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THE TOTAL BLITACT GRAPH OF A GRAPH

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ABSTRACT

In this paper, we introduce the concept of total blitact graph of a graph and obtain the results determining the number of points and lines in this graph. We present characterizations of graphs whose total blitact graphs are planar, outerplanar, minimally nonouterplanar and k (k≥2)-minimally nonouterplanar.

Keywords: *inner point number, planar, k-minimally nonouterplanar, middle blict graph, total blitact graph.*

1. INTRODUCTION

In this paper, we consider a graph as finite, undirected without loops and multiple lines. For any undefined term or notation, we refer Kulli [1].

The inner point number $i(G)$ of a planar graph G is the minimum possible number of points not belonging to the boundary of the exterior region in any embedding of G in the plane. Obviously G is planar if and only if $i(G)=0$. A graph G is minimally nonouterplanar if $i(G)=1$, and is k-minimally nonouterplanar ($k\geq2$) if $i(G)=k$. This concept was introduced by Kulli in [2].

If $B=\{u_1, u_2, \ldots, u_r; r \geq 2\}$ is a block of a graph G, then we say that point u_1 and block B are incident with each other, as are u_2 and B and so on. If two distinct blocks B_1 and B_2 are incident with a common cutpoint, then they are adjacent blocks. This idea was introduced in [3]. The blocks, points and lines of a graph are called its members.

The middle blict graph $M_n(G)$ of a graph G is the graph whose set of points is the union of the set of points, lines and blocks of G and in which two points are adjacent if the corresponding lines and blocks of G are adjacent or one corresponds to a point and the other to a line incident with it or one corresponds to block B of G and other to a point v of G and v is in B. This concept was introduced by Kulli and Biradar in [4].

The point block graph $Pb(G)$ of a graph G is the graph whose point set is the union of the set of points and blocks of G and two points are adjacent if the corresponding blocks contain a common cutpoint of G or one corresponds to a block B of G and the other to a point v of G and v is in B. This concept was studied by Kulli and Biradar in [5, 6, 7]. Many other graph valued functions in graph theory were studied, for example, in [8-24].

The following will be useful in the proof of our results.

Theorem A: [4] If G is a connected (p, q) graph whose points have degree d_i and b_i is the number of blocks to which point v_i belongs in G, then the middle blict graph $M_n(G)$ has $(1+q+\sum_{i=1}^{p} b_i)$ *points and* $q+\frac{1}{2}\sum_{i=1}^{p}[d_i^2+b_i(b_i+1)]$ *lines.*

Theorem B: [1, p.197] A graph is planar if and only if it has no subgraph homeomorphic to K_5 or $K_{3,3}$.

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2. TOTAL BLITACT GRAPH OF A GRAPH

The definitions of $M_n(G)$ and $Pb(G)$ inspired us to introduce the following graph valued function. The points, lines and blocks of a graph are called its members.

Definition 1: The total blitact graph $T_n(G)$ of a graph G is the graph whose set of points is the union of the set of points, lines and blocks of G and in which two points are adjacent if the corresponding members of G are adjacent or one corresponds to a point and the other to a line incident with it or one corresponds to block B of G and other to a point v of G and v is in B.

Example 2: In Figure 1, the graph G and its total blitact graph $T_n(G)$ are shown.

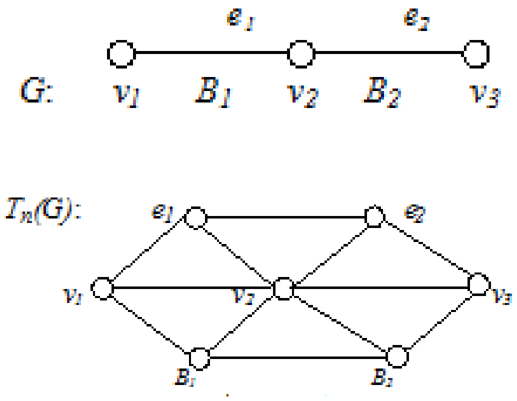


Figure-1

When defining any class of graphs, it is desirable to know the number of points and lines in each; the next theorem determines the same.

Theorem 3: If G is a connected (p, q) graph whose points have degree d_i and b_i is the number of blocks to which point v_i belongs in G, then the total blitact graph $T_n(G)$ has $(1 + q + \sum_{i=1}^{p} b_i)$ *points and* $2q + \frac{1}{2}\sum_{i=1}^{p}[d_i^2 + b_i(b_i+1)]$ *lines.*

Proof: The graphs $M_n(G)$ and $T_n(G)$ have the same number of points and by Theorem A, $T_n(G)$ *has* $(1 + q + \sum_{i=1}^{p} b_i)$ *points.*

The number of lines in $T_n(G)$=*Number of lines in* $M_n(G)$+*Number of lines in G*

$$= q + \frac{1}{2}\sum_{i=1}^{p}[d_i^2 + b_i(b_i+1)] + q.$$

$$= 2q + \frac{1}{2}\sum_{i=1}^{p}[d_i^2 + b_i(b_i+1)].$$

3. PLANARITY OF THE TOTAL BLITACT GRAPH

In the next theorem, we obtain a characterization of graphs whose total blitact graphs are planar.

Theorem 4: The total blitact graph $T_n(G)$ of a graph G is planar if and only if $\Delta(G) \le 2$.

Proof: Suppose $T_n(G)$ is planar. Assume $\Delta(G)=3$. Then there exists a point v of degree 3. Then v lies on at most three blocks. Then G has a subgraph homeomorphic to G_1 or G_2 or G_3 with respect to the cutpoints (see Fig. 2(a)). Then $T_n((G_1$ or G_2 or $G_3)$ can be drawn in the plane as shown in Fig.2(b). Then $T_n(G_1$ or G_2 or $G_3)$ has a subgraph homeomorphic to K_5 (shown with bold lines and points in Fig. 2(b)). Since $T_n(G_1$ or G_2 or $G_3)$ is a subgraph of $T_n(G)$, $T_n(G)$ has a subgraph homeomorphic to K_5, by Theorem C, $T_n(G)$ is nonplanar, a contradiction.

Conversely, suppose $\Delta(G) \le 2$. Then G is either a path or a cycle. If G is either P_n $(n\ge1)$ or C_n $(n\ge1)$ (see Fig. 3 (a) and (b)), $T_n(G)$ is clearly a planar graph (see Fig.4 (a) and (b)). Hence the proof of the theorem.

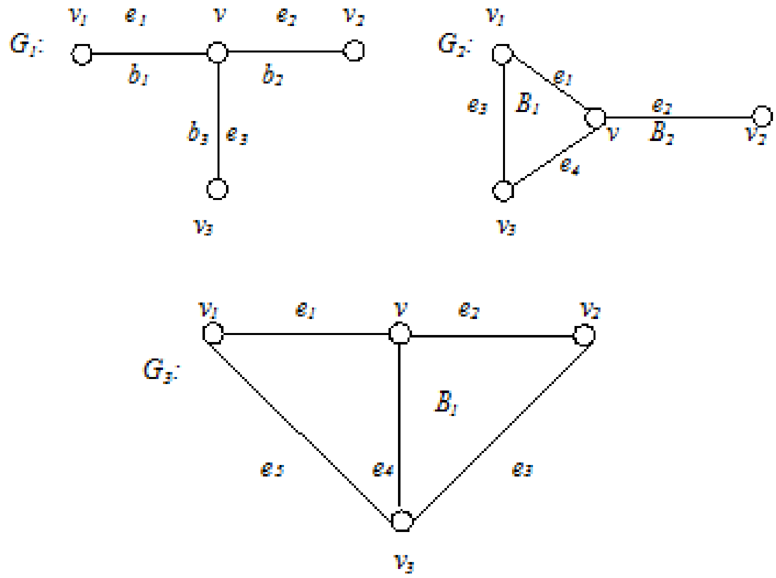


Figure-2 (a)

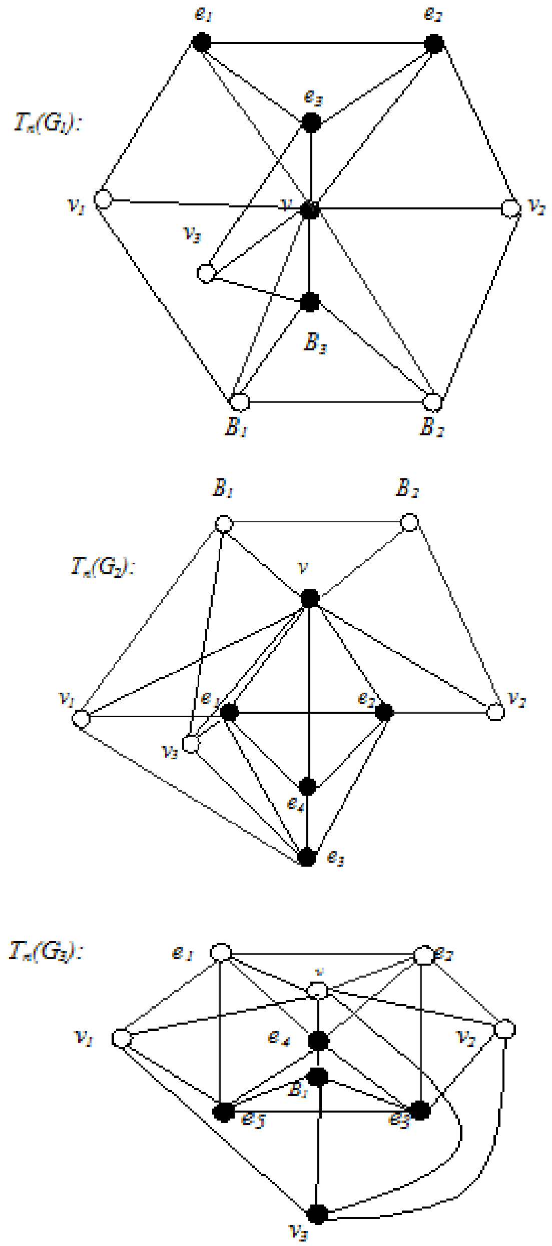


Figura-2 (b)

The following are the simple theorems which we merely state.

Theorem 5: The total blitact graph $T_n(G)$) of a graph G is outerplanar if and only if $G = P_n$, $n \leq 2$.

Theorem 6: The total blitact graph $T_n(G)$ of a graph G is minimally nonouterplanar if and only if $G = P_3$.

Theorem 7: The total blitact graph $T_n(G)$ of a graph G is 2-minimally nonouterplanar if and only if $G = P_4$.

Theorem 8: The total blitact graph $T_n(G)$ of a graph G is 3-minimally nonouterplanar if and only if G is either P_5 or C_3.

Theorem 9: The total blitact graph $T_n(G)$ of a graph G is 4-minimally nonouterplanar if and only if $G = P_6$.

In the next theorem, we present a characterization of graphs whose total blitact graphs are k $(k \geq 5)$ – minimally nonouterplanar.

Theorem 10: The total blitact graph $T_n(G)$ of a graph G is k-minimally nonouterplanar ($k \geq 5$) if and only if G is either P_{k+2} or C_{k-1}.

Proof: Suppose G is either P_{k+2} or C_{k-1}, $(k \geq 5)$. To prove the result we use mathematical induction on k.

Suppose $k = 5$. Then it is easy to see that $T_n(P_7 \text{ or } C_4)$ is 5-minimally nonouterplanar.

Assume the result is true for $k = m$. That is if G is either P_{m+2} *or* C_{m-1}, $T_n(G)$ is m-minimally nonouterplanar.

Suppose $k = m+1$. Then G is either P_{m+3} or C_m. Then we have to prove $T_n(G)$ is $(m+1)$ – minimally nonouterplanar. We consider the following cases.

Case-1: Let v be an endpoint of G and let $G = P_{m+3}$, delete from G the point v. The resulting graph $G_1 = P_{m+2}$. By inductive hypothesis $T_n(G_1)$ is m-minimally nonouterplanar.

Let $e_i = (v_i, v_j)$ be an endline of G_1. Then b_i is an endblock incident with the cutpoint v_i, since the line and block coincide in a path. The points e_i, b_i and v_j in $T_n(G_1)$ are on the boundary of the exterior region on some cycle C, since $T_n(G_1)$ is m-minimally nonouterplanar. Now rejoin the point v to the point v_j of G_1 resulting the graph G.

Let $e_j = (v_j, v)$ be the endline and b_j is an end block incident with the cutpoint v_j. The formation of $T_n(G)$ is an extension of $T_n(G_1)$ with additional points e_j, b_j and v such that e_j is joined to e_i and both are joined to v_j and v. Similarly b_j is joined to b_i and both are joined to v_j and v_i. Since e_i, b_i and v_j are on C, the points corresponding to e_i, b_i, v_j and v together with the cycle C produces a subgraph homeomorphic from K_4 which has an inner point. Therefore, $T_n(G)$ is $(m+1)$- minimally nonouterplanar.

Case-2: Let v_m be a point of G and let $G = C_m$, delete from G the point v_m by deleting the lines $e_{m-1} = (v_{m-1}, v_m)$ and $e_m = (v_m, v_1)$ which are incident with v_m, resulting the graph $G_1 = C_{m-1}$. By inductive hypothesis $T_n(G_1)$ is m-minimally nonouterplanar. Now rejoin the point v_m to the points v_{m-1} and v_1 of G_1 by joining the lines e_{m-1} and e_m, resulting the graph G. The formation of $T_n(G)$ is an extension of $T_n(G_1)$ with additional points v_m and e_m, where e_m is joined to the points e_{m-1}, v_m, v_1 and e_1 and also v_m is joined to the points e_{m-1} and B_1, where B_1 represents the point corresponding to the single block of G. Then the points corresponding to the points e_1, e_{m-1}, e_m, v_1 together with the point v_m produces a subgraph homeomorphic from K_4 which has an inner point. Therefore, $T_n(G)$ is $(m+1)$ – minimally nonouterplanar.

Conversely, suppose $T_n(G)$ is k – minimally nonouterplanar. Then by Theorem 4, $\Delta(G) \leq 2$, since $T_n(G)$ is planar. Then G is either a path or a cycle. We consider the following cases.

Case-1: Suppose G is a path. We consider the following subcases.

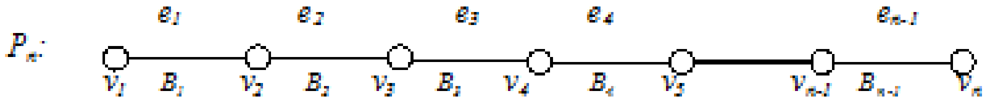
Subcase-1.1: Assume $G = P_{k+1}$, $k \geq 5$. In particular, let $k=5$, then $G=P_6$ and by Theorem 9, $T_n(G)$ is 4-minimally nonouterplanar, a contradiction.

Subcase-1.2: Assume $G = P_{k+3}$. In particular, let $k=5$, then $G=P_8$ and from Fig. 4(a), it is observed that $T_n(P_8)$ is 6-minimally nonouterplanar, again a contradiction.

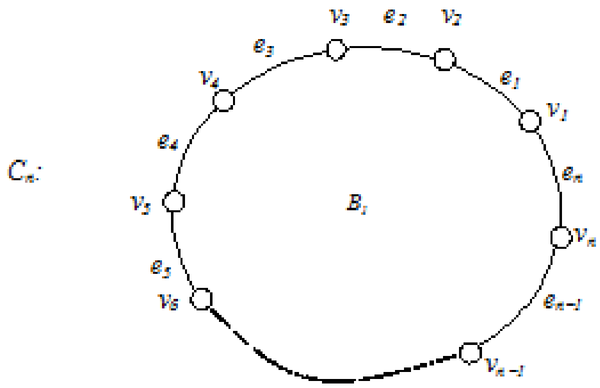
Case-2: Suppose G is a cycle. We consider the following subcases.

Subcase 2.1: Assume $G = C_{k-2}$, $k \geq 5$. In particular, let k=5, then $G=C_3$ and by Theorem 8, $T_n(G)$ is 3-minimally nonouterplanar, a contradiction.

Subcase 2.2: Assume $G = C_k$. In particular, let k=5, then $G=C_5$ and from Fig. 4(b), it is observed that $T_n(C_5)$ is 6-minimally nonouterplanar, again a contradiction. Thus from the above cases we conclude that G is either P_{k+2} or C_{k-1}. Hence the proof of the theorem.



(a)



(b)

Figure-3

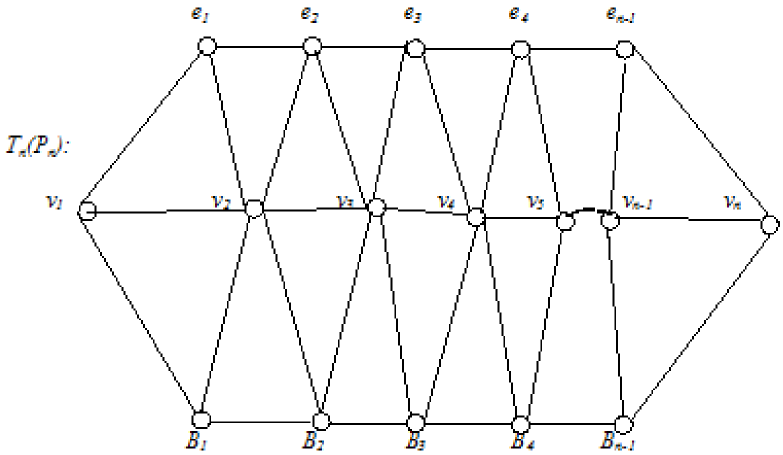


Figure- 4(a)

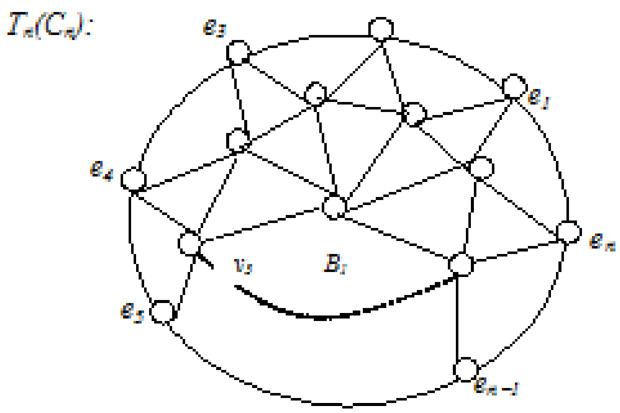


Figure 4(b)

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