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SOME NOTIONS OF NEARLY OPEN SETS IN FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce a new class of sets, namely fuzzy α^*-open sets and fuzzy α^*-closed sets. Further we define fuzzy α-interior and fuzzy α*-closure and discuss their properties. Finally, we relate fuzzy α*-open sets and fuzzy α*-closed sets with some other sets in fuzzy topological spaces.*

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Keywords: *fuzzy α*-open set, fuzzy α*-closed set, fuzzy α*-interior, fuzzy α*-closure.*

1 INTRODUCTION

Zadeh, in [7] introduced the concept of fuzzy sets. The study of fuzzy topology was introduced by Chang [4]. In 1991, A.S.Bin shahna [3] introduced α-open sets in fuzzy topological spaces. After Bin shahna's work, many mathematicians turned their attention to generalizing various concepts in fuzzy topology by considering fuzzy α-open sets instead of fuzzy open sets. The concept of fuzzy generalized closed sets was introduced by S.S.Thakur [6]. In this paper, we define a new class of sets, namely fuzzy α^*-open sets and fuzzy α*-closed sets. Further we define fuzzy α*-interior and fuzzy α*-closure and discuss their properties. Finally, we relate fuzzy α*-open sets and fuzzy α*-closed sets with some other sets in fuzzy topological spaces.

2. PRELIMINARIES

Throughout this paper X and Y denote fuzzy topological spaces (X, τ) and (Y, σ) on which no separation axioms are assumed. Let A be a subset of a space X. The closure of A and the interior of A are denoted by Cl(A) and Int(A) respectively. The following concepts are used in the sequel.

Definition 2.1: [3] A subset A of a fuzzy topological space (X, τ) is said to be a **fuzzy pre-open** if A ≤ Int(Cl(A)) and a **fuzzy pre-closed** if Cl(Int(A)) ≤ A.

Definition 2.2: [1] A subset A of a fuzzy topological space (X, τ) is said to be a **fuzzy semi-open** if A ≤ Cl(Int(A)) and a **fuzzy semi-closed** if Int(Cl(A)) ≤ A.

Definition 2.3: [3] A subset A of a fuzzy topological space (X, τ) is said to be a **fuzzy α-open** if A ≤ Int(Cl(Int(A))) and a **fuzzy α-closed** if Cl(Int(Cl(A))) ≤ A.

Definition 2.4: [6] A subset A of a fuzzy topological space (X, τ) is said to be **fuzzy generalized closed** (briefly g-closed) if Cl(A) ≤ U whenever A ≤ U and U is fuzzy open in X.

Definition 2.5: [6] A subset A of a fuzzy topological space (X, τ) is said to be **fuzzy generalized open** (briefly g-open) if its complement is g-closed in X.

Definition 2.6: [2] Let A be a subset of a fuzzy topological space (X, τ), then the **fuzzy generalized closure** of A is defined as the intersection of all fuzzy g-closed sets in X containing A and is denoted by $Cl^*(A)$.

Definition 2.7: [2] Let A be a subset of a fuzzy topological space (X, τ), then the **fuzzy generalized interior** of A is defined as the union of all fuzzy g-open sets in X that are contained A and is denoted by $Int^*(A)$.

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Definition 2.8: [5] A subset A of a fuzzy topological space (X, τ) is said to be **fuzzy generalized α-closed** if αCl(A) ≤ U whenever A ≤ U and U is fuzzy α-open in (X, τ).

Definition 2.9: [5] A subset A of a fuzzy topological space (X, τ) is said to be **fuzzy α-generalized closed** if αCl(A) ≤ U whenever A ≤ U and U is fuzzy open in (X, τ).

The **fuzzy α-interior** [3] of a subset A of a fuzzy topological space (X, τ) is the union of all fuzzy open sets contained in A and is denoted by αInt(A). The **fuzzy semi-interior** [1] of A and **fuzzy pre-interior** [3] of A are analogously defined and that are respectively denoted by sInt(A) and pInt(A).

The fuzzy α-closure [3] of a subset A of a fuzzy topological space (X, τ) is the intersection of all fuzzy closed sets containing A and is denoted by αCl(A). The **fuzzy semi-closure** [1] of A and **fuzzy pre-closure** [3] of A are analogously defined and that are respectively denoted by sCl(A) and pCl(A).

3. FUZZY α*-OPEN SETS

Definition 3.1: A subset A of a fuzzy topological space (X, τ) is called **fuzzy** α***-open set** if A ≤ Int*(Cl(Int*(A))). The collection of all fuzzy α*-open sets in (X, τ) is denoted by α*O(X, τ).

Lemma 3.2: If there exists fuzzy g-open set V such that V ≤ A ≤ Int*(Cl(V)), then A is fuzzy α*-open.

Proof: Since V is fuzzy g-open, Int*(V) = V. Therefore, A ≤ Int*(Cl(V)) = Int*(Cl(Int*(V))) ≤ Int*(Cl(Int*(A))). Hence A is fuzzy α*-open.

Theorem 3.3: Every fuzzy open set is fuzzy α*-open.

Proof: Let A be a fuzzy open set in X. Every fuzzy open set is fuzzy α-open. Then
A ≤ Int(Cl(Int(A))) ≤ Int*(Cl(Int*(A))). Hence A is fuzzy α*-open.

Remark 3.4: The converse of the above theorem need not be true as seen from the following example.

Example 3.5: Let X = {a, b} and τ = {0, 1, α_1}. The fuzzy sets are defined as $\alpha_1(a) = 0.5$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.5$, $\alpha_3(b) = 0.5$. Clearly α_3 is fuzzy α*-open but not fuzzy open.

Theorem 3.6: Let {Aα} be a collection of fuzzy α*-open sets in a fuzzy topological space X. Then ∨Aα is fuzzy α*-open.

Proof: Since Aα is fuzzy α*-open for each α. Then Aα ≤ Int*(Cl(Int*(Aα))). This implies ∨Aα ≤ ∨(Int*(Cl(Int*(Aα)))) ≤ (Int*(∨Cl(Int*(Aα)))) ≤ (Int*(Cl(∨Int*(Aα)))) ≤ (Int*(Cl(Int*(∨Aα)))). Hence ∨Aα is fuzzy α*-open.

Remark 3.7: The intersection of two fuzzy α*-open sets need not be fuzzy α*-open is shown in the following example.

Example 3.8: Let X = {a, b} and τ = {0, 1, α_1}. The fuzzy sets are defined as $\alpha_1(a) = 0.5$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.4$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.5$, $\alpha_3(b) = 0.6$, $\alpha_4(a) = 0.6$, $\alpha_4(b)=0.4$, $\alpha_5(a) = 0.4$, $\alpha_5(b) = 0.4$. Clearly $\alpha_1$and α_2 are fuzzy α*-open sets but $\alpha_1 \wedge \alpha_2 = \alpha_5$ is not fuzzy α*-open.

Theorem 3.9: Every fuzzy α-open set is fuzzy α*-open.

Proof: Let A be a fuzzy α-open set. Then A ≤ Int(Cl(Int(A))) ≤ Int*(Cl(Int*(A))). Hence A is fuzzy α*-open.

Remark 3.10: The converse of the above theorem need not be true as seen from the following example.

Example 3.11: Let X = {a, b} and τ = {0, 1, α_1}. The fuzzy sets are defined as $\alpha_1(a) = 0.5$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.5$, $\alpha_3(b) = 0.5$. Clearly α_3 is fuzzy α*-open but not fuzzy α-open.

Theorem 3.12: Every fuzzy g-open set is fuzzy α*-open.

Proof: Let A be a fuzzy g-open set. Then Int*(A) = A. Therefore Int*(A) ≤ Cl(Int*(A)).

Then Int*(Int*(A)) ≤ Int*(Cl(Int*(A))) ⇒ Int*(A) ≤ Int*(Cl(Int*(A))) ⇒ Int*(A) = A ≤ Int*(Cl(Int*(A))). Hence A is fuzzy α*-open.

Remark 3.13: The converse of the above theorem need not be true as seen from the following example.

Example 3.14: Let X = {a, b} and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.6$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$ and $\alpha_3(a) = 0.6$, $\alpha_3(b) = 0.4$. Clearly α_2 is fuzzy α*-open but not fuzzy g-open.

Theorem 3.15: If a subset A is fuzzy α*-open and B is fuzzy open, then A∨B is fuzzy α*-open.

Proof: Proof follows from theorem 3.3 and theorem 3.6.

Theorem 3.16: If a subset A is fuzzy α*-open and B is fuzzy α-open, then A ∨ B is fuzzy α*-open.

Proof: Proof follows from theorem 3.9 and theorem 3.6.

Theorem 3.17: If a subset A is fuzzy α*-open and B is fuzzy g-open, then A ∨ B is fuzzy α*-open.

Proof: Proof follows from theorem 3.12 and theorem 3.6.

Remark 3.18: The concept of fuzzy α*-open sets and fuzzy semi-open sets are independent as shown in the following examples.

Example 3.19: Let X = {a, b} and $\tau = \{0, 1, \alpha_2\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.6$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.6$, $\alpha_3(b) = 0.4$. Clearly α_1 is fuzzy α*-open but not fuzzy semi-open.

Example 3.20: Let X = {a, b} and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.5$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.5$, $\alpha_3(b) = 0.5$. Clearly α_2 is fuzzy semi open but not fuzzy α*-open.

Remark 3.21: The concept of fuzzy α*-open sets and fuzzy α-generalized open sets are independent as shown in the following examples.

Example 3.22: Let X = {a, b} and $\tau = \{0, 1, \alpha_2\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.6$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.6$, $\alpha_3(b) = 0.4$. Clearly α_2 is fuzzy α*-open but not fuzzy α-generalized open.

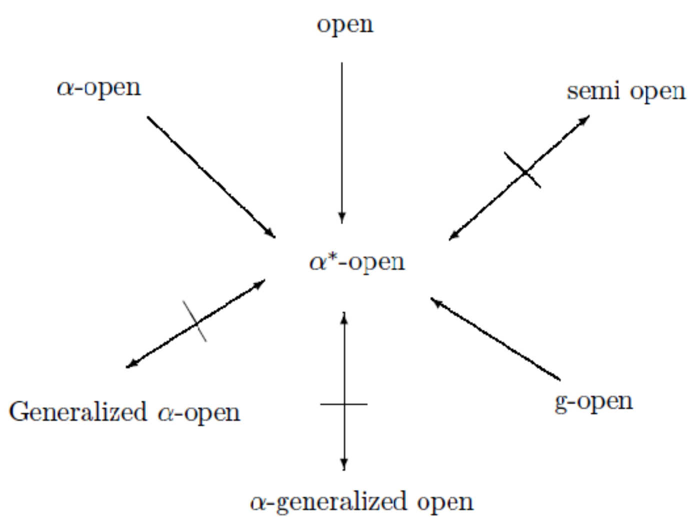
Example 3.23: Let X = {a, b} and $\tau = \{0, 1, \alpha_2\}$. The fuzzy sets are defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.4$, $\alpha_1(c) = 0.4$, $\alpha_2(a) = 0.3$, $\alpha_2(b) = 0.6$, $\alpha_2(c) = 0.4$, $\alpha_3(a) = 0.7$, $\alpha_3(b) = 0.6$, $\alpha_3(c) = 0.6$. Clearly α_3 is fuzzy α-generalized open but not fuzzy α*-open.

Remark 3.24: The concept of fuzzy α*-open sets and fuzzy generalized α-open sets are independent as shown in the following examples.

Example 3.25: Let X = {a, b} and $\tau = \{0, 1, \alpha_2\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.6$, $\alpha_2(a) = 0.5$, $\alpha_2(b) = 0.6$, $\alpha_3(a) = 0.6$, $\alpha_3(b) = 0.4$. Clearly α_2 is fuzzy α*-open but not fuzzy generalized α-open.

Example 3.26: Let X = {a, b} and $\tau = \{0, 1, \alpha_2\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.4$, $\alpha_1(c) = 0.4$, $\alpha_2(a) = 0.3$, $\alpha_2(b) = 0.6$, $\alpha_2(c) = 0.4$, $\alpha_3(a) = 0.7$, $\alpha_3(b) = 0.6$, $\alpha_3(c) = 0.6$. Clearly α_3 is fuzzy generalized α-open but not fuzzy α*-open.

Remark 3.27: From the above theorems and remarks, we have the following implication diagram.



Definition 3.28: Let A be a subset of a fuzzy topological space (X, τ), then **fuzzy α*-interior** of A is defined as the union of all fuzzy α*-open sets in X that are contained in A and is denoted by α*Int(A).

Theorem 3.29: If A is any subset of a fuzzy topological space (X, τ), α*Int(A) is fuzzy α*-open. Infact, α*Int(A) is the largest fuzzy α*-open set contained in A.

Proof: Proof follows from the definition 3.28 and theorem 3.3.

Theorem 3.30: Let A be a subset of a fuzzy topological space (X, τ). Then A is fuzzy α*-open if and only if α*Int(A) = A.

Proof: If A is fuzzy α*-open then α*Int(A) = A. Conversely, let α*Int(A) = A, by theorem 3.29, α*Int(A) is fuzzy α*-open. Hence A is fuzzy α*-open.

Theorem 3.31: Let A and B are subsets of a fuzzy topological space (X, τ), then the following conditions are hold:

a) α*Int(φ) = φ
b) α*Int(X) = X
c) α*Int(A) ≤ A
d) If A ≤ B, then α*Int(A) ≤ α*Int(B)
e) A ≤ Int(A) ≤ αInt(A) ≤ α*Int(A)
f) α*Int(A) ∨ α*Int(B) ≤ α*Int(A ∨ B)
g) α*Int(A) ∧ α*Int(B) ≥ α*Int(A ∧ B)

Proof: a), b), c), d) follows from the definition 3.28 and e) follows from theorem 3.9. From d) α*Int(A) ≤ α*Int(A∨B) and α*Int(B) ≤ α*Int(A∨B).

⟹ α*Int(A) ∨ α*Int(B) ≤ α*Int(A ∨ B). Hence f) follows.

Again from d) α*Int(A) ≥ α*Int(A ∧ B) and α*Int(B) ≥ α*Int(A ∧ B).

⟹ α*Int(A) ∧ α*Int(B) ≥ α*Int(A ∧ B). Hence g) follows.

4. α *-CLOSED SETS

Definition 4.1: A subset A of a fuzzy topological space (X, τ) is called **fuzzy α*-closed set** if its complement is α*-open. The collection of all fuzzy α*-closed sets in (X, τ) is denoted by α*C(X, τ).

Lemma 4.2: If there exists an fuzzy g-closed set F such that Cl*(Int(F)) ≤ A ≤ F, then A is fuzzy α*-closed.

Proof: Since F is fuzzy g-closed, Cl*(F) = F. Therefore, Cl*(Int(Cl*(A))) ≤ Cl*(Int(Cl*(F))) = Cl*(Int(F)) ≤ A. Hence A is fuzzy α*-closed.

Theorem 4.3: A subset A of a fuzzy topological space (X, τ) is fuzzy α*-closed if and only if Cl*(Int(Cl*(A))) ≤ A.

Proof: Let A be a fuzzy α*-closed set. Then 1−A is fuzzy α_-open. By definition 1 − A ≤ Int*(Cl(Int*(1 − A))). That is 1 − A ≤ 1 – Cl*(Int(Cl*(A))). Hence Cl*(Int(Cl*(A))) ≤ A. Conversely, suppose Cl*(Int(Cl*(A))) ≤ A.

Then 1 − A ≤ 1 – Cl*(Int(Cl*(A))). That is 1 − A ≤ Int*(Cl(Int*(1 − A))). This shows that 1 − A is fuzzy α*-open. Then A is fuzzy α*-closed.

Theorem 4:4. If {Aα} is a collection of fuzzy α*-closed sets in fuzzy topological space (X, τ), then ∧Aα is fuzzy α*-closed.

Proof: Let Aα be a fuzzy α*-closed in X ⟹ 1 − Aα is fuzzy α*-open in X ⟹ By theorem 3.6, ∨(1 − Aα) is fuzzy α*-open in X ⟹ 1 − ∧Aα is fuzzy α*-open in X. Hence ∧Aα is fuzzy α*-closed.

Remark 4.5: The union of fuzzy α*-closed sets need not be α*-closed as seen from the following example.

Example 4.6: Let X = {a, b} and τ = {0, 1, α_2, α_5, α_6}. The fuzzy sets are defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.6$, $\alpha_2(a) = 0.4$, $\alpha_2(b) = 0.5$, $\alpha_3(a) = 0.6$, $\alpha_3(b) = 0.5$, $\alpha_4(a) = 0.6$, $\alpha_4(b) = 0.6$, $\alpha_5(a) = 0.6$, $\alpha_5(b) = 0.7$, $\alpha_6(a) = 0.6$, $\alpha_6(b) = 0.4$.

Clearly α_1 and α_3 are fuzzy α*-closed sets but $\alpha_1 \vee \alpha_3 = \alpha_4$ is not fuzzy α*-closed.

Theorem 4.7: Every fuzzy closed set is fuzzy α^*-closed

Proof: Let A be a fuzzy closed set in X. Then $1 - A$ is fuzzy open in X. By theorem 3.3, $1 - A$ is fuzzy α^*-open

$\Rightarrow$ A is fuzzy α^*-closed.

Remark 4.8: The converse of the above theorem need not be true as seen from the following example.

Example 4.9: Let X = {a, b} and $\tau = \{0, 1, \alpha_2\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.6$, $\alpha_2(b) = 0.5$. Clearly α_1 is fuzzy α^*-closed but not fuzzy closed.

Theorem 4.10: If a subset A of a fuzzy topological space X is fuzzy α^*-closed and B is fuzzy closed then $A \wedge B$ is fuzzy α^*-closed.

Proof: Proof follows from theorem 4.7 and theorem 4.4

Theorem 4.11: Every fuzzy α-closed set is fuzzy α^*-closed.

Proof: Let A be a fuzzy α-closed. Then $1-A$ is fuzzy α-open. By theorem 3.9, $1 - A$ is fuzzy α^*-open. Hence A is fuzzy α^*-closed.

Remark 4.12: The converse of the above theorem need not be true as seen from the following example.

Example 4.13: Let X = {a, b} and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.6$, $\alpha_1(b) = 0.5$, $\alpha_2(a) = 0.7$, $\alpha_2(b) = 0.8$. Clearly α_2 is fuzzy α^*-closed but not fuzzy α-closed.

Theorem 4.14: Every fuzzy g-closed set is fuzzy α^*-closed.

Proof: Let A be a fuzzy g-closed set. Then $1-A$ is fuzzy g-open set. By theorem 3.12, $1 - A$ is fuzzy α^*-open

$\Rightarrow$ A is fuzzy α^*-closed.

Remark 4.15: The converse of the above theorem need not be true as seen from the following example.

Example 4.16: Let X = {a, b} and $\tau = \{0, 1, \alpha_3\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.6$, $\alpha_2(a) = 0.6$, $\alpha_2(b) = 0.5$, $\alpha_3(a) = 0.4$, $\alpha_3(b) = 0.5$. Clearly α_3 is fuzzy α^*-closed but not fuzzy g-closed.

Theorem 4.17: If a subset A of a fuzzy topological space X is fuzzy α^*-closed and B is fuzzy α-closed, then $A \wedge B$ is fuzzy α^*-closed.

Proof: Proof follows from theorem 4.11 and theorem 4.4 .

Theorem 4.18: If a subset A of a fuzzy topological space X is fuzzy α^*-closed and B is fuzzy g-closed, then $A \wedge B$ is fuzzy α^*-closed.

Proof: Proof follows from theorem 4.14 and theorem 4.4 .

Remark 4.19: The concept of fuzzy α^*-closed sets and fuzzy semi-closed sets are independent as shown in the following examples.

Example 4.20: Let X = {a, b} and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.1$, $\alpha_2(b) = 0.9$. Clearly α_2 is fuzzy α^*-closed but not fuzzy semi-closed.

Example 4.21: Let X = {a, b} and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.4$, $\alpha_2(a) = 0.1$, $\alpha_2(b) = 0.9$. Clearly α_1 is fuzzy semi closed but not fuzzy α^*-closed.

Remark 4.22: The concept of fuzzy α^*-closed sets and fuzzy α-generalized closed sets are independent as shown in the following examples.

Example 4.23: Let X = {a, b} and $\tau = \{0, 1, \alpha_1\}$ The fuzzy sets are defined as $\alpha_1(a) = 0.4$, $\alpha_1(b) = 0.5$, $\alpha_1(c) = 0.6$, $\alpha_2(a) = 0.3$, $\alpha_2(b) = 0.3$, $\alpha_2(c) = 0.5$, $\alpha_3(a) = 0.7$, $\alpha_3(b) = 0.5$, $\alpha_3(c) = 0.5$. Clearly α_2 is fuzzy α^*-closed but not fuzzy α-generalized closed.

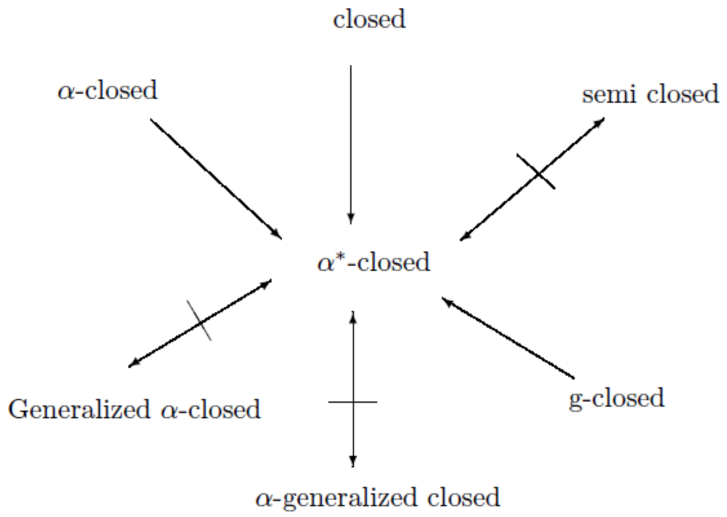
Example 4.24: Let X = {a, b} and τ = {0, 1, α_2} The fuzzy sets are defined as α_1(a) = 0.3, α_1(b) = 0.4, α_1(c) = 0.4, α_2(a) = 0.3, α_2(b) = 0.6, α_2(c) = 0.4, α_3(a) = 0.7, α_3(b) = 0.6, α_3(c) = 0.6. Clearly α_1 is fuzzy α- generalized closed but not fuzzy α*-closed.

Remark 4.25: The concept of fuzzy α*-closed sets and fuzzy generalized α- closed sets are independent as shown in the following examples.

Example 4.26: Let X = {a, b} and τ = {0, 1, α_1} The fuzzy sets are defined as α_1(a) = 0.6, α_1(b) = 0.5, α_2(a) = 0.7, α_2(b) = 0.8. Clearly α_2 is fuzzy α*-closed but not fuzzy generalized α-closed.

Example 4.27: Let X = {a, b} and τ = {0, 1, α_2} The fuzzy sets are defined as α_1(a) = 0.3, α_1(b) = 0.4, α_1(c) = 0.4, α_2(a) = 0.3, α_2(b) = 0.6, α_2(c) = 0.4, α_3(a) = 0.7, α_3(b) = 0.6, α_3(c) = 0.6. Clearly α_1 is fuzzy generalized α-closed but not fuzzy α*-closed.

Remark 4.28: From the above theorems and remarks, we have the following implication diagram.



Definition 4.29: Let A be a subset of a fuzzy topological space (X, τ). Then **fuzzy α*-closure** of A is defined as the intersection of all fuzzy α*-closed sets containing A and denoted by α*Cl(A).

Theorem 4.30: Let A be a subset of a fuzzy topological space (X, τ). Then A is fuzzy α*-closed if and only if α*Cl(A) = A.

Proof: Suppose A is fuzzy α*-closed. Then by definition 4.29, α*Cl(A) = A. Conversely, suppose α*Cl(A) = A. Then by theorem 4.4, A is fuzzy α*-closed.

Theorem 4.31: Let A and B are subsets of a fuzzy topological space (X, τ), then the following conditions are hold:

a) α*Cl(φ) = φ
b) α*Cl(X) = X
c) A ≤ α*Cl(A)
d) If A ≤ B, then α*Cl(A) ≤ α*Cl(B)
e) A ≤ α*Cl(A) ≤ αCl(A) ≤ Cl(A)
f) α*Cl(A) ∨ α*Cl(B) ≤ α*Cl(A ∨ B)
g) α*Cl(A) ∧ α*Cl(B) ≥ α*Cl(A ∧ B)

Proof: a), b), c), d) follows from the definition 4.29 and e) follows from theorem 4.7.

From d) α*Cl(A) ≤ α*Cl(A ∨ B) and α*Cl(B) ≤ α*Cl(A ∨ B)

⇒ α*Cl(A) ∨ α*Cl(B) ≤ α*Cl(A ∨ B). Hence f) follows.

Again from d) α*Cl(A) ≥ α*Cl(A ∧ B) and α*Cl(B) ≥ α*Cl(A ∧ B)

⇒ α*Cl(A) ∧ α*Cl(B) ≥ α*Cl(A ∧ B). Hence g) follows.

REFERENCES

1. K.K.Azad, On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl.,82(1981), 14-32.
2. G.Balasubramanian and P.Sundaram, On some generalizations of fuzzy continuous functions, Fuzzy Sets and Systems, 86(1)(1997)93-100.
3. A.S.Bin shahna, On fuzzy strong semicontinuity and fuzzy precontinuity, Fuzzy Sets and Systems, 44(1991), 303-308.
4. C.L.Chang, Fuzzy topological spaces, J.Math.Anal.Appl., 24(1968)182-190.
5. R.K.Saraf, M.Caldas and S.Mishra, Results via Fgα-closed sets and Fαgclosed sets, preprint.
6. S.S.Thakur and R.Malviya, Generalized closed sets in fuzzy topological spaces, Math.Note 38(1995), 137-140.
7. L.A.Zadeh, Fuzzy sets, Inform.contr., 8(1965), 338-353.

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