2016) can be considered as a special case from present results.

Key words: *Weighted exponential distribution, maximum likelihood estimation, order statistics, transmutation map, reliability function.*

1. INTRODUCTION

The weighted exponential distribution being a competitor to the weibull, gamma and Generalized exponential distributions has received appreciable usage in the fields of engineering and medicine. The weighted exponential distribution has been rigorously explored in the area of probability distribution theory. For instance, Alqallaf *et al.* (2015) used five different estimation methods (maximum likelihood, moments, L-moments, ordinary least squares and weighted least squares) to estimate the parameters of the weighted exponential distribution. Oguntunde (2015) also generalized the weighted exponential distribution using the Exponentiated family of distributions to propose the exponentiated weighted exponential distribution.

Some other weighted distributions have also been defined in the literature, for example, the weighted inverted exponential distribution, Hussian (2013), the weighted weibull distribution, Mahdy (2013) and Shahbaz *et al*. (2010), the weighted multivariate normal distribution, Kim (2008), the weighted inverse weibull distribution, Kersey (2010), a weighted three parameter weibull distribution (Essam and Mohamed, 2013). Oguntunde *et al.* (2016) introduced two-parameter weighted exponential distribution based on a modified weighted version of Azzalini's (1985) approach, the cumulative distribution function (cdf) of the weighted exponential distribution can be defined by

$$G(x) = 1 - e^{-(\beta+1)\alpha x},\ \alpha > 0, \beta \succ 0, x \succ 0, \tag{1}$$

where, α is a scale parameter, β is a shape parameter. The corresponding probability density function (pdf) is given by

$$g(x) = (\beta+1)\alpha\, e^{-(\beta+1)\alpha x},\ \alpha > 0, \beta \succ 0, x \succ 0, \tag{2}$$

The aim of this paper is to provide more flexible extension of the weighted exponential distribution using the transmutation map technique introduced by Shaw and Buckley (2007) called the transmuted weighted exponential distribution. According to the quadratic rank transmutation Map (QRTM) approach a random variable X is said to have transmuted distribution if its cdf is given by

$$F(x) = (1+\lambda)G(x) - \lambda G^2(x),\ |\lambda| \le 1, \tag{3}$$

where $G(x)$ is the cdf of the base distribution. An extensive information about the quadratic rank transmutation map is given in Shaw and Buckley (2007). Observe that at $\lambda = 0$ we have the distribution of the base random variable.

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Many authors deal with the generalization of some well-known distributions. Aryal and Tsokos (2009) defined the transmuted generalized extreme value distribution and they studied some basic mathematical characteristics of transmuted Gumbel probability distribution and it has been observed that the transmuted Gumbel can be used to model climate data. Also Aryal and Tsokos (2011) presented a new generalization of the Weibull distribution called the transmuted Weibull distribution. Khan and King (2013) introduced the transmuted modified Weibull distribution which extended recent developments on the transmuted Weibull distribution by Aryal and Tsokos (2009). Recently, Afify *et al.* (2015) introduced the transmuted Weibull Lomax and studied its mathematical properties. In the present study we will provide mathematical formulations of the transmuted weighted exponential distribution and some of its properties. We will also provide possible area of applications.

The rest of this article is organized as follows, In Section 2 we demonstrate transmuted probability distribution. In Section 3, we find the reliability functions of the subject model. The statistical properties include quantile functions, moments and moment generating functions are derived in Section 4. The minimum, maximum and order statistics models are discussed in Section 5. Section 6 we demonstrate the maximum likelihood estimates and the asymptotic confidence intervals of the unknown parameters. In section 7, the transmuted weighted exponential distribution is applied to a real data set. Finally, in Section 8, we provide some conclusion.

2. TRANSMUTED WEIGHTED EXPONENTIAL DISTRIBUTION

In this section we studied the transmuted weighted exponential distribution and the sub-models of this distribution. Now using (1) and (3) we have the cdf of transmuted weighted exponential distribution.

$$F(x)=\left[1-e^{-\alpha(\beta+1)x}\right]\left[1+\lambda e^{-\alpha(\beta+1)x}\right]. \triangleright$$

where α is a scale parameter and β is a shape parameters representing the different patterns of the transmuted weighted exponential distribution and λ is the transmuted parameter. Hence, the pdf of transmuted weighted exponential distribution with parameters α, β and λ is

$$f(x)=\alpha(\beta+1)\, e^{-\alpha(\beta+1)x}\left[1-\lambda+2\lambda e^{-\alpha(\beta+1)x}\right].$$

Figure 1and 2 illustrates some of the possible shapes of the pdf and cdf of a transmuted weighted exponential distribution for selected values of parameters $\alpha, \beta,$ and λ .

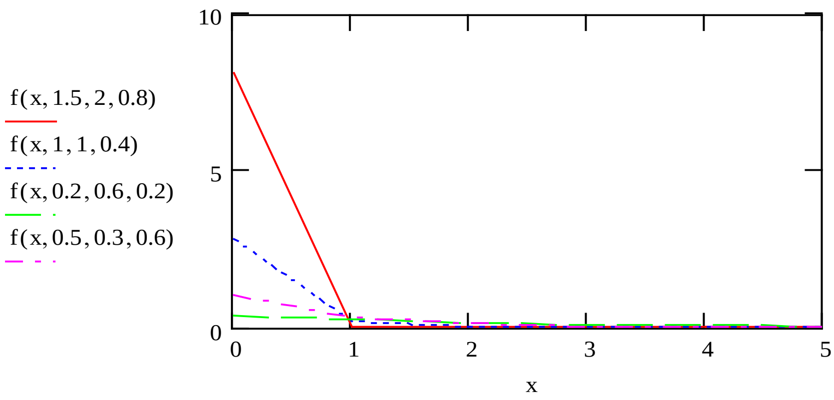

Figure-1: The pdf's of various transmuted weighted exponential distributions.

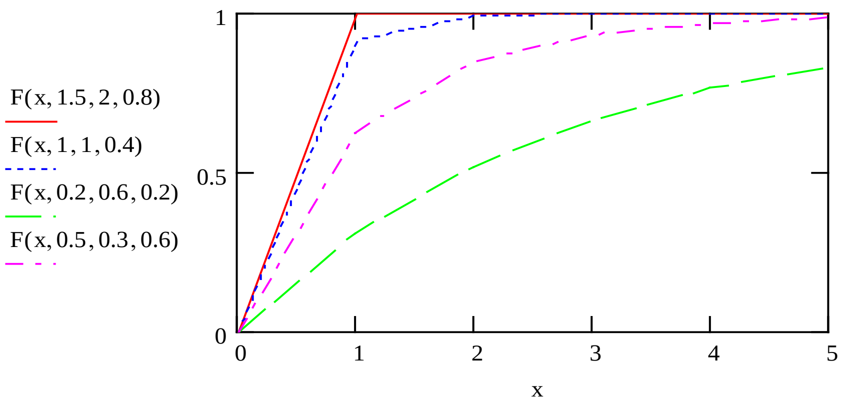

Figure-2: The cdf of various transmuted weighted exponential distributions.

Note that the transmuted weighted exponential distribution is an extended model to analyze more complex data and it generalizes some of widely used distributions. In particular, for $\beta = 0$ we have the transmuted exponential distribution as discussed in Shaw and Buckley (2007). The weighted exponential distribution is clearly a special case for $\lambda = 0$ as discussed in Oguntunde *et al.* (2016). When $\beta = \lambda = 0$ then the resulting distribution is an exponential distribution with parameter α .

3. RELIABILITY ANALYSIS

The reliability function $R(t)$, which is the probability of an item not failing prior to some time t, is defined by $R(t) = 1 - F(t)$. The reliability function of a transmuted weighted exponential distribution can be a useful characterization of life time data analysis. It is defined as

$$R(t) = 1 - F(t)$$
$$= 1 - \left\{\left[1 - e^{-\alpha(\beta+1)t}\right]\left[1 + \lambda e^{-\alpha(\beta+1)t}\right]\right\}$$

The other characteristic of interest of a random variable is the hazard rate function defined by

$$h(t) = \frac{f(t)}{1 - F(t)},$$

which is an important quantity characterizing life phenomenon. It can be loosely interpreted as the conditional probability of failure, given it has survived to the time t. The hazard rate function for a transmuted weighted exponential distribution is defined by

$$h(t) = \frac{\alpha(\beta+1)\, e^{-\alpha(\beta+1)t}\left[1 - \lambda + 2\lambda e^{-\alpha(\beta+1)t}\right]}{1 - \left\{\left[1 - e^{-\alpha(\beta+1)t}\right]\left[1 + \lambda e^{-\alpha(\beta+1)t}\right]\right\}}.$$

Figure 3 and 4 illustrates some of the possible shapes of the hazard rate function and survival function of transmuted weighted exponential distribution for selected values of the parameters $\alpha, \beta,$ and λ .

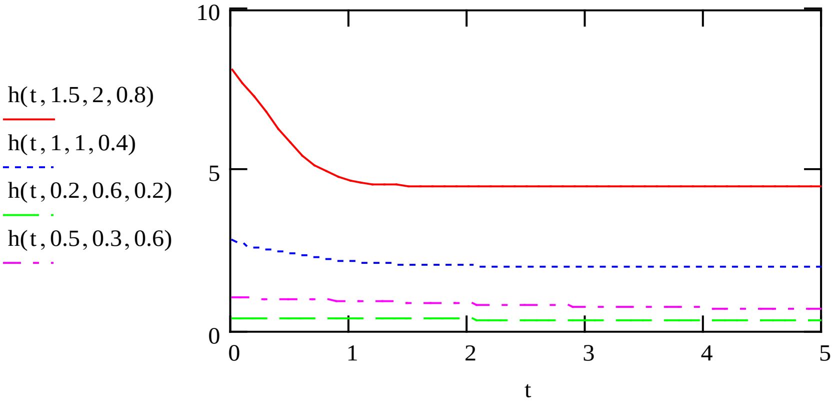

Figure-3: The hazard function of various transmuted weighted exponential distributions

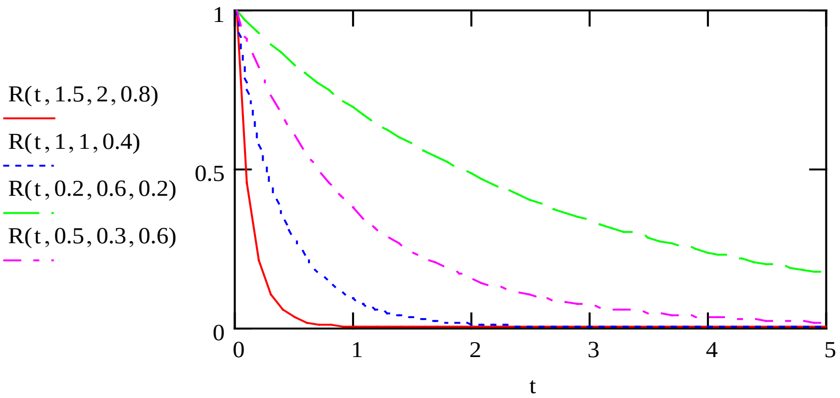

Figure-4: The survival function of various transmuted weighted exponential distributions.

4. STATISTICAL PROPERTIES

In this section, the statistical properties of the transmuted weighted exponential distribution is studied. Specifically the quantile function and random number generation, moments and moment generating function.

4.1 Quantile function and Median

The quantile function x_q of the transmuted weighted exponential distribution is real solution of the following equation

$$x_q = \frac{1}{\alpha(\beta+1)}\left\{-\ln\left[1-\left(\frac{1+\lambda-\sqrt{(1+\lambda)^2-4\lambda q}}{2\lambda}\right)\right]\right\}, \quad 0 \le q \le 1, \tag{4}$$

The above equation has no closed form solution in x_q, so we have to use a numerical technique such as a Newton-Raphson method to get the quantile. By putting $q = 0.5$ in Equation (4) we can get the median of transmuted weighted exponential distribution.

4.2 Random Number Generation

The random number generation of the transmuted weighted exponential distribution is defined by the following relation

$$x_q = \frac{1}{\alpha(\beta+1)}\left\{-\ln\left[1-\left(\frac{1+\lambda-\sqrt{(1+\lambda)^2-4\lambda u}}{2\lambda}\right)\right]\right\},$$

where u is a uniform distribution (0,1).

4.3 Moments

The r[th] non-central moment of X , denoted by $\mu_r^{'}$, of the transmuted weighted exponential distribution is given by the following theorem.

Theorem 1: If X is a continuous random variable has the transmuted weighted exponential distribution, then the r[th] non-central moment of X is given by

$$\mu_r^{'} = E[X^r] = \frac{\Gamma(r+1)}{[\alpha\,(\beta+1)]^r}\left[1-\lambda+\frac{\lambda}{2^r}\right]$$

Proof: The r[th] non-central moment is given by

$$\mu_r^{'} = \int_0^\infty x^r \, f(x)\, dx$$

$$= \alpha(\beta+1)\int_0^\infty x^r e^{-\alpha(\beta+1)x}\left(1-\lambda+2\lambda e^{-\alpha(\beta+1)x}\right) dx$$

$$= \alpha(\beta+1)\left[(1-\lambda)\int_0^\infty x^r e^{-\alpha(\beta+1)x}\, dx + (2\lambda)\int_0^\infty x^r\, e^{-2\alpha(\beta+1)x}\, dx\right]$$

Let $[\alpha(\beta+1)x] = y$, then $x = \frac{y}{[\alpha(\beta+1)]}$, therefore

$$\mu_r^{'} = \alpha(\beta+1)\left[(1-\lambda)\frac{\Gamma(r+1)}{[\alpha(\beta+1)]^{r+1}} + (2\lambda)\frac{\Gamma(r+1)}{[2\alpha(\beta+1)]^{r+1}}\right]$$

$$= \frac{\Gamma(r+1)}{[\alpha\,(\beta+1)]^r}\left[1-\lambda+\frac{\lambda}{2^r}\right].$$

Which completes the proof. Therefore, the expected value μ and variance σ^2 of a transmuted weighted exponential random variable X are, respectively, given by

$$\mu = E[X] = \frac{1}{[\alpha\,(\beta+1)]}\left[1-\frac{\lambda}{2}\right],$$

and the variance is

$$\sigma^2 = Var[X] = \frac{1}{[\alpha\ (\beta+1)]^2}\left[1-\frac{\lambda}{2}-\frac{\lambda^2}{4}\right].$$

The skewness and kurtosis measures can be obtained from the expressions

$$\text{Skewness}[X] = \frac{\mu_3^{'} - 3\mu\,\mu_2^{'} + 2\mu^3}{\sigma^3},$$

$$\text{kurtosis}[X] = \frac{\mu_4^{'} - 4\mu\,\mu_3^{'} + 6\mu^2\,\mu_2^{'} - 3\mu^4}{\sigma^4},$$

Upon substituting for the moments.

4.4 Moment Generating Function

In this subsection we derived the moment generating function of the transmuted weighted exponential distribution.

Theorem 2: If X is a continuous random variable has the transmuted weighted exponential distribution, then the moment generating function of X is given by

$$M_X(t) = \sum_{r=0}^{\infty} \frac{t^r\Gamma(r+1)}{r!\,[\alpha\,(\beta+1)]^r}\left[1-\lambda+\frac{\lambda}{2^r}\right]$$

Proof: The moment generating function of the random variable X is given by

$$M_X(t) = E\left[e^{tx}\right] = \int_0^{\infty} e^{tx}\ f(x)\,dx$$

$$= \int_0^{\infty}\left(1+t\,x+\frac{t^2\,x^2}{2!}+\ldots+\frac{t^n\,x^n}{n!}+\ldots\right)\ f(x)\ dx$$

$$= \sum_{r=0}^{\infty}\frac{t^r\ E\left(X^r\right)}{r!}$$

$$= \sum_{r=0}^{\infty} \frac{t^r\Gamma(r+1)}{r!\,[\alpha\,(\beta+1)]^r}\left[1-\lambda+\frac{\lambda}{2^r}\right].$$

Which completes the proof.

5. ORDER STATISTICS

In fact, the order statistics have many applications in reliability and life testing. Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous population with cdf $F_X(x)$ and pdf $f_X(x)$. Let $X_{(1)} \le X_{(2)} \le \ldots \le X_{(n)}$ denote the order statistics obtained from this sample then the pdf of $X_{(j)},\ j = 1,2,\ldots,n$ is given by

$$f_{X_{(j)}}(x) = \frac{n!}{(j-1)!\,(n-j)!}\,f_X(x)\left[F_X(x)\right]^{j-1}\left[1-F_X(x)\right]^{n-j}.$$

The pdf of the j^th order statistic for transmuted weighted exponential distribution is given by

$$f_{X_{(j)}}(x) = \frac{n!}{(j-1)!\,(n-j)!}\,\alpha\,(\beta+1)e^{-\alpha(\beta+1)x}\left[1-\lambda+2\,\lambda\,e^{-\alpha(\beta+1)x}\right]$$

$$\times\left[\left\{1-e^{-\alpha(\beta+1)x}\right\}\left\{1+\lambda\,e^{-\alpha(\beta+1)x}\right\}\right]^{j-1}\left[1-\left\{\left(1-e^{-\alpha(\beta+1)x}\right)\left(1+\lambda\,e^{-\alpha(\beta+1)x}\right)\right\}\right]^{n-j}$$

The pdf of the largest order statistic $X_{(n)} = \max(X_1, X_2, \ldots, X_n)$ is therefore:

$$f_{X_{(n)}}(x) = n\,\alpha\,(\beta+1)e^{-\alpha(\beta+1)x}\left[1-\lambda+2\,\lambda\,e^{-\alpha(\beta+1)x}\right]\left[\left\{1-e^{-\alpha(\beta+1)x}\right\}\left\{1+\lambda\,e^{-\alpha(\beta+1)x}\right\}\right]^{n-1}.$$

and the pdf of the smallest order statistic $X_{(1)} = \min(X_1, X_2, \ldots, X_n)$ is given by:

$$f_{X_{(j)}}(x) = n\,\alpha\,(\beta+1)e^{-\alpha(\beta+1)x}\left[1-\lambda+2\,\lambda\,e^{-\alpha(\beta+1)x}\right]\left[1-\left\{\left(1-e^{-\alpha(\beta+1)x}\right)\left(1+\lambda\,e^{-\alpha(\beta+1)x}\right)\right\}\right]^{n-1}.$$

6. ESTIMATION AND INFERENCE

In this section we discuss the maximum likelihood estimators (MLEs) and inference for the parameters (α, β, λ) of the transmuted weighted exponential distribution. Let $(X_1, X_2, ..., X_n)$ be a random sample of size n from this distribution with unknown parameter vector $\nu = (\alpha, \beta, \lambda)^T$. The likelihood function for ν, $l(\nu; x)$ can be written as

$$l(\nu; x) = \alpha^n (\beta+1)^n \; e^{-\alpha(\beta+1)\sum_{i=1}^{n} x_i} \prod_{i=1}^{n} \left[1-\lambda+2\lambda e^{-\alpha(\beta+1)x_i}\right].$$

Then, the log-likelihood function, ℓ, becomes:

$$\ell = n\ln\alpha + n\ln(\beta+1) - \alpha(\beta+1)\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \ln\left[1-\lambda+2\lambda e^{-\alpha(\beta+1)x_i}\right].$$

Therefore the score vector is $U(\nu) = \frac{\partial \ell}{\partial \nu} = \left(\frac{\partial \ell}{\partial \alpha}, \frac{\partial \ell}{\partial \beta}, \frac{\partial \ell}{\partial \lambda}\right)^T$. Let $P_i = e^{-\alpha(\beta+1)x_i}$

$$\frac{\partial \ell}{\partial \alpha} = \frac{n}{\alpha} - (\beta+1)\sum_{i=1}^{n} x_i - \sum_{i=1}^{n}\left[\frac{\left(2(\beta+1)\lambda x_i P_i\right)}{(1-\lambda+2\lambda P_i)}\right],$$

$$\frac{\partial \ell}{\partial \beta} = \frac{n}{(\beta+1)} - \alpha\sum_{i=1}^{n} x_i - \sum_{i=1}^{n}\left[\frac{\left(2\alpha\lambda x_i P_i\right)}{(1-\lambda+2\lambda P_i)}\right],$$

and

$$\frac{\partial \ell}{\partial \lambda} = \sum_{i=1}^{n}\left[\frac{(2P_i-1)}{(1-\lambda+2\lambda P_i)}\right].$$

We can find the estimates of the unknown parameters by setting the score vector to zero, $U(\nu) = 0$, and solving them simultaneously yields the MLEs $\hat{\alpha}, \hat{\beta},$ and $\hat{\lambda}$. These equations cannot be solved analytically and statistical software can be used to solve them numerically by means of iterative techniques such as the Newton-Raphson algorithm. For the three parameters transmuted weighted exponential distribution all the second order derivatives exist. Thus we have the inverse dispersion matrix is given by

$$\begin{pmatrix} \hat{\alpha} \\ \hat{\beta} \\ \hat{\lambda} \end{pmatrix} \sim N\left[\begin{pmatrix} \alpha \\ \beta \\ \lambda \end{pmatrix}, \begin{pmatrix} \hat{V}_{\alpha\alpha} & \hat{V}_{\alpha\beta} & \hat{V}_{\alpha\lambda} \\ \hat{V}_{\beta\alpha} & \hat{V}_{\beta\beta} & \hat{V}_{\beta\lambda} \\ \hat{V}_{\lambda\alpha} & \hat{V}_{\lambda\beta} & \hat{V}_{\lambda\lambda} \end{pmatrix}\right]$$

$$V^{-1} = -E\begin{pmatrix} V_{\alpha\alpha} & V_{\alpha\beta} & V_{\alpha\lambda} \\ V_{\beta\alpha} & V_{\beta\beta} & V_{\beta\lambda} \\ V_{\lambda\alpha} & V_{\lambda\beta} & V_{\lambda\lambda} \end{pmatrix} \tag{5}$$

where

$$V_{\alpha\alpha} = \frac{\partial^2 \ell}{\partial \alpha^2}, V_{\beta\beta} = \frac{\partial^2 \ell}{\partial \beta^2}, V_{\lambda\lambda} = \frac{\partial^2 \ell}{\partial \lambda^2}, V_{\alpha\beta} = \frac{\partial^2 \ell}{\partial \alpha \partial \beta}, V_{\alpha\lambda} = \frac{\partial^2 \ell}{\partial \alpha \partial \lambda}, \text{and } V_{\beta\lambda} = \frac{\partial^2 \ell}{\partial \beta \partial \lambda}.$$

By solving this inverse dispersion matrix these solutions will yield asymptotic variance and covariances of these MLs for $\hat{\alpha}, \hat{\beta},$ and $\hat{\lambda}$. Using (5), we approximate $100(1-\gamma)\%$ confidence intervals for $\alpha, \beta,$ and λ are determined respectively as

$$\hat{\alpha} \pm z_{\frac{\gamma}{2}}\sqrt{\hat{V}_{\alpha\alpha}}, \; \hat{\beta} \pm z_{\frac{\gamma}{2}}\sqrt{\hat{V}_{\beta\beta}} \text{ and } \hat{\lambda} \pm z_{\frac{\gamma}{2}}\sqrt{\hat{V}_{\lambda\lambda}}$$

where z_γ is the upper $100\gamma_{the}$ percentile of the standard normal distribution.

7. APPLICATION

In this section, the flexibility and potentiality of the transmuted weighted exponential distribution are examined using three real data sets to show that the transmuted weighted exponential distribution can be a better model than one based on the weighted exponential distribution. the analysis involved in this research was performed with Mathcad software package.

Data set 1: This data has been previously used by Ghitany *et al.* (2008) and Alqallaf *et al.* (2015). It represents the waiting time (measured in min) of 100 bank customers before service is being rendered. The data is as follows:

0.8	0.8	1.3	1.5	1.8	1.9	2.1	2.6	2.7	4.6	6.2	8.6
3.3	3.5	3.6	4.0	4.1	4.2	4.3	4.3	4.4	3.1	8.2	6.2
4.7	4.8	4.9	4.9	5.0	5.3	5.7	5.7	6.1	18.9	10.9	4.7
6.3	6.7	6.9	7.1	7.1	7.1	7.4	7.6	7.7	13.1	13.3	3.2
8.6	8.6	8.8	8.8	8.9	8.9	9.6	9.7	9.8	10.7	19.0	
11.0	11.1	11.2	11.2	11.5	11.9	12.5	12.9	13.0	8.0	19.9	
13.7	13.9	14.1	15.4	15.4	17.3	18.1	18.2	18.4	6.2	13.6	
20.6	21.3	21.4	21.9	23.0	27.0	33.1	38.5	2.9	4.4	11.0	

The estimates of the unknown parameters for the distributions by the maximum likelihood method,-2log-likelihood, Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), and the Consistent Akaike Information Criteria (CAIC) are used to compare the candidate distributions. The best distribution corresponds to lower for 2log-likelihood, AIC, BIC, and CAIC statistics values.

Table-1: maximum likelihood estimates (MLs) and their corresponding standard errors (in parentheses) of the model parameter, AIC, BIC, and CAIC values for waiting time (measured in min) of 100 bank customers.

Distributions	MLs			Measures			
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	-2ℓ	AIC	BIC	CAIC
transmuted weighted exponential	0.067 (0.013)	0.517 (0.300)	-0.700 (0.672)	653.831	659.831	659.831	660.081
weighted exponential	0.004 (0.002)	20.389 (10.790)		658.042	662.042	664.042	662.292

From Table 1,we observe that the transmuted weighted exponential distribution is a competitive distribution compared with another distribution. In fact, based on the values of the statistics we observe that the transmuted weighted exponential distribution provides the best fit for these data.

Data set 2: This data represents the lifetime of 20 electronic components. The data has been previously used by Teimouri and Gupta (2013) and Nasiru (2015). The data is as follows:

0.03	0.22	0.73	1.41	1.52	1.80	2.38	2.87	3.14	5.09
0.12	0.35	0.79	1.25	1.79	1.94	2.40	2.99	3.17	4.72

Table-2: MLs and their corresponding standard errors (in parentheses) of the model parameter, AIC, BIC, and CAIC values for the lifetime of 20 electronic components.

Distributions	MLs			Measures			
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	-2ℓ	AIC	BIC	CAIC
transmuted weighted exponential	0.357 (0.173)	0.455 (0.700)	-0.250 (0.611)	65.991	71.991	69.894	73.491
weighted exponential	21.761 (8.279)	-0.976 (0.010)		68.414	72.414	71.016	73.120

Data set 3: Data is from an accelerated life test of 59 conductors. It has been previously used by Nasiri *et al.* (2011). The data is as follows:

6.545	9.289	7.543	6.956	6.492	5.459	8.120	4.706	8.687	2.997
8.591	6.129	11.038	5.381	6.958	4.288	6.522	4.137	7.459	7.495
6.573	6.538	5.589	6.087	5.807	6.725	8.532	6.663	6.369	7.024
8.336	9.218	7.945	6.869	6.352	4.700	6.948	9.254	5.009	7.489
7.398	6.033	10.092	7.496	4.531	7.974	8.799	7.683	7.224	7.365
6.923	5.640	5.434	7.937	6.515	6.476	6.071	10.491	5.923	

Table-3: maximum likelihood estimates and their corresponding standard errors (in parentheses) of the model parameter, AIC, BIC, and CAIC values for the accelerated life test of 59 conductors.

Distributions	MLs			Measures			
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	-2ℓ	AIC	BIC	CAIC
transmuted weighted exponential	0.098 (0.079)	0.991 (1.603)	-0.704 (0.829)	319.819	325.819	330.444	326.255
weighted exponential	34.558 (11.604)	-0.995 (0.001)		364.418	350.418	367.960	350.632

From the above results, it is evident that the transmuted weighted exponential distribution is the best distribution for fitting these data sets compared to the weighted exponential distribution, and it is a strong competitor to other distribution commonly used in literature for fitting lifetime data.

8. CONCLUSION

Here we propose a new model, the so-called the transmuted weighted exponential distribution which extends the weighted exponential distribution in the analysis of data with real support. It is provides larger flexibility in modeling real data. We derive expansions for moments and for the moment generating function. The estimation of parameters is approached by the method of maximum likelihood, also the information matrix is derived. An application of transmuted weighted exponential distribution to real data show that the new distribution can be used quite effectively to provide better fits than weighted exponential distribution.

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