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ON THE INDEPENDENT END EQUITABLE DOMINATION IN GRAPH

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ABSTRACT

An equitable dominating set S in a graph G is called independent equitable dominating set if the subgraph induced by S is connected. In this paper we introduce the Independence end equitable domination in graphs. The values of this concept are found for some families of graphs relations with some other domination parameters are presented. Also we define the IED-graph. Necessary and sufficient conditions for any graph to be IED-graph are established.

Mathematics Subject Classification (2000): *05C50.*

Key words: *Independent end equitable domination, Independent equitable dominating graph, IED- graph.*

1. INTRODUCTION

All the graphs considered here are finite and undirected with no loops and multiple edges. As usual $p=|V|$ and $q=|E|$ denote the number of vertices and edges of a graph G, respectively. In general, we use $\langle X\rangle$ to denote the subgraph induced by the set of vertices X and $N(v)$ and $N[v]$ denote the open and closed neighbourhoods of a vertex v, respectively. A vertex $v\in(G)$ is called pendant vertex or end vertex if $\deg(v)=1$.

A set D of vertices in a graph G is a dominating set if every vertex in $V-D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G.

A subset D of $V(G)$ is called an equitable dominating set of a graph G if for every $u\in(V-D)$, there exists a vertex $v\in D$ such that $uv\in E(G)$ and $|deg(u)-deg(v)|\leq 1$. The minimum cardinality of such a dominating set is denoted by $\gamma_e(G)$ and is called equitable domination number of G.

The equitable open neighbourhood of u in G denoted by $N_e(u)$ is defined as $N_e(u)=\{v\in V / v\in N(u), |deg(u)-deg(v)|\leq 1\}$ and $u\in I_e$ if and only if $N_e(u)=\varnothing$. The cardinality of $N_e(u)$ is denoted by $deg_e(u)$. The maximum and minimum equitable degree of a vertex in G are denoted respectively by $\Delta_e(G)$ and $\delta_e(G)$. That is $\Delta_e(G)=max_{u\in V(G)}|N_e(u)|$, $\delta_e(G)=min_{u\in V(G)}|N_e(u)|$. An edge $e=uv$ called equitable edge if $|deg(u)-deg(v)|\leq 1\}$, for more details about equitable domination number see [1] and [5]. Recently in [5] new variant in domination is introduced called end equitable domination in graphs. An equitable dominating set D is said to be an end equitable dominating set of G if D contains all the end vertices of $V(G)$.The minimum cardinality of an end equitable dominating set is called the end equitable domination number of G and is denoted by $\gamma_{ee}(G)$.

For terminology and notations not specifically defined here, we refer reader to [3]. For more details about parameters of domination number, we refer to [3], [4] and [6].

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2. THE INDEPENDENT END EQUITABLE DOMINATION NUMBER OF GRAPHS

Definition 2.1: *An equitable dominating set D of a graph G is called independent equitable dominating set if D is independent set in G that equivalent to the induced subgraph $< D >$ is totally disconnected. The minimum cardinality of an independent equitable dominating set is called the independence equitable domination number of G and denoted by $\gamma_{ie}(G)$.*

Remark 2.2: *The independence equitable domination parameter not defined for any graph. For example the graph $k_{1,n}$, $n \geq 3$ has no independence equitable domination number.*

Remark 2.2, motivated us to make classification to the graph by defining new types of graph as the following definition.

Definition 2.3: *A graph G is called independent equitable dominating graph if the graph G has an independent equitable dominating set. And in short G is called IED-graph.*

Example 2.4: *Let G as in Figure 1. Then clearly G is not IED-graph.*

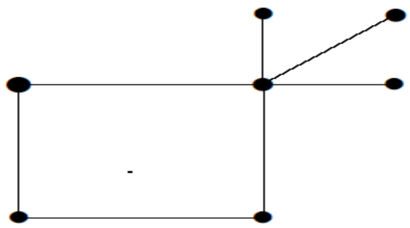

Figure-1: G is not IED-graph

One question will arise here. When the graph G is IED-graph?

We get the answer in the following theorem.

Theorem 2.5: *A graph G is called IED-graph if and only if G has at least one equitable dominating set D such that $V - D$ is covering set in G.*

Proof: Suppose the graph G is IED-graph and let the set $D \subseteq V(G)$ is the independent equitable dominating set. We want to prove that $V - D$ is covering set that means any edge of G contains one of its end vertices in the set $V - D$. Suppose $V - D$ is not covering set in G. Then at least there exists one edge say $e = uv$ all its end vertices u and v are not in $V - D$.Then u and v belong to D that means D is not independent set in G, which is contradiction. Hence, $V - D$ is covering set.

Conversely, suppose that there exist at least one equitable dominating set D where $V - D$ is covering set, we want to prove that G is IED-graph. First D is equitable dominating set remaining only to prove that D is independent equitable dominating set. Suppose that D is not independent set that means there exist at least two vertices in D they are adjacent say uv , that implies that edge uv has no end vertex in $V - D$.

Therefore, $V - D$ is not covering set which is contradiction.

Hence, D is independent equitable dominating set of G.

Proposition 2.6: *Let G be not IED-graph. Then G has at least one edge not equitable edge.*

Proof: Let G be not IED-graph. Then there is no equitable dominating set in G which its compliment is covering set , that means doesn't exists any equitable dominating set D which is independent, in other words any equitable dominating set in G is not independent. Therefore any equitable dominating set contains at least one edge.

If D be any equitable dominating set in G then D has at least one edge say $t = uv$. Now if uv is an equitable edge then $D - \{u\}$ or $D - \{v\}$ is an equitable dominating set and similarly if D has no edge which is not equitable edge we will delete one vertex of the edge till we will get independent equitable dominating set which is contradiction since G is not IED-graph.

Hence, G has at least one edge which is not equitable edge.

The converse of Proposition 2.6 is not always true that means if G has edge which is not equitable edge doesn't mean that the graph G is not IED-graph, see the following example.

Example 2.7: *Let G be as in Figure 2.*

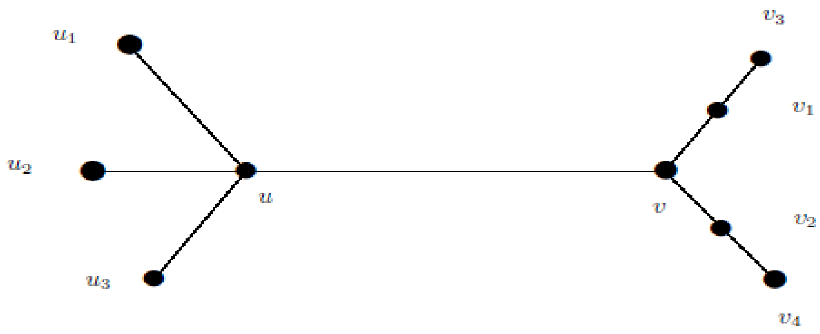

Figure-2: Graph has edge which is not equitable, but G is IED-graph.

From the Figure 2, the edges uu_1, uu_2, uu_3 are not equitable edge but G is IED-graph and the set $\{u_1, u_2, u_3, v, v_3, v_4\}$ is independent equitable dominating set.

Proposition 2.8: *For any k-regular or $(k, k+1)$ bi regular graph G then the graph G is IED-graph.*

Proof: If G is k-regular or $(k, k+1)$ biregular graph then clearly all the edges in G are equitable edge then by Proposition 2.6, we deduce that G is IED-graph.

Converse of Proposition 2.8 is not true.

Definition 2.9: *Let $G = (V, E)$ be a graph. An end equitable dominating set D is called independent end equitable dominating set if D is independent set the minimum cardinality of an independent end equitable dominating set is called independent end equitable domination number and denoted by $\gamma_{iee}(G)$.*

Example 2.10: Let G be a graph as in Figure 3.

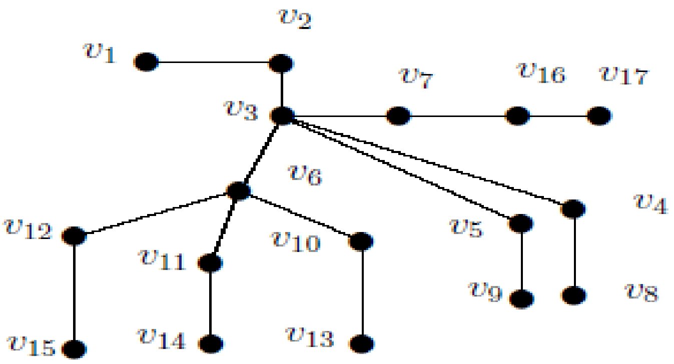

Figure-3: An independent equitable dominating set which contains all the end vertices in G.

In Figure 3, the set $\{v_1, v_8, v_9, v_{13}, v_{14}, v_{15}, v_{17}, v_{16}, v_6\}$ is an independent end equitable dominating set. The set $\{v_1, v_8, v_9, v_{13}, v_{14}, v_{15}, v_3, v_{16}\}$ is independent equitable dominating set. It is easy to see that $\gamma_{ie}(G) = 8$. But $\gamma_{iee}(G) = 9$. From the definition of the independent end equitable dominating set clearly this set is not defined always and by study the definition obviously the independent end equitable dominating set in a graph G is exist if G has an independent equitable dominating set that means the graph is IED-graph.

In other words the independent end equitable dominating set is an independent equitable dominating set which contains all the end vertices in$G$.

Proposition 2.11: *Let G be IED-graph. Then*

(i) $\gamma_{ie}(G) \leq \gamma_{iee}(G)$.
(ii) $\gamma_{ee}(G) \leq \gamma_{iee}(G)$.

Proof:
(i) Let G be IED-graph and let D be the minimum independent end equitable dominating set in G. $|D| = \gamma_{iee}(G)$. From the definition it is obviously D is also independent equitable dominating set in G.
Hence, $\gamma_{ie}(G) \leq |D| = \gamma_{iee}(G)$.

(ii) Similarly if S is any minimum independent end equitable dominating set then $\gamma_{iee}(G) = |S|$ and S also is an equitable end dominating set.
Hence, $\gamma_{ee}(G) \leq |S| = \gamma_{iee}(G)$.

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