International Journal of Mathematical Archive-7(3), 2016, 157-165
Available online through www.ijma.info ISSN 2229 – 5046

SOME NEW RESULTS ON 1-NEAR MEAN CORDIAL LABELING OF GRAPHS

A. RAJA RAJESWARI*[1], S. SARAVANA KUMAR[2]

[1]M.Phil Scholar, Department of Mathematics, Sri S. R. N. M. College, Sattur, (T.N.), India.

[2]Department of Mathematics, Sri. S. R. N. M. College, Sattur, (T.N.), India.

(Received On: 09-03-16; Revised & Accepted On: 29-03-16)

ABSTRACT

Let G = (V, E) be a simple graph. A surjective function f: V(G) →{0, 1, 2} is said to be a 1-Near Mean Cordial Labeling if for each edge uv, the induced map

$$f^*(uv) = \begin{cases} 0 & \text{if } \frac{f(u)+f(v)}{2} \text{ is an integer} \\ 1 & \text{otherwise} \end{cases}$$

Satisfies the condition $|e_f(0) - e_f(1)| \leq 1$ *where* $e_f(0)$ *is the number of edges with 0 label and* $e_f(1)$ *is the number of edges with 1 label.*

G is said to be a 1-Near Mean Cordial Graph if it has a 1- Near Mean Cordial Labeling. In this paper ,we proved that wheel, complete bipartite, helm, closed helm, flower, sunflower, and $S(K_{1,n})$ *are 1- Near Mean Cordial Graphs.*

Keywords: *1-Near Mean Cordial Labeling, 1-Near Mean Cordial Graph.*

1. INTRODUCTION

All graphs considered here are finite, simple and undirected. Gallian [2] has given a dynamic survey of labeling. For graph theoretic terminologies and notations we follow Harary [3].The concept of mean cordial labeling was introduced by Raja Ponraj, Muthirulan Sivakumar and Murugesan Sundaram in the year 2012 [1,4,5,7]. Let f be a function from V (G) to {0, 1, 2}. For each edge uv of G, assign the label$\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called a mean cordial labeling of G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, i, j∈{0,1,2} where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with x(x=0,1,2) respectively. A graph with a mean cordial labeling is called Mean Graph. K.Palani, J.Rejila Jeya Surya [6] introduced a new concept called 1-Near Mean Cordial Labeling and investigated some standard graphs.

2. PRELIMINARIES

We define the concept of 1-Near Mean Cordial Labeling as follows,

Let G = (V, E) be a simple graph. A surjective function f: V(G) → {0, 1, 2} said to be a 1-Near Mean Cordial Labeling if for each edge uv, the induced map

$$f^*(uv) = \begin{cases} 0 & \text{if } \frac{f(u)+f(v)}{2} \text{ is an integer} \\ 1 & \text{otherwise} \end{cases}$$

Satisfies the condition $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ is the number of edges with 0 label and $e_f(1)$ is the number of edges with 1 label.

G is said to be a 1-Near Mean Cordial Graph if it has a 1- Near Mean Cordial Labeling. we proved that wheel, complete bipartite, helm, closed helm, flower, sunflower, and $S(K_{1,n})$ are 1- Near Mean Cordial Graphs.

Definition 2.1: A graph C_n+K_1 is called a **wheel** with n spokes and is denoted by W_n.

Corresponding Author: A. Raja Rajeswari[1]*
[1]M.Phil Scholar, Department of Mathematics, Sri S. R. N. M. College, Sattur, (T.N.), India.

Definition 2.2: A graph G is called a **complete bipartite graph** $K_{m,n}$ with bipartition V (G) = $V_1 \cup V_2$ where $V_1 = \{x_1, x_2, ..., x_m\}$ and $V_2 = \{y_1, y_2, ..., y_n\}$ and all vertices in V_1 are adjacent to all vertices in V_2 but no vertices in V_1 and V_2.

Definition 2.3: The **helm** H_n, is the graph obtained from a wheel by attaching a pendant edge at each vertex of the n- cycle.

Definition 2.4: A **closed helm** CH_n, is a graph obtained from a helm by joining each pendent vertex to the central vertex of the helm.

Definition 2.5: A **flower** Fl_n, is the graph obtained from a helm graph by joining each pendant vertex to the central vertex of the helm graph.

Definition 2.6: The **sunflower graph** v[n, s, t] is the resultant graph obtained from the flower graph of wheels W_n by adding n-1 pendant edges to the central vertex.

Definition 2.7: For each vertex v of a graph G take a new vertex v_0. join v_0 to all the vertices of G adjacent to v. The graph S(G) thus obtained is called **splitting graph** of G.

3. MAIN RESULTS

Theorem 3.1: The wheel W_n is a 1-Near Mean Cordial Graph.

Proof: Let G = (V, E) be a simple graph.

Let G be W_n.

Let $V(G) = \{u, v_i: 1 \leq i \leq n\}$ and $E(G) = \{[(uv_i) : 1 \leq i \leq n] \cup [(v_iv_{i+1}) : 1 \leq i \leq n-1] \cup [v_nv_1]\}$

Define f: V (G) → {0, 1, 2} by
f(u) = 1
$f(v_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 2 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$

The induced edge labeling are
$f^*(uv_i)=1, \quad 1 \leq i \leq n$
$f^*(v_iv_{i+1}) = 0, \quad 1 \leq i \leq n-1$
$f^*(v_nv_1)=0$

Here, $e_f(0) = e_f(1) = n$

Hence the graph satisfies the condition $|e_f(0) - e_f(1)| \leq 1$

Therefore, the wheel W_n is a 1-near mean cordial graph.

Illustration 1: The 1- near mean cordial graph of W_5 is shown in the figure 1

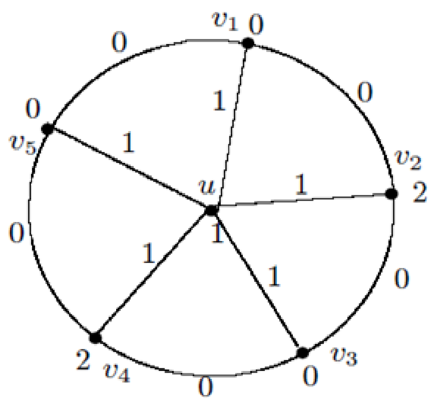

Figure 1 : W_5

Theorem 3.2: The complete bipartite graph, $K_{m,n}$ is a 1-Near Mean Cordial Graph.

Proof: Let G = (V, E) be a simple graph.

Let G be $K_{m,n}$

Let V (G) = $\{u_i : 1 \le i \le m,\ v_j : 1 \le j \le n\}$ and E(G)=$\{(u_iv_j : 1 \le i \le m, 1 \le j \le n\}$

Define f: V (G) → {0, 1, 2} by

$$f(u_i) = \begin{cases} 0 & i \equiv 1 \ mod\ 4 \\ 1 & i \equiv 0,2 \ mod\ 4 \\ 2 & i \equiv 3 \ mod\ 4 \end{cases} \quad 1 \le i \le m,$$

$$f(v_j) = \begin{cases} 0 & i \equiv 0 \ mod\ 2 \\ 1 & i \equiv 1 \ mod\ 2 \end{cases} \quad 1 \le j \le n,$$

The induced edge labeling are

Case-(i): when m is even and n is even or odd

$$f^*(u_{2i-1}v_j) = \begin{cases} 0 & j \equiv 0 \ mod\ 2 \\ 1 & j \equiv 1 \ mod\ 2 \end{cases} \quad 1 \le i \le \frac{m}{2}, 1 \le j \le n$$

$$f^*(u_{2i}v_j) = \begin{cases} 0 & j \equiv 1 \ mod\ 2 \\ 1 & j \equiv 0 \ mod\ 2 \end{cases} \quad 1 \le i \le \frac{m}{2}, 1 \le j \le n$$

Here, $e_f(0) = e_f(1) = mn$

Case-(ii): when m is odd and n is even or odd

$$f^*(u_{2i-1}v_j) = \begin{cases} 0 & j \equiv 0 \ mod\ 2 \\ 1 & j \equiv 1 \ mod\ 2 \end{cases} \quad 1 \le i \le \frac{m+1}{2},\ 1 \le j \le n$$

$$f^*(u_{2i}v_j) = \begin{cases} 0 & j \equiv 1 \ mod\ 2 \\ 1 & j \equiv 0 \ mod\ 2 \end{cases} \quad 1 \le i < \frac{m+1}{2},\ 1 \le j \le n$$

$$\text{Here, } e_f(0) = \begin{cases} \frac{mn-1}{2} & n \text{ is odd} \\ \frac{mn}{2} & n \text{ is even} \end{cases}$$

$$e_f(1) = \begin{cases} \frac{mn+1}{2} & n \text{ is odd} \\ \frac{mn}{2} & n \text{ is even} \end{cases}$$

Hence the graph satisfies the condition $| e_f(0) - e_f(1) | \le 1$

Therefore, the complete bipartite graph, $K_{m,n}$, is a 1-near mean cordial graph.

Illustration 2: The 1-near mean cordial graph of $K_{4,3}$ and $K_{3,3}$ are shown in the figure 2(a) and figure 2(b)

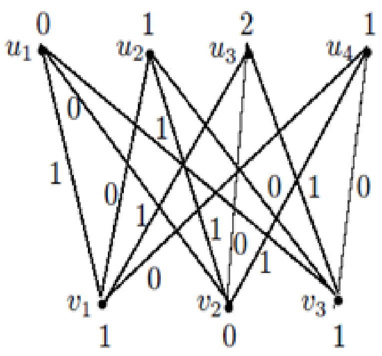

Figure 2(a): $K_{4,3}$

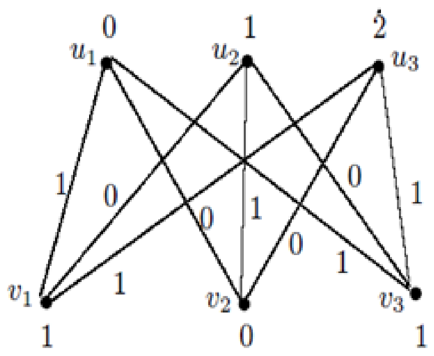

Figure 2(b): $K_{3,3}$

Theorem 3.3: The helm H_n is a 1-Near Mean Cordial Graph.

Proof: Let G= (V, E) be a simple graph.

Let G be H_n.

Let $V(G) = \{u, v_i : 1 \leq i \leq n, w_i : 1 \leq i \leq n\}$ and

$E(G) = \{[(uv_i), (w_iv_i) : 1 \leq i \leq n] \cup [(v_iv_{i+1}) : 1 \leq i \leq n-1] \cup [v_nv_1]\}$

Define $f : V(G) \to \{0, 1, 2\}$ by

$f(u) = 1$

$$f(v_i) = \begin{cases} 0 & i \equiv 1 \ mod\ 2 \\ 2 & i \equiv 0 \ mod\ 2 \end{cases} \qquad 1 \leq i \leq n$$

$$f(w_i) = \begin{cases} 1 & i \equiv 0 \ mod\ 2 \\ 2 & i \equiv 1 \ mod\ 2 \end{cases} \qquad 1 \leq i \leq n$$

The induced edge labeling are
$f^*(uv_i) = 1, \quad 1 \leq i \leq n$

$f^*(v_iv_{i+1}) = 0, \quad 1 \leq i \leq n-1$

$f^*(v_nv_1) = 0$

$$f^*(w_iv_i) = \begin{cases} 0 & i \equiv 1 \ mod\ 2 \\ 1 & i \equiv 0 \ mod\ 2 \end{cases} \qquad 1 \leq i \leq n$$

$$\text{Here, } e_f(0) = \begin{cases} \frac{3n+1}{2} & n \text{ is odd} \\ \frac{3n}{2} & n \text{ is even} \end{cases}$$

$$e_f(1) = \begin{cases} \frac{3n-1}{2} & n \text{ is odd} \\ \frac{3n}{2} & n \text{ is even} \end{cases}$$

Hence the graph satisfies the condition $|e_f(0) - e_f(1)| \leq 1$

Therefore, the helm H_n is a 1-near mean cordial graph.

Illustration 3: The 1-near mean cordial graph of H_5 is shown in the figure 3

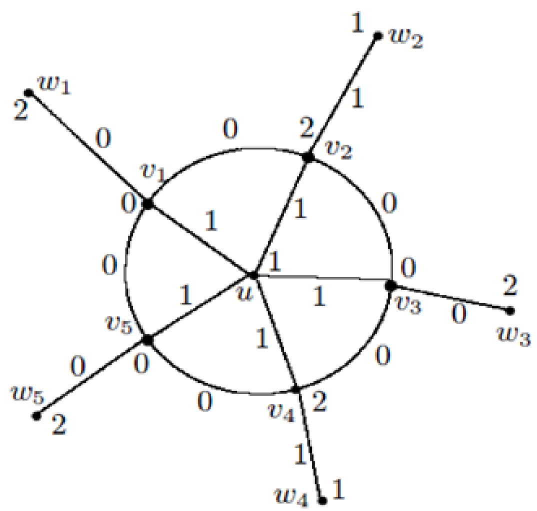

Figure 3 : H_5

Theorem 3.4: A closed helm CH_n is a 1-Near Mean Cordial Graph.

Proof: Let G = (V, E) be a simple graph.

Let G be CH_n.

Let V (G) = {u, v_i: $1 \leq i \leq n$, w_i : $1 \leq i \leq n$} and

E(G) = {[(uv_i), (w_iv_i) : $1 \leq i \leq n$] ∪ [(v_iv_{i+1}), (w_iw_{i+1}) : $1 \leq i \leq n-1$] ∪ [(v_nv_1), (w_nw_1)]}

Define f: V (G) → {0, 1, 2} by

f(u) = 1

$f(v_i)= \begin{cases} 0 & i \equiv 1 \; mod \; 2 \\ 2 & i \equiv 0 \; mod \; 2 \end{cases}$ $\quad 1 \leq i \leq n$

$f(w_i) = 1$, $\quad 1 \leq i \leq n$

The induced edge labeling are
$f^*(uv_i) = 1$, $\quad 1 \leq i \leq n$
$f^*(v_iw_i) = 1$, $\quad 1 \leq i \leq n$
$f^*(v_iv_{i+1}) = 0$, $\quad 1 \leq i \leq n-1$
$f^*(w_iw_{i+1}) = 0$, $\quad 1 \leq i \leq n-1$
$f^*(v_nv_1) = 0$
$f^*(w_nw_1) = 0$

Here, $e_f(0) = e_f(1) = 2n$

Hence the graph satisfies the condition $| e_f(0) - e_f(1) | \leq 1$

Therefore, the closed helm CH_n is a 1-near mean cordial graph.

Illustration 4: The 1-near mean cordial graph of CH_4 is shown in the figure 4,

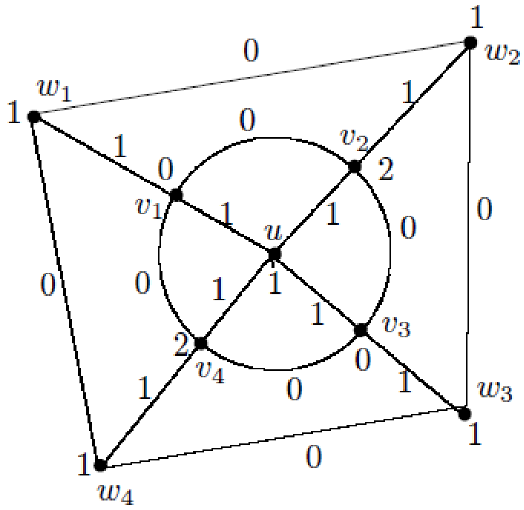

Figure 4 : CH_4

Theorem 3.5: A flower graph Fl_n is a 1- Near Mean Cordial Graph.

Proof: Let G = (V, E) be a simple graph.

Let G be Fl_n.

Let V (G) = {u, v_i: $1 \leq i \leq n$, w_i : $1 \leq i \leq n$} and

E(G) = {[(uv_i), (uw_i), (w_iv_i) : $1 \leq i \leq n$] ∪ [(v_iv_{i+1}) : $1 \leq i \leq n-1$] ∪ (v_nv_1)}

Define f : V (G) → {0, 1, 2} by

f(u) = 1

$f(v_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 2 & i \equiv 0 \bmod 2 \end{cases}$ $1 \le i \le n$

$f(w_i) = \begin{cases} 1 & i \equiv 0 \bmod 2 \\ 2 & i \equiv 1 \bmod 2 \end{cases}$ $1 \le i \le n$

The edge induced labeling are,
$f^*(uv_i) = 1$, $1 \le i \le n$

$f^*(v_iv_{i+1}) = 0$, $1 \le i \le n-1$

$f^*(v_nv_1) = 0$,

$f^*(uw_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases}$ $1 \le i \le n$

$f^*(w_iv_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases}$ $1 \le i \le n$

Here, $e_f(0) = e_f(1) = 2n$

Hence the graph satisfies the condition $| e_f(0) - e_f(1) | \le 1$

Therefore, a flower graph Fl_n is a 1-near mean cordial graph.

Illustration 5: The 1-near mean cordial graph of Fl_5 is shown in the figure 5,

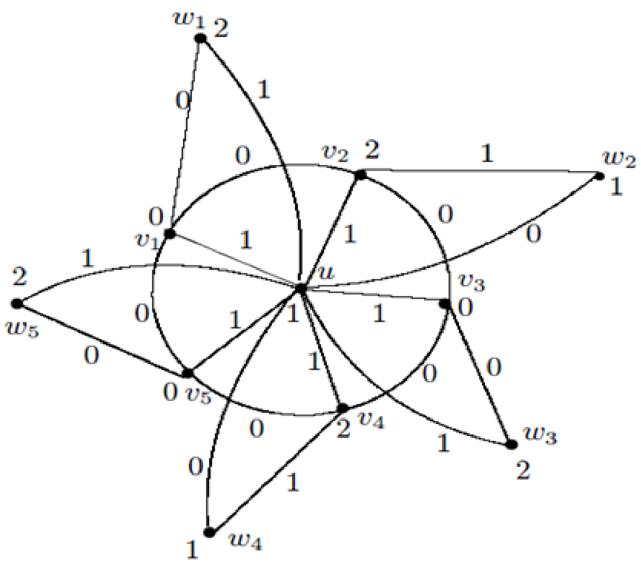

Figure 5 : Fl_5

Theorem 3.6: The sunflower graph S_n is a 1-Near Mean Cordial Graph.

Proof: Let G = (V, E) be a simple graph.

Let G be S_n.

Let $V(G) = \{u, v_i : 1 \le i \le n, w_i : 1 \le i \le n, x_i : 1 \le i \le n\}$ and

$E(G) = \{[(uv_i), (uw_i), (ux_i), (w_iv_i) : 1 \le i \le n] \cup [(v_iv_{i+1}) : 1 \le i \le n-1] \cup (v_nv_1)\}$

Define f : V (G) → {0, 1, 2} by

$f(u) = 1$

$f(v_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 2 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$

$f(w_i) = \begin{cases} 1 & i \equiv 0 \bmod 2 \\ 2 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$

$f(x_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$

The edge induced labeling are,

$f^*(uv_i) = 1, \quad 1 \leq i \leq n$

$f^*(v_iv_{i+1}) = 0, \quad 1 \leq i \leq n - 1$

$f^*(v_nv_1) = 0,$

$f^*(uw_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$

$f^*(w_iv_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$

$f^*(ux_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$

Here, $e_f(0) = \begin{cases} \frac{5n+1}{2} & n \text{ is odd} \\ \frac{5n}{2} & n \text{ is even} \end{cases}$

$e_f(1) = \begin{cases} \frac{5n-1}{2} & n \text{ is odd} \\ \frac{5n}{2} & n \text{ is even} \end{cases}$

Hence the graph satisfies the condition $| e_f(0) - e_f(1) | \leq 1$

Therefore, the sunflower graph S_n is a 1-near mean cordial graph.

Illustration 6: The 1-near mean cordial graph of S_4 is shown in the figure 6

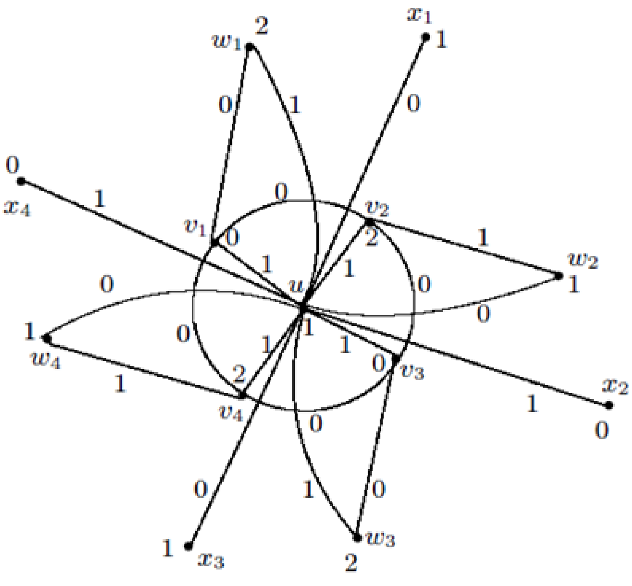

Figure 6 : S_4

Theorem 3.7: The splitting graph $S(K_{1,n})$ is a 1- Near Mean Cordial Graph
.
Proof: Let G = (V, E) be a simple graph.

Let G be $S(K_{1,n})$.

Let $V(G) = \{u, v, u_i : 1 \leq i \leq n, v_i : 1 \leq i \leq n\}$ and $E(G) = \{(uu_i), (uv_i), (vu_i) : 1 \leq i \leq n\}$

Define $f: V(G) \rightarrow \{0, 1, 2\}$ by

$f(u) = 1$

$f(v) = 0$

$$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 3 \\ 1 & i \equiv 1 \bmod 3 \\ 2 & i \equiv 2 \bmod 3 \end{cases} \quad 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

The edge induced labeling are,

$$f^*(uu_i) = \begin{cases} 0 & i \equiv 1 \bmod 3 \\ 1 & i \equiv 0,2 \bmod 3 \end{cases} \quad 1 \leq i \leq n$$

$$f^*(vu_i) = \begin{cases} 0 & i \equiv 0,2 \bmod 3 \\ 1 & i \equiv 1 \bmod 3 \end{cases} \quad 1 \leq i \leq n$$

$$f^*(uv_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

Here, $$e_f(0) = \begin{cases} \frac{3n+1}{2} & n \text{ is odd} \\ \frac{3n}{2} & n \text{ is even} \end{cases}$$

$$e_f(1) = \begin{cases} \frac{3n-1}{2} & n \text{ is odd} \\ \frac{3n}{2} & n \text{ is even} \end{cases}$$

Hence the graph satisfies the condition $|e_f(0) - e_f(1)| \leq 1$

Therefore, the splitting graph $S(K_{1,n})$ is a 1-near mean cordial graph.

Illustration 7: The 1- near mean cordial graph of $S(K_{1,4})$ is shown in the figure 7,

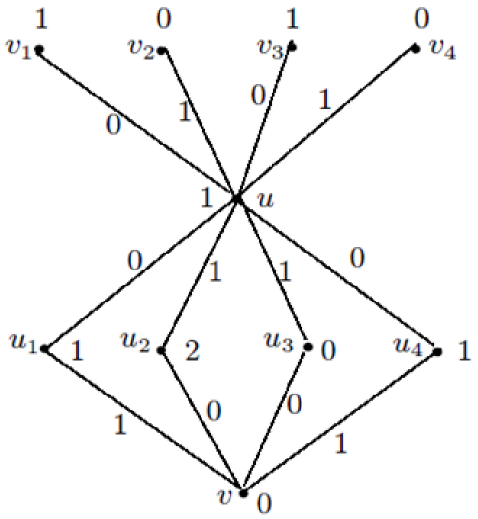

Figure 7 : $S(K_{1,4})$

REFERENCES

1. Albert Williami, Indra Rajasingh and Roy.S, Mean Cordial Labeling of Certain Graphs, Journal of Computer and Mathematical Sciences, vol 4, Issue 4, 31 Augest, 2013 pages (201-321)
2. Gallian.J.A, A Dynamic Survey of Graph Labeling. The Electronic Journal of Combinatorics 6, #D4, 5S6, 2001.
3. Harary.F, Graph Theory, Addison-Wesley Publishing Company Inc, USA, 1969
4. Nellai Murugan.A and Esther.G, Path related Mean Cordial Graphs, Journal of Global Research in Mathematical Archives, ISSN:2320-5822, Volume 11,No.3, March 2014
5. Nellai Murugan.A and Esther.G, Some Results on Mean Cordial Graphs, International Journal of Mathematics Trends and Technology, ISSN:2231- 5373, Volume 11, No.2, July 2014
6. Palani.K, Rejila Jeya Surya.J, 1-Near Mean Cordial Labeling of Graphs, IJMA-6(7), Jully 2015 PP15-20
7. Raja Ponraj, Muthirulan Sivakumar and Murugesan Sundaram, Mean Cordial Labeling of Graphs, Open Journal of Discrete Mathematics, 2012, 2, 145-148.

Source of support: Nil, Conflict of interest: None Declared