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COMMUTATIVITY OF ASSOCIATIVE RINGS WITH (X, Y²) - (Y², X), YX²Y=XY²X

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ABSTRACT

In this paper we have mainly focused on some theorems related to commutativity of associative and non associative rings. We prove that if R is an associative ring with unity satisfying ***$(x, y^2) - (y^2, x)$****. $\forall x, y \in R$, $n \geq 2$ and* ***$xy^3=y^2xy$*** *$\forall x, y \in R$, $n \geq 2$. Then R is commutative ring and also I have mainly obtained two principles for a non associative ring to be a commutative ring.*

Key words: *Ring with unity, Associative ring, Non-associative ring.*

INTRODUCTION

The object of this note to investigate the commutativity of the associative and non associative rings satisfying condition '.' Such that $y(yx) = y(xy)$ $\forall$ x, y $\in$ R and $(yx)x = (xy)x$ $\forall$ x, y $\in$ R,

PRELIMINARIES

Definition:

(i) A non empty set R together with two binary operations + and . is said to be a ring (Associative ring) if (R, +) is an abelian group and (R, .) is a semi group satisfying distributive laws.
(ii) In a ring R if there exists an element '1' in R such that a.1=1.a = a for all a $\in$R then R is said to be a ring with unity.

Theorem 1: R is an Associative Ring with unity 1 then R is Commutative such that $(x, y^2) - (y^2, x)$ belongs to z(R).

Proof: Given that $xy^2 = y^2x$
Replacing y by y+1,
$x(y+1)^2 = (y+1)^2x \in Z(R)$
$x(y^2+2y+1) = (y^2+2y+1)x \in Z(R)$
$xy^2+2xy+x = y^2x+2yx+x \in Z(R)$
$2xy = 2yx \ \forall x, y, \in Z(R)$
$yx = xy \ \forall x, y, \in R$
Hence R is Commutative ring.

Theorem 2: Let R is an Associative Ring with unity and R is Commutative then $xy^3 = y^2xy$.

Proof: Given that $xy^3=y^2xy$
Replacing y by y+1, $x(y+1)^3 = (y+1)^2 x (y+1) \in Z(R)$
$x(y+1)(y^2+2y+1) = (y^2+2y+1)(xy+x)$
$(xy+x)(y^2+2y+1) = y^2xy+2yxy+xy \in Z(R)$
$xy^3+2xy^2+xy+xy^2+2xy+x = y^2xy+2yxy+xy \in Z(R)$
$2xy = 2yx$ [from 1, $xy^2=y^2x$]
$yx = xy \ \forall x, y, \in R$
Hence R is Commutative ring.

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Theorem 3: Let R be a prime ring with $yx^2y = xy^2x$ in Z(R) for every x, y in R. Then R is a commutative ring.

Proof: Given that**,** $yx^2y = xy^2x$

Replacing x by x+1,

$y(x+1)^2y = (x+1)(y^2)(x+1) \in Z(R)$

$y(x^2+2x+1)y = (xy^2+y^2)(x+1) \in Z(R)$

$yx^2y+2yxy+y^2 = xy^2x+xy^2+y^2x+y^2$

$2yxy = xy^2+y^2x$

(From the theorem $y^nx = y^{n-1}xy$)

$yxy = xy^2 \in Z(R)$

Replacing y by y+1,

$(y+1)(xy+x) = x(y^2+2y+1)$

We get,

$yxy+yx+xy+x = xy^2+2xy+x$

$yx = xy \ \forall x, y, \in R$

Hence R is Commutative ring.

Theorem 4: If R is an Associative Ring with unity 1 then R is Commutative if and only if $x^3yx = x^4y$ for all x, y belongs to R**.**

Proof: Given that , $x^3yx = x^4y$

Replacing x by x+1,

$(x+1)^3y(x+1) = (x+1)^4y \in Z(R)$

$(x+1)(x+1)(x+1)\, y\, (x+1) = (x+1)^2(x+1)^2y \in Z(R)$

$(x^2+2x+1)(xy+y)\, (x+1) = (x^2+2x+1)\, (x^2+2x+1)y \in Z(R)$

$(x^3y+x^2y+2x^2y+2xy+xy+y)\, (x+1) = (x^2+2x+1)\, (x^2y+2xy+y$

$3x^2yx+3xyx+yx = 3x^2y+3x^3y+xy$

[by the theorem, $x^ny = x^{n-1}yx$]

$yx = xy \ \forall x, y, \in R$

Hence R is Commutative ring.

Theorem 5: If R is ring with unity 1 satisfying $[(xy)^2 - xy, x]=0$ then R is commutative.

Proof: Given that, $[(xy)^2-xy, x] = 0$

$[(xy)^2-xy]x = x[(xy)^2-xy]$

Replacing x by x+1, $[(x+1)\, y]^2 - (x+1)\, y](x+1) = (x+1)\, [\, ((x+1)y)^2-(x+1)y] \in Z(R)$

$[(xy+y)^2 - (xy+y)](x+1) = (x+1)\, [(xy+y)^2 – (xy+y)\,]$

$[(xy+y)\, (xy+y)-(xy+y)]\, (x+1) = (x+1)\, [(xy+y)\, (xy+y) - (xy+y)]$

$(xy)(yx) + y(xy)x-yx = x(xy)y + (xy)\, (xy)-xy \in Z(R)$

Replacing x by x+1,

$[(x+1)y]\, [y(x+1)]+y((x+1)yx+1)-y(x+1) = (x+1)\, ((x+1)y)y+((x+1)y)((x+1)y-(x+1)y$

$(xy+y)(yx+y)+(yxy+y^2)(x+1)-yx-y = (x+1)\, (xy+y)y+(xy+y)\, (xy+y) – xy-y$

$Y(yx)+yxy = (xy)y+y(xy)$

By replacing y by y+1, we get $(yx)y = (xy)\, y \in Z(R)$

$yx = xy \ \forall x, y, \in R$

Hence R is Commutative ring

Theorem 6: If R be a non-associative ring with unity 1 satisfying y(yx)=y(xy) for all x,y belongs to R then R is commutative.

Proof: Given that, y(yx) = y(xy)

Replacing y by y+1, $(y+1)\, [yx+x] = (y+1)\, [xy+x] \in Z(R)$

$y(yx)+yx+yx+x = y(xy)+yx+xy+x$

$yx = xy \ \forall x, y, \in R$

Hence R is Commutative ring

Theorem 7: If R be a non-associative ring with unity 1 satisfying (yx)x = (xy)x for all x,y belongs to R then R is commutative.

Proof: Given that, (yx)x = (xy)x

Replacing x by x+1, [y(x+1)] (x+1) = [(x+1)y](x+1) ϵZ(R)

(yx+y) (x+1) = (xy+y) (x+1) ϵZ(R)

(yx)x+yx+yx+y) = (xy)x+xy+yx+y ϵZ(R)

yx = xy ∀ x, y, ϵR

Hence R is Commutative ring

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