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UNSTEADY NATURAL CONVECTION MHD FLOW THROUGH POROUS MEDIA PAST AN ACCELERATED VERTICAL PLATE IN A THERMALLY STRATIFIED FLUID

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ABSTRACT

Unsteady natural convection MHD flow through porous media past an accelerated vertical plate in a thermally stratified fluid is studied here. The fluid considered is gray, absorbing-emitting radiation but a non-scattering medium. The governing equations involved in the present analysis are solved by the Laplace-transform technique. The velocity, skin friction and Nusselt number are studied for different parameters like Prandtl number, thermal Grashof number, magnetic field parameter, permeability parameter, stratification parameter and time.

Key Words: *Unsteady flow, MHD, Stratified fluid, Porous media.*

MS Classification 2000: *76R99, 76W05, 76S05.*

INTRODUCTION

Study of MHD flow with heat and mass transfer plays an important role in chemical, mechanical and biological Sciences. Some important applications are cooling of nuclear reactors, liquid metals fluid, power generation system and aero dynamics. The response of laminar skin friction and heat transfer to fluctuations in the stream velocity was studied by Lighthill [6]. Free convection effects on the oscillating flow past an infinite vertical porous plate with constant suction - I, was studied by Soundalgekar [11] which was further improved by Vajravelu *et al.* [13]. Further researches in these areas were done by Gupta *et al.* [1], Jaiswal *et al.* [4] and Soundalgekar *et al.* [12] by taking different models. Some effects like radiation and mass transfer on MHD flow were studied by Muthucumaraswamy *et al.* [7-8] and Prasad *et al.* [9]. Radiation effects on mixed convection along a vertical plate with uniform surface temperature were studied by Hossain and Takhar [3]. Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux was studied by Jha, Prasad and Rai [5].

On the other hand, Radiation and free convection flow past a moving plate were considered by Raptis and Perdikis [10]. We are considering radiation effects on MHD flow through porous media past an impulsively started vertical oscillating plate with variable mass diffusion. The results are shown with the help of graphs (Fig-1 to Fig-7) and table-1.

MATHEMATICAL ANALYSIS

In this paper we have considered the flow of unsteady viscous incompressible fluid. The x-axis is taken along the plate in the upward direction and y - axis is taken normal to the plate. Initially the fluid and plate are at the same temperature. A transverse magnetic field B_0, of uniform strength is applied normal to the plate. The viscous dissipation and induced magnetic field has been neglected due to its small effect. Initially, the fluid and plate are at the same temperature T_∞ in the stationary condition. At time $t^{'} > 0$, the temperature of the plate is raised to T_w and the concentration level near the plate is raised linearly with respect to time.

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The flow modal is as under:

$$\frac{\partial u'}{\partial t'} = g\beta\left(T - T_\infty\right) + \upsilon\frac{\partial^2 u'}{\partial x^2} - \frac{\sigma B_0^2 u'}{\rho} - \upsilon\frac{u'}{K_p} \tag{1}$$

$$\frac{\partial T}{\partial t'} = -\gamma u' + \alpha\frac{\partial^2 T}{\partial x^2} \tag{2}$$

where $\gamma = \frac{dT_\infty}{dz} + \frac{g}{C_p}$.

Here $\frac{dT_\infty}{dz}$ is thermal stratification and $\frac{g}{C_p}$ is pressure work term. The environment is statically stable, neutral or unstable if $\gamma >, \; = \text{ or } < 0$. We consider the cases of stable and neutral conditions only.

The following boundary conditions have been assumed:

$$\begin{aligned} &t' \le 0: \quad u' = 0, \quad T = T_\infty \quad \text{for all the values of } x \\ &t' > 0: \quad u' = At', \quad T = T_w \quad \text{at} \quad x = 0 \\ &\qquad\quad u' \to 0, \quad T = T_\infty \quad \text{as} \quad x \to 0 \end{aligned} \tag{3}$$

where $A(>0)$ is the constant acceleration,\where the symbols are**:** C_p - specific heat at constant pressure, T - temperature of the fluid near the plate, T_∞ - temperature of the mainstream fluid, t' - time, ρ - density, g - acceleration due to gravity, α - thermal diffusivity, β - volumetric coefficient of thermal expansion, υ - kinematic viscosity, B_0 - external magnetic field, σ - Stefan– Boltzmann constant, γ - thermal stratification parameter, u' - velocity of the fluid in the z - direction and K_p - permeability of porous medium.

Introducing the following non- dimensional quantities**:**

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},\; t = At',\; G_r = \frac{g\beta\left(T_w - T_\infty\right)}{A},\; \xi = \left(\frac{A}{\upsilon}\right)^{\frac{1}{2}} x$$

$$M = \frac{\sigma B_0^2}{\rho A},\; P_r = \frac{\mu C_p}{k},\; S = \frac{\gamma}{A\left(T_w - T_\infty\right)},\; K = \frac{k_p A}{\upsilon}, \mu = \rho\upsilon \tag{4}$$

where u -dimensionless velocity, θ -dimensionless temperature, G_r –Grashof number, ξ -dimensionless coordinate normal to the plate, P_r - Prandtl number, t -dimensionless time, M - magnetic field parameter, k - thermal conductivity of the fluid, K - permeability parameter of porous medium and S - non-dimensional stratification parameter.

Equations (1) and (2) leads to

$$\frac{\partial u}{\partial t} = G_r\theta - \left(M + \frac{1}{K}\right)u + \frac{\partial^2 u}{\partial \xi^2} \tag{5}$$

$$\frac{\partial \theta}{\partial t} = -Su + \frac{1}{P_r}\frac{\partial^2 \theta}{\partial \xi^2} \tag{6}$$

and the boundary conditions (3) reduces to:

$$\left.\begin{aligned} &t \le 0: \quad u = 0, \quad \theta = 0 \quad \text{for all the values of } \xi \\ &t > 0: \quad u = t, \quad \theta = 1 \quad \text{at} \quad \xi = 0 \\ &\qquad\quad u \to 0, \quad \theta \to 0 \quad \text{as} \quad \xi \to \infty \end{aligned}\right\} \tag{7}$$

METHOD OF SOLUTIONS

Using the Laplace transform technique, equations (5) and (6) subject to (7) is transformed to, taking $P_r = 1$, $V =$ Laplace transform of u and $\varphi =$ Laplace transform of θ

$$\left[D^4 - 2\left(s + M^*\right)D^2 + \left(s + M^*\right)^2 + H^2\right]\varphi = 0 \tag{8}$$

where $M^* = \frac{1}{2}\left(M + \frac{1}{K}\right)$ and $H^2 = SG_r - M^{*2}$.

Therefore solution is given by

$$\varphi = \frac{1}{2}\left[\frac{e^{-\xi\sqrt{s+M^*+iH}}}{s} + \frac{e^{-\xi\sqrt{s+M^*-iH}}}{s}\right] + \frac{iS}{2H}\left[\frac{e^{-\xi\sqrt{s+M^*-iH}}}{s^2} - \frac{e^{-\xi\sqrt{s+M^*+iH}}}{s^2}\right] . \tag{9}$$

and $$V = \frac{1}{2}\left[\frac{e^{-\xi\sqrt{s+M^*+iH}}}{s^2} + \frac{e^{-\xi\sqrt{s+M^*-iH}}}{s^2}\right] + \frac{iH}{2S}\left[\frac{e^{-\xi\sqrt{s+M^*+iH}}}{s} - \frac{e^{-\xi\sqrt{s+M^*-iH}}}{s}\right] \tag{10}$$

Using Hetnarski's algorithm [2] for inverse Laplace Transform of φ and V, therefore solutions for φ and V are given as:

$$\begin{aligned}\theta = &\frac{1}{4}\left[e^{-\xi\sqrt{M^*+iH}} erfc\left(\frac{\xi}{2\sqrt{t}} - \sqrt{\left(M^* + iH\right)t}\right)\left\{1 - \frac{iSt}{H} - \frac{iS\xi}{2H\sqrt{M^* + iH}}\right\}\right] \\ &+ \frac{1}{4}\left[e^{\xi\sqrt{M^*+iH}} erfc\left(\frac{\xi}{2\sqrt{t}} + \sqrt{\left(M^* + iH\right)t}\right)\left\{1 - \frac{iSt}{H} + \frac{iS\xi}{2H\sqrt{M^* + iH}}\right\}\right] + cc\end{aligned} \tag{11}$$

$$\begin{aligned}u = &\frac{1}{4}\left[e^{-\xi\sqrt{M^*+iH}} erfc\left(\frac{\xi}{2\sqrt{t}} - \sqrt{\left(M^* + iH\right)t}\right)\left\{t + \frac{iH}{S} - \frac{\xi}{2\sqrt{M^* + iH}}\right\}\right] \\ &+ \frac{1}{4}\left[e^{\xi\sqrt{M^*+iH}} erfc\left(\frac{\xi}{2\sqrt{t}} + \sqrt{\left(M^* + iH\right)t}\right)\left\{t + \frac{iH}{S} + \frac{\xi}{2\sqrt{M^* + iH}}\right\}\right] + cc\end{aligned} \tag{12}$$

Where cc is the complex conjugate and $M^* + iH = A$ and $M^* - iH = B$.

SKIN FRICTION

Skin friction is given by

$$\tau = -\left(\frac{\partial u}{\partial \xi}\right)_{\xi=0} \tag{13}$$

$$\begin{aligned}\tau = &\frac{e^{-M^* t}}{\sqrt{\pi t}}\left(tCos(Ht) + \frac{H}{S}Sin(Ht)\right) \\ &+ \frac{1}{2}\sqrt{M^* + iH}\left(t + \frac{iH}{S} + \frac{1}{2(M^* + iH)}\right) erf\left(\sqrt{(M^* + iH)t}\right) + cc\end{aligned} \tag{14}$$

NUSSELT NUMBER

Nusselt number is given by

$$N_u = -\left(\frac{\partial \theta}{\partial \xi}\right)_{\xi=0} \tag{15}$$

$$N_u = \frac{e^{-M^* t}}{\sqrt{\pi t}}\left(Cos(Ht) - \frac{St}{H} Sin(Ht) \right) + \frac{1}{2}\sqrt{M^* + iH}\left\{ 1 - \frac{iSt}{H} + \frac{iS}{2H(M^* + iH)} \right\} erf\left(\sqrt{(M^* + iH)t} \right) + cc \tag{16}$$

RESULT AND DISCUSSION

Figures 1 and 2 represent the temperature profile at time t = 0.4. Figure -1 shows that when the value of S is increased the temperature decreases. Similarly in Figure - 2, it is observed that when the value of magnetic field increases the temperature of the plate decreases. Other parameters are kept constant in both figures. Figure-3 represents the temperature profile for different values of time parameter. It shows that the value of time t is increased the temperature decreases.

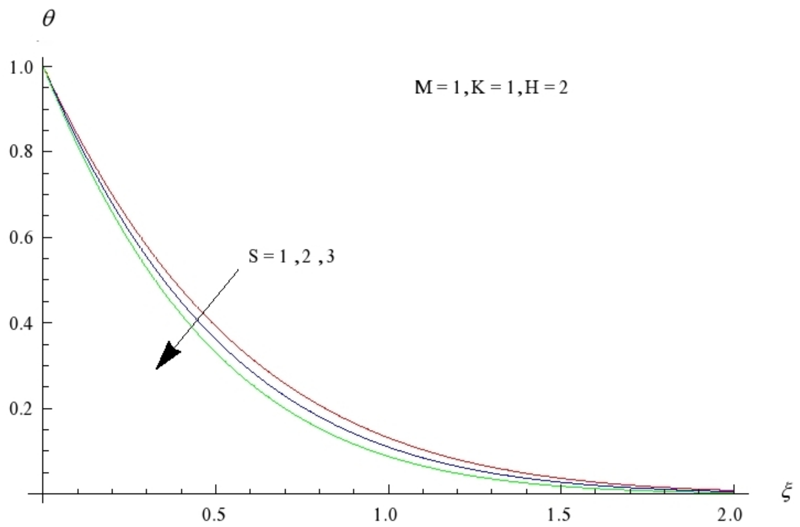

Figure-1: Temperature Profile at $t = 0.4$

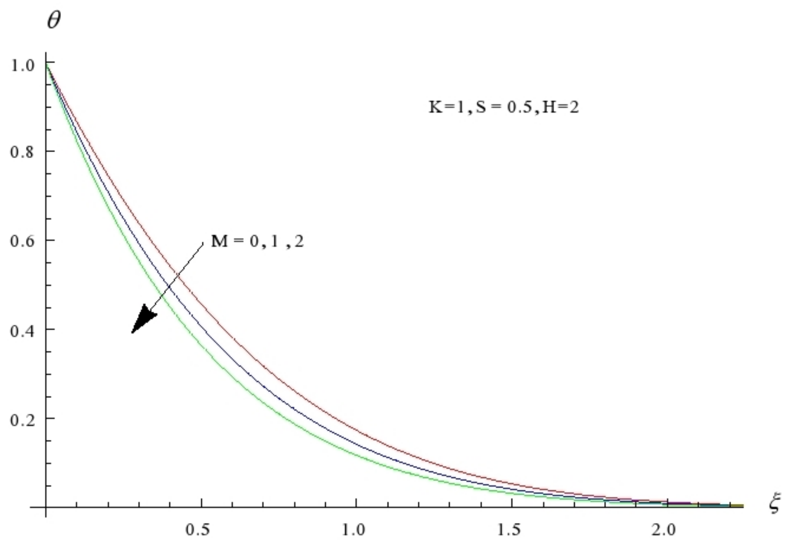

Figure-2: Temperature Profile at $t = 0.4$

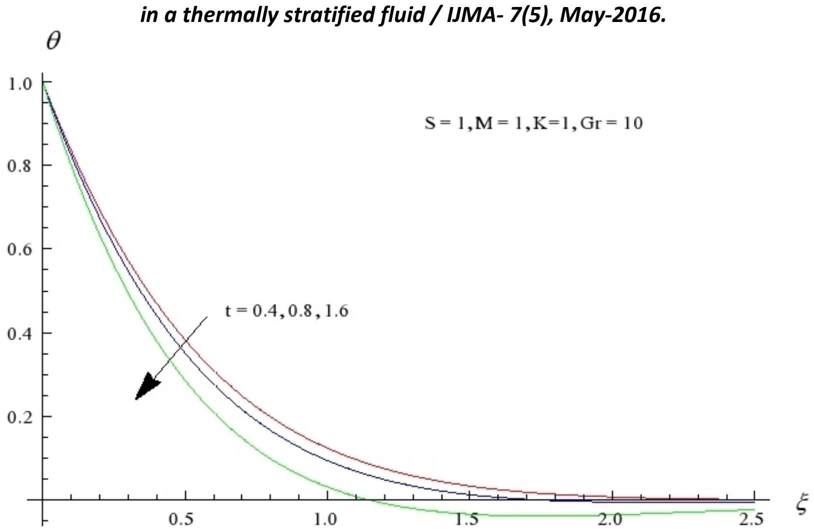

Figure-3: Temperature Profile for different values of t

The velocity profile for different parameters like magnetic field parameter, permeability parameter, thermal stratification parameter and time are shown in figure - 4 to figure - 7. Figure – 4 shows that when the value of S increased the velocity decreases (Keeping magnetic field, thermal Grashof number and permeability parameter constant). But in figure- 5 shows that, keeping the value of K and S constant and increased the velocity of magnetic field the velocity decreases at $t = 0.4$. In figure -6, it is cleared from figure that when permeability parameter K is increased the velocity increases. In figure -7, it is observed that when time t is increased the velocity increases.

Table (1) represents the values of skin friction and Nusselt number.

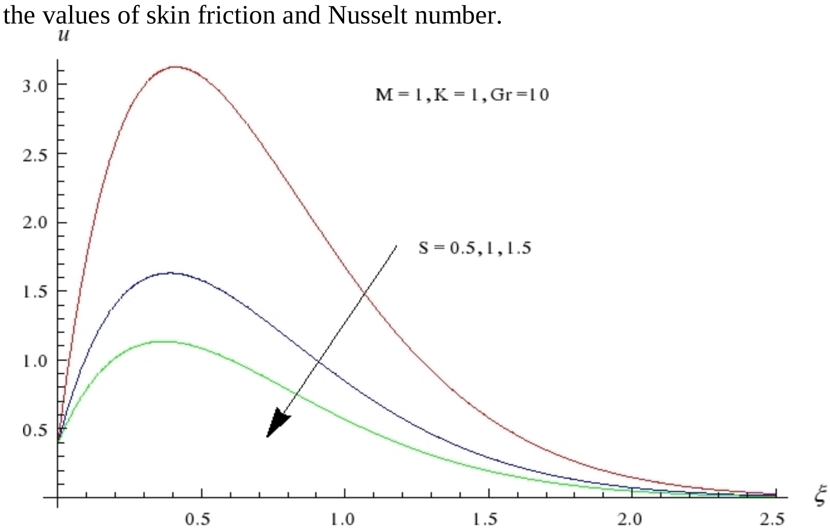

Figure-4: Velocity Profile at $t = 0.4$

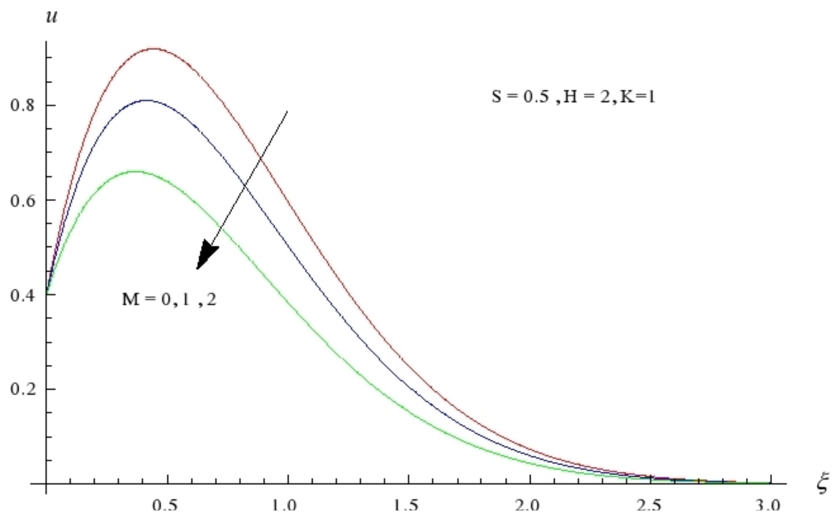

Figure-5: Velocity Profile at $t = 0.4$

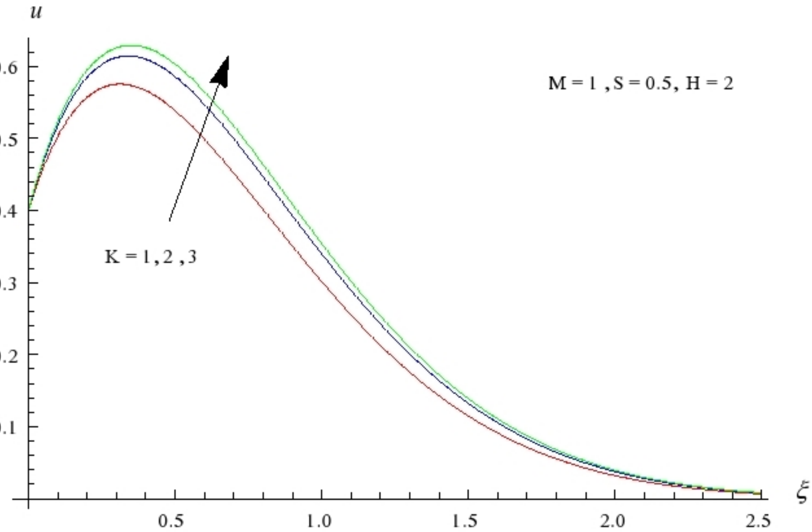

Figure-6: Velocity Profile at $t = 0.4$

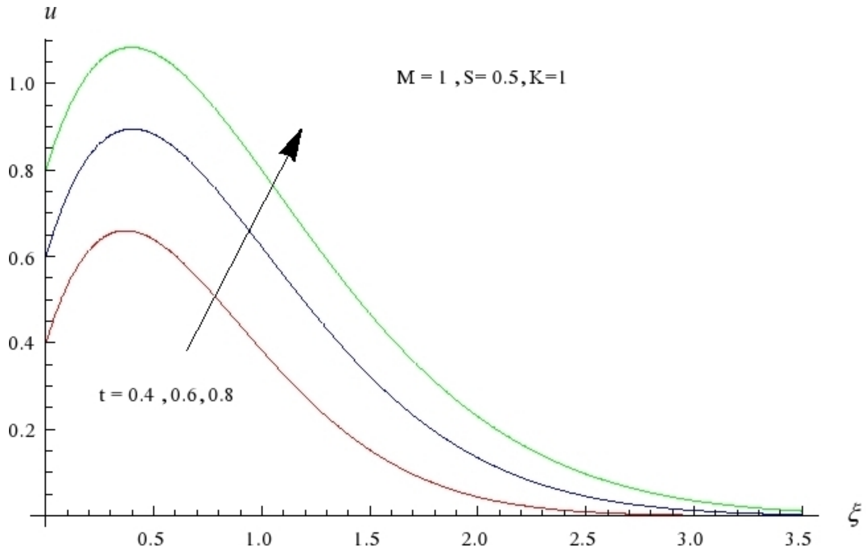

Figure-7: Velocity Profile for values of t

Table-1: Skin friction and Nusselt number for different parameters

M	K	G_r	S	t	τ	N_u
0	1	10	0.5	0.4	-2.7428	1.09968
1	1	10	0.5	0.4	-232712	1.24792
1	1	10	1	0.4	-2.3165	1.42555
2	1	10	1	0.4	-1.88472	1.54337
2	1	5	1	0.4	-0.41729	1.46015
2	1	5	0.5	0.4	-0.12692	1.33586
1	1	5	0.5	0.4	-0.72012	1.98460
1	1	5	0.5	0.8	-0.74336	1.20093
1	2	10	0.5	0.4	-0.25318	1.74730
1	4	10	0.5	0.4	-0.30360	1.70896

The values of skin friction and the Nusselt number are tabulated in table-1. When the values of M and S are increased (keeping other parameters constant) the values of skin friction and the Nusselt number are also increased. But if values of G_r and t are increased the values of skin friction and Nusselt number get decreased (keeping other parameters constant).

CONCLUSION

In this section a theoretical analysis has been done to study the unsteady natural convection MHD flow through porous media past an accelerated vertical plate in a thermally stratified fluid**.** Solutions for the model have been derived by using Laplace – transform techniques. Some conclusions of the study are as below:

1- Velocity increases with the increase in S, G_r, K and t and decreases with increase in M.
2- Temperature of the fluid decreases when M and S are increased.
3- Skin friction increases when magnetic field parameter, stratification parameter, permeability parameter and time are increased but decreases when thermal Grashof number is increased.
4- Nusselt number increases when magnetic field parameter, thermal Grashof number, permeability parameter, stratification parameter and time are increased.

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