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ON ψg*- OPEN MAPS AND ψg*- HOMEOMORPHISMS IN TOPOLOGICAL SPACES

K. BALA DEEPA ARASI*[1], G. SUGANYA[2]

[1]Assistant Professor of Mathematics,
A. P. C. Mahalaxmi College for Women, Thoothukudi, (T.N.), India.

[2] M. Phil Scholar, A. P. C. Mahalaxmi College for Women, Thoothukudi, (T.N.), India.

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ABSTRACT

In this paper, we define new class of functions namely ψg – open maps and we prove some of their basic properties. Also, we introduce a new class of ψg* – homeomorphisms and we prove some of their relationship among other homeomorphisms. Throughout this paper f: (X, τ) → (Y, σ) is a function from a topological space (X, τ) to a topological space (Y, σ).*

Keywords: *closed set, ψg* - closed sets, ψg* - continuous functions, ψg* - irresolute functions, ψg* - open maps, ψg* - closed maps and ψg* - homeomorphisms.*

1. INTRODUCTION

N.Levine [14] introduced the concept of generalized closed sets and studied their properties in 1970. By considering the concept of g-closed sets many concepts of topology have been generalized and interesting results have been obtained by several mathematician. Veerakumar [28] introduced and studied ψ-closed sets. Veerakumar [27] introduced g*-closed sets in topological spaces and studied their properties. We introduced ψg* -closed sets [3] and studied their properties in 2015. K.Balachandran *et al.* [26] introduced the concept of generalized continuous maps in Topological spaces. We introduced ψg*-continuous maps [4] in topological spaces and studied their properties.

Now, we introduce a new version of maps ψg* – open maps and ψg* – homeomorphisms. And, also we prove some properties of these functions and establish the relationships between ψg* – homeomorphisms and other homeomorphisms.

2. PRELIMINARIES

Throughout this paper (X, τ) (or simply X) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X, τ), Cl(A), Int(A) and A^c denote the closure of A, interior of A and the complement of A respectively. We are giving some definitions.

Definition 2.1: A subset A of a topological space (X,τ) is called

1. a semi-open set[15] if $A \subseteq Cl(Int(A))$.
2. an α-open set[19] if $A \subseteq Int(Cl(Int(A)))$.
3. a regular open set[25] if $A = Int(Cl(A))$.
4. a semi pre-open set[1] if $A \subseteq Cl(Int(Cl(A)))$.

The complement of a semi–open (resp.α–open, regular–open, semi pre-open) set is called semi-closed (resp.α–closed, regular–closed, semi pre-closed) set.

Corresponding Author: K. Bala Deepa Arasi[1],*
[1]Assistant Professor of Mathematics,
A. P. C. Mahalaxmi College for Women, Thoothukudi, (T.N.), India.

The intersection of all semi-closed (resp.α-closed, regular-closed, semi pre-closed) sets of X containing A is called the semi-closure (resp.α-closure, regular-closure, semi pre-closure) of A and is denoted by sCl(A) (resp.αCl(A), rCl(A), spCl(A)). The family of all semi-open (resp. α-open, regular-open, semi pre-open) subsets of a space X is denoted by SO(X) (resp. αO(X), rO(X), spO(X)).

Definition 2.2: A subset A of a topological space (X, τ) is called

1) a generalized closed set (briefly g-closed)[14] if Cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
2) a sg-closed set[6] if sCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in X.
3) a gs-closed set[2] if sCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
4) a αg-closed set[16] if αCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
5) a gr*-closed set[12] if rCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in X.
6) a g*-closed set[27] if Cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open in X.
7) a g**-closed set[20] if Cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g*-open in X.
8) a g*s-closed set[22] if sCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is gs-open in X.
9) a (gs)*-closed set[10] if Cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is gs-open in X.
10) a gsp-closed set[9] if spCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
11) a ψ-closed set[28] if sCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is sg-open in X.
12) a ψg-closed set [23] if ψCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
13) a ψg*-closed set [3] if ψCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g*-open in X.

The complement of a g-closed (resp. sg-closed, gs-closed, αg-closed, gr*–closed, g*-closed, g**-closed, g*s-closed, (gs)*-closed, gsp-closed, ψ-closed, ψg-closed and ψg*-closed) set is called g-open (resp. sg-open, gs-open, αg-open, gr*–open, g*-open, g**-open, g*s-open, (gs)*-open, gsp-open, ψ-open, ψg-open and ψg*-open) set.

Definition 2.3: ψCl(A) is defined as the intersection of all ψ-closed sets containing A.

Definition 2.4: A function f: (X, τ) → (Y, σ) is called a

1. continuous [29] if f^{-1}(V) is closed in X for every closed set V in Y.
2. semi-continuous [15] if f^{-1}(V) is semi-closed in X for every closed set V in Y.
3. α-continuous [7] if f^{-1}(V) is α-closed in X for every closed set V in Y.
4. regular continuous[18] if f^{-1}(V) is regular closed in X for every closed set V in Y.
5. g-continuous [26] if f^{-1}(V) is g-closed in X for every closed set V in Y.
6. αg-continuous [11] if f^{-1}(V) is αg-closed in X for every closed set V in Y.
7. gr*-continuous[13] if f^{-1}(V) is gr*-closed in X for every closed set V in Y.
8. g*-continuous[27] if f^{-1}(V) is g*-closed in X for every closed set V in Y.
9. g**-continuous[20] if f^{-1}(V) is g**-closed in X for every closed set V in Y.
10. g*s-continuous [21] if f^{-1}(V) is g*s-closed in X for every closed set V in Y.
11. (gs)*-continuous [10] if f^{-1}(V) is (gs)*-closed in X for every closed set V in Y.
12. gsp-continuous [9] if f^{-1}(V) is gsp-closed in X for every closed set V in Y.
13. ψg-continuous [24] if f^{-1}(V) is ψg-closed in X for every closed set V in Y.
14. ψg*-continuous[4] if f^{-1}(V) is ψg*-closed in X for every closed set V in Y.

Definition 2.5: A function f: (X, τ) → (Y, σ) is called a

1. open map[29] if f(V) is open in (Y, σ) for every open set V in (X, τ).
2. Semi-open map [5] if f(V) is semi-open in (Y, σ) for every open set V in (X, τ).
3. α-open map[7] if f(V) is α-open in (Y, σ) for every open set V in (X,τ).
4. regular open map[18] if f(V) is regular open in (Y,σ) for every open set V in (X, τ).
5. g-open map[26] if f(V) is g-open in (Y, σ) for every open set V in (X, τ).
6. αg-open map[11] if f(V) is αg-open in (Y, σ) for every open set V in (X, τ).
7. gr*-open map[13] if f(V) is gr*-open in (Y, σ) for every open set V in (X, τ).
8. g*-open map[27] if f(V) is g*-open in (Y, σ) for every open set V in (X, τ).
9. g**-open map[20] if f(V) is g**-open in (Y, σ) for every open set V in (X, τ).
10. g*s-open map[21] if f(V) is g*s-open in (Y, σ) for every open set V in (X, τ).
11. (gs)*-open map [10] if f(V) is (gs)*-open in (Y, σ) for every open set V in (X, τ).
12. ψg-open map[24] if f(V) is ψg-open in (Y,σ) for every open set V in (X, τ).

Definition 2.6: A bijection f: (X, τ) → (Y, σ) is called a

1. homeomorphism[29] if f is both continuous map and open map
2. semi-homeomorphism[5] if f is both semi-continuous map and semi-open map
3. α-homeomorphism[8] if f is both α-continuous map and α-open map
4. regular-homeomorphism[18] if f is both regular continuous map and regular open map

5. g-homeomorphism [17] if f is both g-continuous map and g-open map
6. gr*-homeomorphism [13] if f is both gr*-continuous map and gr*-open map.
7. g*-homeomorphism [27] if f is both g*-continuous map and g*-open map.
8. g*s-homeomorphism [21] if f is both g*s-continuous map and g*s-open map.
9. (gs)*-homeomorphism [10] if f is both (gs)*-continuous map and (gs)*-open map.

Definition 2.7: A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is said to be g*- irresolute function [27] if the inverse image of every g*-closed set in (Y, σ) is g*- closed set in (X, τ).

Remark 2.8: The family of all ψg* – open subsets of a space X is denoted by ψg*-O(X). The family of all ψg* – closed subsets of a space X is denoted by ψg*-C(X).

Definition 2.9: A Space (X, τ) is called a
a. $T_{\psi g^*}$-space [3] if every ψg*-closed set in it is closed.
b. ${}_gT_{\psi g^*}$-space [3] if every ψg*-closed set in it is g-closed.

3. ψg*– OPEN MAPS AND ψg*– CLOSED MAPS

We introduce the following definitions.

Definition 3.1: Let X and Y be two topological spaces. A map f: $(X, \tau) \rightarrow (Y, \sigma)$ is called ψg* – open map if for each ope n set V of X, f(V) is ψg* – open set in Y.

Definition 3.2: Let X and Y be two topological sapces. A map f: $(X, \tau) \rightarrow (Y, \sigma)$ is called ψg* – closed map if for each closed set V of X, f(V) is ψg* – closed set in Y.

Example 3.3: Let X = Y = {a, b, c}
$\tau = \{X, \phi, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{b\}\}$

Define a map f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = b, f(b) = a, f(c) = c.

Then f is ψg* – open map, since the image of a open set {a, c} in (X, τ) is {b, c} which is ψg*-open set in (Y, σ).

Proposition 3.4:
a. Every open map is ψg*-open map.
b. Every semi-open map is ψg*- open map.
c. Every α-open map is ψg*- open map.
d. Every regular open map is ψg*- open map.
e. Every g-open map is ψg*- open map.
f. Every αg-open map is ψg*- open map.
g. Every gr*-open map is ψg*- open map.
h. Every g*-open map is ψg*- open map.
i. Every g**-open map is ψg*- open map.
j. Every g*s-open map is ψg*- open map.
k. Every (gs)*-open map is ψg*- open map.

Proof:
a. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an open map and V be an open set in X. Since f is an open map, f(V) is an open set in Y. By Proposition 3.4 in [3], f(V) is a ψg*-open set in (Y, σ).Therefore, f is ψg*-open map.
b. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an semi-open map and V be an open set in X. Since f is an semi-open map, f(V) is an semi-open set in Y. By Proposition 3.6 in [3], f(V) is a ψg*-open set in (Y, σ).Therefore, f is ψg*-open map.
c. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an α-open map and V be an open set in X. Since f is an α-open map, f(V) is an α-open set in Y. By Proposition 3.8 in [3], f(V) is a ψg*-open set in (Y, σ). Therefore, f is ψg*-open map.
d. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an regular open map and V be an open set in X. Since f is an regular open map, f(V) is an regular open set in Y. By Proposition 3.10 in [3], f(V) is a ψg*-open set in (Y, σ). Therefore, f is ψg*-open map.
e. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an g-open map and V be an open set in X. Since f is an g-open map, f(V) is an g-open set in Y. By Proposition 3.12 in [3], f(V) is a ψg*-open set in (Y, σ). Therefore, f is ψg*-open map.
f. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an αg-open map and V be an open set in X. Since f is an αg-open map, f(V) is an αg-open set in Y. By Proposition 3.14 in [3], f(V) is a ψg*-open set in (Y, σ). Therefore, f is ψg*-open map.
g. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an gr*-open map and V be an open set in X. Since f is an gr*-open map, f(V) is an gr*-open set in Y. By Proposition 3.16 in [3], f(V) is a ψg*-open set in (Y, σ). Therefore, f is ψg*-open map.
h. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an g*-open map and V be an open set in X. Since f is an g*-open map, f(V) is an g*-open set in Y. By Proposition 3.18 in [3], f(V) is a ψg*-open set in (Y, σ). Therefore, f is ψg*-open map.

i. Let f: (X, τ) → (Y, σ) be an g**-open map and V be an open set in X. Since f is an g**-open map, f(V) is an g**-open set in Y. By Proposition 3.20 in [3], f(V) is a ψg*-open set in (Y, σ). Therefore, f is ψg*-open map.
j. Let f: (X, τ) → (Y, σ) be an g*s-open map and V be an open set in X. Since f is an g*s-open map, f(V) is an g*s-open set in Y. By Proposition 3.22 in [3], f(V) is a ψg*-open set in (Y, σ). Therefore, f is ψg*-open map.
k. Let f: (X, τ) → (Y, σ) be an (gs)*-open map and V be an open set in X. Since f is an (gs)*-open map, f(V) is an (gs)*-open set in Y. By Proposition 3.24 in [3], f(V) is a ψg*-open set in (Y, σ). Therefore, f is ψg*-open map.

The following examples show that the converse of the above proposition need not be true.

Example 3.5:

a. Let X = Y = {a, b, c},
τ = {X, ϕ, {a}} and σ = {Y, ϕ, {c}, {a, b}}.
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
O(X) = {X, ϕ, {a}}
O(Y) = {Y, ϕ, {c}, {a, b}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}
Since the image of an open set {a} in (X, τ) is {a} which is ψg*-open set but not open set in (Y, σ), f is ψg*-open map but not open map.
b. Let X = Y = {a, b, c},
τ = {X, ϕ, {a}} and σ = {Y, ϕ, {c}, {a, b}}.
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
O(X) = {X, ϕ, {a}}
Semi-O(Y) = {Y, ϕ, {c}, {a, b}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}
Since the image of an open set {a} in (X, τ) is {a} which is ψg*-open set but not semi-open set in (Y, σ), f is ψg*-open map but not semi-open map.
c. Let X = Y = {a, b, c},
τ = {X, ϕ, {a}, {b}, {a, b}} and σ = {Y, ϕ, {b}, {a, c}}.
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
O(X) = {X, ϕ, {a},{b}, {a, b}}
α-O(Y) = {Y, ϕ, {b}, {a, c}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}
Since the image of an open set {a},{a, b} in (X, τ) are {a},{a, b} which is ψg*-open set but not α-open set in (Y, σ), f is ψg*-open map but not α-open map.
d. Let X = Y = {a, b, c},
τ = {X, ϕ, {b}, {a, c}} and σ = {Y, ϕ, {c}, {b, c}}.
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
O(X) = {X, ϕ, {b}, {a, c}}
regular-O(Y) = {Y, ϕ}
ψg*-O(Y) = {Y, ϕ, {b}, {c}, {b, c}, {a, c}}
Since the image of an open set {b},{a, c} in (X, τ) are {b},{a, c} which is ψg*-open set but not regular-open set in (Y, σ), f is ψg*-open map but not regular-open map.
e. Let X = Y = {a, b, c},
τ = {X, ϕ, {b}, {a, c} and σ = {Y, ϕ, {c}, {b, c}}.
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
O(X) = {X, ϕ, {b}, {a, c}}
g-O(Y) = {Y, ϕ, {b}, {c}, {b, c}}
ψg*-O(Y) = {Y, ϕ, {b}, {c}, {b, c}, {a, c}}
Since the image of an open set {a, c} in (X, τ) is {a, c} which is ψg*-open set but not g-open set in (Y, σ), f is ψg*-open map but not g-open map.
f. Let X = Y = {a, b, c},
τ = {X, ϕ, {b}, {a, c}} and σ = {Y, ϕ, {a}, {b}, {a, b}}.
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
O(X) = {X, ϕ, {b}, {a, c}}
αg-O(Y) = {Y, ϕ, {a}, {b}, {a, b}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {a, b}, {b, c}, {a, c}}
Since the image of an open set {a, c} in (X, τ) is {a, c} which is ψg*-open set but not αg-open set in (Y, σ), f is ψg*-open map but not αg-open map.
g. Let X = Y = {a, b, c},
τ = {X, ϕ, {b}, {a, c}} and σ = {Y, ϕ, {c}, {b, c}}.
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
O(X) = {X, ϕ, {b},{a, c}}
gr*-O(Y) = {Y, ϕ, {b}, {c}, {b, c}}

ψg*-O(Y) = {Y, ϕ, {b}, {c}, {b, c}, {a, c}}
Since the image of an open set {a, c} in (X, τ) is {a, c} which is ψg*-open set but not gr*-open set in (Y, σ), f is ψg*-open map but not gr*-open map.

h. Let X = Y = {a, b, c},
τ = {X, ϕ, {b}, {a, b}} and σ = {Y, ϕ, {a}}.
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
O(X) = {X, ϕ, {b},{a, b}}
g*-O(Y) = {Y, ϕ, {a}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {c}, {a, b}, {a, c}}
Since the image of an open set {b},{a, b} in (X, τ) are {b},{a, b} which is ψg*-open set but not g*-open set in (Y, σ), f is ψg*-open map but not g*-open map.

i. Let X = Y = {a, b, c},
τ = {X, ϕ, {a}, {b, c}} and σ = {Y, ϕ, {b}, {a, b}}.
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
O(X) = {X, ϕ, {a}, {b, c}}
g**-O(Y) = {Y, ϕ, {a}, {b}, {a, b}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {a, b}, {b, c}}
Since the image of an open set {b, c} in (X, τ) is {b, c} which is ψg*-open set but not g**-open set in (Y, σ), f is ψg*-open map but not g**-open map.

j. Let X = Y = {a, b, c},
τ = {X, ϕ, {a}} and σ = {Y, ϕ, {b}, {a, b}}.
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
O(X) = {X, ϕ, {a}}
g*s-O(Y) = {Y, ϕ, {b}, {a, b}, {b, c}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {a, b}, {b, c}}
Since the image of an open set {a} in (X, τ) is {a} which is ψg*-open set but not g*s-open set in (Y, σ), f is ψg*-open map but not g*s-open map.

k. Let X = Y = {a, b, c},
τ = {X, ϕ, {a}} and σ = {Y, ϕ, {b}, {a, b}}.
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c.
O(X) = {X, ϕ, {a}}
(gs)*-O(Y) = {Y, ϕ, {b}, {a, b}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {a, b}, {b, c}}
Since the image of an open set {a} in (X, τ) is {a} which is ψg*-open set but not (gs)*-open set in (Y, σ), f is ψg*-open map but not (gs)*-open map.

Proposition 3.6: Every ψg*-open map is ψg-open map.

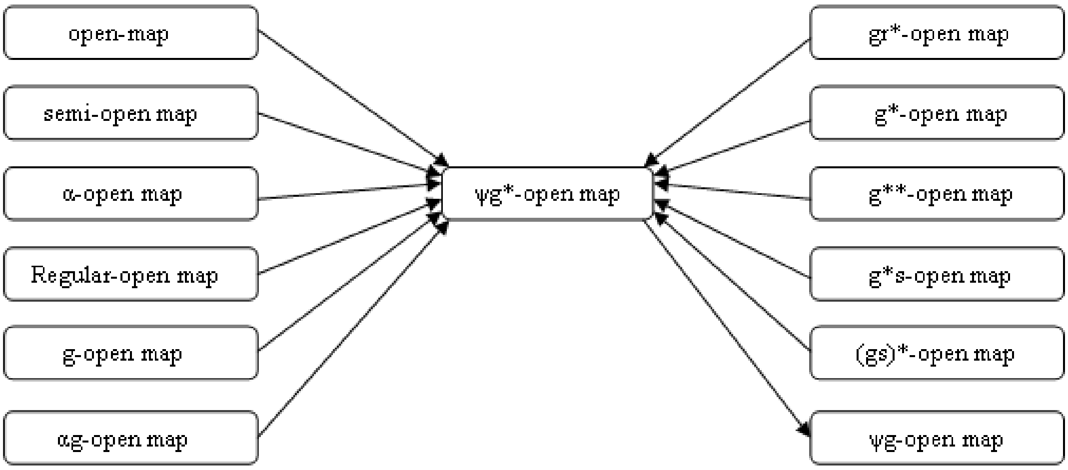
Proof: Let f: (X, τ) → (Y, σ) be a ψg*-open map and V be an open set in X. Since f is ψg*-open map, f(V) is ψg*-open set in Y. By Proposition 3.28 in [3], f(V) is ψg-open set in (Y, σ). Therefore, f is ψg-open map.

Example 3.7: Let X = Y = {a, b, c, d}
τ = {X, ϕ, {c}, {c, d},{b, c, d}} and σ = {Y, ϕ, {d}, {a, b}, {a, b, d}}.

Define a map f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c, f(d) = d.
O(X) = {X, ϕ, {c}, {c, d}, {b, c, d}}
ψg-O(Y) = {Y, ϕ, {a}, {b}, {d}, {a, b}, {a, d}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {d}, {a, b}, {a, d}, {b, d}, {c, d}, {a, b, c}, {a, b, d}}

Since the image of an open set {c, d},{b, c, d} in (X, τ) is {c, d},{b, c, d} which is ψg-open set but not ψg*-open set in (Y, σ), f is ψg-open map but not ψg*-open map.

Remark 3.8: The following diagram shows the relationships of ψg*-continuous functions with other known existing functions. A → B represents A implies B but not conversely.



4. ψg* – HOMEOMORPHISM

We introduce the following definition.

Definition 4.1: A bijection f: (X, τ) → (Y, σ) is called a ψg*-homeomorphism if f is both ψg*-continuous map and ψg*-open map.

That is, both f and f^{-1} are ψg*-continuous map.

Example 4.2:
Let X = Y = {a, b, c}
τ = {X, ϕ, {a}} and σ = {Y, ϕ, {b}}
Define a map f: (X, τ) → (Y, σ) by f(a) = b, f(b) = c, f(c) = a
ψg*-C(X) = {X, ϕ, {b}, {c}, {a, b}, {b, c}, {a, c}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {c}, {a, b}, {b, c}}
C(Y) = {Y, ϕ, {a, c}}
O(X) = {X, ϕ, {a}}

Here, the inverse image of a closed set {a, c} in Y is {b, c} which is ψg*-closed set in X and the image of an open set {a} in X is {b} which is ψg*-open in Y. Hence, f is ψg*-homeomorphism.

Proposition 4.3:
a) Every homeomorphism is ψg*-homeomorphism
b) Every semi-homeomorphism is ψg*-homeomorphism
c) Every α-homeomorphism is ψg*-homeomorphism
d) Every regular-homeomorphism is ψg*-homeomorphism
e) Every g-homeomorphism is ψg*-homeomorphism
f) Every gr*-homeomorphism is ψg*-homeomorphism
g) Every g*-homeomorphism is ψg*-homeomorphism
h) Every g*s-homeomorphism is ψg*-homeomorphism
i) Every (gs)*-homeomorphism is ψg*-homeomorphism

Proof:
a) Let f: (X, τ) → (Y, σ) be a homeomorphism. Then f is continuous and open map. By Proposition 3.5(a) in [4] and Proposition 3.4(a), f is ψg*-continuous and ψg*-open map. Hence, f is ψg*-homeomorphism.
b) Let f: (X, τ) → (Y, σ) be a semi-homeomorphism. Then f is semi-continuous and semi-open map. By Proposition 3.5(b) in [4] and Proposition 3.4(b), f is ψg*-continuous and ψg*-open map. Hence, f is ψg*-homeomorphism.
c) Let f: (X, τ) → (Y, σ) be a α-homeomorphism. Then f is α-continuous and α-open map. By Proposition 3.5(c) in [4] and Proposition 3.4(c), f is ψg*-continuous and ψg*-open map. Hence, f is ψg*-homeomorphism.
d) Let f: (X, τ) → (Y, σ) be a regular-homeomorphism. Then f is regular-continuous and regular-open map. By Proposition 3.5(d) in [4] and Proposition 3.4(d), f is ψg*-continuous and ψg*-open map. Hence, f is ψg*-homeomorphism.
e) Let f: (X, τ) → (Y, σ) be a g-homeomorphism. Then f is g-continuous and g-open map. By Proposition 3.5(e) in [4] and Proposition 3.4(e), f is ψg*-continuous and ψg*-open map. Hence, f is ψg*-homeomorphism.

f) Let f: (X, τ) → (Y, σ) be a gr*-homeomorphism. Then f is gr*-continuous and gr*-open map. By Proposition 3.5(g) in [4] and Proposition 3.4(g), f is ψg*-continuous and ψg*-open map. Hence, f is ψg*-homeomorphism.
g) Let f: (X, τ) → (Y, σ) be a g*-homeomorphism. Then f is g*-continuous and g*-open map. By Proposition 3.5(h) in [4] and Proposition 3.4(h), f is ψg*-continuous and ψg*-open map. Hence, f is ψg*-homeomorphism.
h) Let f: (X, τ) → (Y, σ) be a g*s-homeomorphism. Then f is g*s-continuous and g*s-open map. By Proposition 3.5(j) in [4] and Proposition 3.4(j), f is ψg*-continuous and ψg*-open map. Hence, f is ψg*-homeomorphism.
i) Let f: (X, τ) → (Y, σ) be a (gs)*-homeomorphism. Then f is (gs)*-continuous and (gs)*-open map. By Proposition 3.5(k) in [4] and Proposition 3.4(k), f is ψg*-continuous and ψg*-open map. Hence, f is ψg*-homeomorphism.

The following examples show that the converse of the above proposition need not be true.

Example 4.4:

a) Let X = Y = {a, b, c}, τ = {X, ϕ, {a}}, σ = {Y, ϕ, {c}, {a ,b}}
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c
ψg*-C(X) = {X, ϕ, {b}, {c}, {a, b}, {b, c}, {a, c}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}
O(X) = {X, ϕ, {a}}
C(Y) = {Y, ϕ, {c}, {a, b}}
Here, the inverse image of a closed set {c},{a, b} in (Y, σ) are {c},{a, b} which is ψg*-closed but not closed in (X, τ). So f is ψg*-continuous but not continuous. Also the image of an open set {a} in (X, τ) is {a} which is ψg*-open set but not open set in (Y, σ). So f is ψg*-open map but not open map. Hence, f is ψg*-homeomorphism but not homeomorphism.
b) Let X = Y = {a, b, c}, τ = {X, ϕ, {a}}, σ = {Y, ϕ, {c}, {a, b}}
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c
ψg*-C(X) = {X, ϕ, {b}, {c}, {a, b}, {b, c}, {a, c}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}
O(X) = {X, ϕ, {a}}
C(Y) = {Y, ϕ, {c}, {a, b}}
semi-C(X) = {X, ϕ, {b}, {c}, {b, c}}
semi-O(Y) = {Y, ϕ, {c}, {a, b}}
Here, the inverse image of a closed set {a, b} in (Y, σ) is {a, b} which is ψg*-closed but not semi-closed in (X, τ). So f is ψg*-continuous but not semi-continuous. Also the image of an open set {a} in (X, τ) is {a} which is ψg*-open set but not semi-open set in (Y, σ). So f is ψg*-open map but not semi-open map. Hence, f is ψg*-homeomorphism but not semi-homeomorphism.
c) Let X = Y = {a, b, c}, τ = {X, ϕ, {b}, {a, c}}, σ = {Y, ϕ, {a}, {b, c}}
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c
ψg*-C(X) = {X, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}
O(X) = {X, ϕ, {b}, {a, c}}
C(Y) = {Y, ϕ, {a}, {b, c}}
α-C(X) = {X, ϕ, {b}, {a, c}}
α-O(Y) = {Y, ϕ, {a}, {b, c}}
Here, the inverse image of a closed set {a},{b, c} in (Y, σ) are {a},{b, c} which is ψg*-closed but not α-closed in (X, τ). So f is ψg*-continuous but not α-continuous. Also the image of an open set {b},{a, c} in (X, τ) is {b}, {a, c} which is ψg*-open set but not α-open set in (Y, σ). So f is ψg*-open map but not α-open map. Hence, f is ψg*-homeomorphism but not α-homeomorphism.
d) Let X = Y = {a, b, c}, τ = {X, ϕ, {b}}, σ = {Y, ϕ, {a}, {b, c}}
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c
ψg*-C(X) = {X, ϕ, {a}, {c}, {a, b}, {b, c}, {a, c}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}
O(X) = {X, ϕ, {b}}
C(Y) = {Y, ϕ, {a}, {b, c}}
regular-C(X) = {X, ϕ}
regular-O(Y) = {Y, ϕ, {a}, {b, c}}
Here, the inverse image of a closed set {a},{b, c} in (Y, σ) are {a},{b, c} which is ψg*-closed but not regular-closed in (X, τ). So f is ψg*-continuous but not regular-continuous. Also the image of an open set {b} in (X, τ) is {b} which is ψg*-open set but not regular-open set in (Y, σ). So f is ψg*-open map but not regular-open map. Hence, f is ψg*-homeomorphism but not regular-homeomorphism.
e) Let X = Y = {a, b, c, d}, τ = {X, ϕ, {c}, {c, d}, {b, c, d}}, σ = {Y, ϕ, {a}, {a, b, d}}
Define a function f: (X, τ) → (Y, σ) by f(a) = b, f(b) = c, f(c) = d, f(d) = a.
ψg*-C(X) = {X, ϕ, {a}, {b}, {d}, {a, b}, {a, c}, {a, d}, {b, d}, {a, b, c}, {a, c, d}, {a, b, d}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {d}, {a, b}, {a, c}, {a, d}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}}

O(X) = {X, ϕ, {c}, {c, d},{b, c, d}}
C(Y) = {Y, ϕ, {c}, {b, c, d}}
g-C(X) = { X, ϕ, {a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, c, d}, {a, b, d}}
g-O(Y) = {Y, ϕ, {a}, {b}, {d}, {a, b}, {a, d}, {b, d}, {a, b, d}}
Here, the inverse image of a closed set {c} in (Y, σ) is {d} which is ψg*-closed but not g-closed in (X, τ). So f is ψg*-continuous but not g-continuous. Also the image of an open set {b, c, d} in (X, τ) is {a, c, d} which is ψg*-open set but not g-open set in (Y, σ). So f is ψg*-open map but not g-open map. Hence, f is ψg*-homeomorphism but not g-homeomorphism.

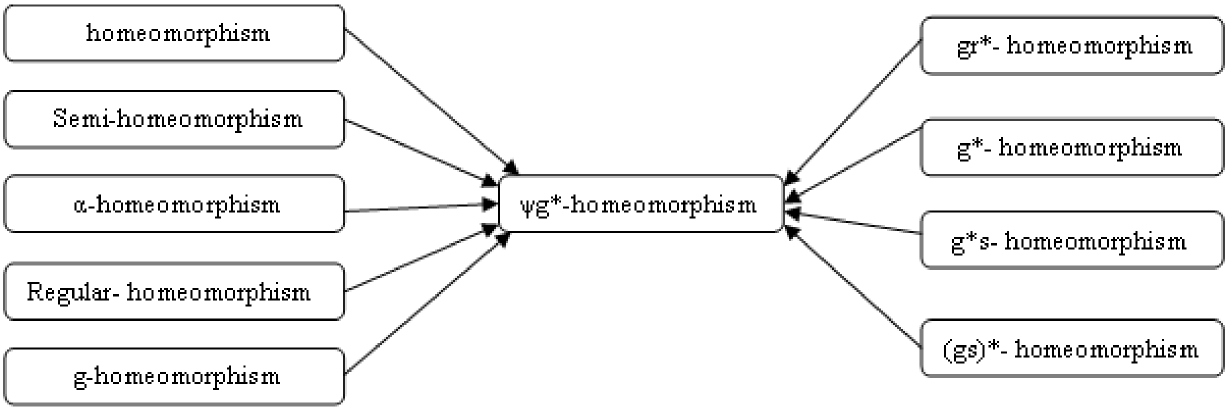
f) Let X = Y = {a, b, c}, τ = {X, ϕ, {c}, {b, c}}, σ = {Y, ϕ, {b}, {a, c}}
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c
ψg*-C(X) = {X, ϕ, {a}, {b}, {a, b}, {a, c}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}
O(X) = {X, ϕ, {c}, {b, c}}
C(Y) = {Y, ϕ, {b}, {a, c}}
gr*-C(X) = {X, ϕ, {a}, {a, b}, {a, c}}
gr*-O(Y) = {Y, ϕ, {b}, {a, c}}
Here, the inverse image of a closed set {b} in (Y, σ) is {b} which is ψg*-closed but not gr*-closed in (X, τ). So f is ψg*-continuous but not gr*-continuous. Also the image of an open set {c},{b, c} in (X, τ) is {c},{b, c} which is ψg*-open set but not gr*-open set in (Y, σ). So f is ψg*-open map but not gr*-open map. Hence, f is ψg*-homeomorphism but not gr*-homeomorphism.

g) Let X = Y = {a, b, c}, τ = {X, ϕ, {c}, {b, c}}, σ = {Y, ϕ, {b}, {a, c}}
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c
ψg*-C(X) = {X, ϕ, {a}, {b}, {a, b}, {a, c}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}
O(X) = {X, ϕ, {c}, {b, c}}
C(Y) = {Y, ϕ, {b}, {a, c}}
g*-C(X) = {X, ϕ, {a}, {a, b}, {a, c}}
g*-O(Y) = {Y, ϕ, {b}, {a, c}}
Here, the inverse image of a closed set {b} in (Y, σ) is {b} which is ψg*-closed but not g*-closed in (X, τ). So f is ψg*-continuous but not g*-continuous. Also the image of an open set {c},{b, c} in (X, τ) is {c},{b, c} which is ψg*-open set but not g*-open set in (Y, σ). So f is ψg*-open map but not g*-open map. Hence, f is ψg*-homeomorphism but not g*-homeomorphism.

h) Let X = Y = {a, b, c}, τ = {X, ϕ, {c}, {b, c}}, σ = {Y, ϕ, {b}, {a, c}}
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c
ψg*-C(X) = {X, ϕ, {a}, {b}, {a, b}, {a, c}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}
O(X) = {X, ϕ, {c}, {b, c}}
C(Y) = {Y, ϕ, {b}, {a, c}}
g*s-C(X) = {X, ϕ, {a}, {b}, {a, b}}
g*s-O(Y) = {Y, ϕ, {b}, {a, c}}
Here, the inverse image of a closed set {a, c} in (Y, σ) is {a,c} which is ψg*-closed but not g*s-closed in (X, τ). So f is ψg*-continuous but not g*s-continuous. Also the image of an open set {c},{b, c} in (X, τ) are {c},{b, c} which is ψg*-open set but not g*s-open set in (Y, σ). So f is ψg*-open map but not g*s-open map. Hence, f is ψg*-homeomorphism but not g*s-homeomorphism.

i) Let X = Y = {a, b, c}, τ = {X, ϕ, {c}, {b, c}}, σ = {Y, ϕ, {b}, {a, c}}
Define a function f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b, f(c) = c
ψg*-C(X) = {X, ϕ, {a}, {b}, {a,b}, {a, c}}
ψg*-O(Y) = {Y, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}}
O(X) = {X, ϕ, {c}, {b, c}}
C(Y) = {Y, ϕ, {b}, {a, c}}
(gs)*-C(X) = {X, ϕ, {a}, {a, b}}
(gs)*-O(Y) = {Y, ϕ, {b}, {a, c}}
Here, the inverse image of a closed set {b},{a, c} in (Y, σ) are {b}, {a, c} which is ψg*-closed but not (gs)*-closed in (X, τ). So f is ψg*-continuous but not (gs)*-continuous. Also the image of an open set {c},{b, c} in (X, τ) are {c},{b, c} which is ψg*-open set but not (gs)*-open set in (Y, σ). So f is ψg*-open map but not (gs)*-open map. Hence, f is ψg*-homeomorphism but not (gs)*-homeomorphism.

Remark 4.5: The following diagram shows the relationships of ψg*-homeomorphism with other known existing homeomorphisms. A → B represents A implies B but not conversely.



Proposition 4.6: For any bijection f: (X, τ) → (Y, σ) the following statements are equivalent

a) Its inverse map f^{-1}: Y → X is ψg*-continuous
b) f is a ψg*-open map
c) f is a ψg*-closed map.

Proof:

(a) ==> (b):

Let G be any open set in X.

Since f^{-1} is ψg*-continuous, f(G) is ψg*-open in Y. So, f is a ψg*-open map.

(b) ==> (c):

Let F be any closed set in X. Then F^c is open in X. Since f is ψg*-open, f(F^c) is ψg*-open in Y. So f(F) is ψg*-closed in Y. Therefore, f is a ψg*-closed map.

(c) ==> (a):

Let F be any closed set in X. Since f is a ψg*-closed map, f(F) is closed in Y. So $(f^{-1})^{-1}$ (F) is closed in Y. Therefore, f^{-1} is ψg*-continuous.

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