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A COMMON FIXED POINT THEOREM FOR SIX MAPPINGS IN INTUITIONISTIC FUZZY METRIC SPACE

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ABSTRACT

In the present paper, a common fixed point theorem for six mappings satisfying a rational inequality has been obtained in intuitionistic fuzzy metric space, the result generalized the result given by Q. H. khan [14] in intuitionistic fuzzy metric space.

Keywords: *Complete Intuitionistic fuzzy metric space, compatible mapping, Weakly Compatible mapping, Coincidence point.*

Mathematics Subject classification: *Primary 47H10 and Secondary 54H25.*

1. INTRODUCTION

The concept of fuzzy sets was initially investigated by Zadeh [12] as a new way to represent vagueness in everyday life. Subsequently, it was developed by many authors and used in various fields. To use this concept in Topology and Analysis, several researchers have defined Fuzzy metric space in various ways. Atanassov [2] introduced the concept of Intuitionistic fuzzy sets by generalizing the notion of fuzzy set by treating membership as a fuzzy logical value has to be consistent (in the sense $\gamma_A(x) + \mu_A(x) \geq 1$). $\gamma_A(x)$ *and* $\mu_A(x)$ denotes degree of membership and degree of non–membership, respectively. All results hold of fuzzy sets can be transformed intuitionistic fuzzy sets but converse need not be true. In 2004, Park [6] defined the notion of intuitionistic fuzzy metric space with the help of continuous t- norm and continuous t-conorm. Since the intuitionistic fuzzy metric space has extra conditions see [3], [11] modified the idea of intuitionistic fuzzy metric space and presented the new notion of intuitionistic fuzzy metric space with the help of continuous t- norm and continuous t-conorm.

We recall some definitions and known results in intuitionistic fuzzy metric space.

2. BASIC DEFINITIONS AND PRELIMINARIES

Definition 2.1: [9] A binary operation *:[0,1]x[0,1]→[0,1] is called a *t-norm* * satisfies the following conditions:

i. * is continuous,
ii. * is commutative and associative,
iii. a * 1 =a for all a ∈ [0, 1],
iv. a * b ≤ c * d whenever a ≤ c and b ≤ d for all a, b, c, d ∈ [0,1].

Examples of *t-norm* a * b = ab and a * b=min{a, b}

Definition 2.2: [9]A binary operation ◊:[0,1]x[0,1]→[0,1] is said to be continuous *t-conorm* if it satisfied the following conditions:

i. ◊ is associative and commutative,
ii. a ◊ 0 = a for all a ∈ [0,1],
iii. ◊ is continuous,
iv. a ◊ b ≤ c ◊ d whenever a ≤ c and b ≤ d for each a, b, c, d ∈ [0,1]

Examples of *t-conorm* a ◊ b = min(a+b , 1) and a ◊ b = max(a, b)

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Remark 2.1: [1] The concept of triangular norms (*t-norm) and* triangular conorms (*t-conorm)* are knows as axiomatic skeletons that we use for characterizing fuzzy intersections and union respectively.

Definition 2.3: [1] A 5- tuple (X, M, N, *,◊) is called intuitionistic fuzzy metric space if X is an arbitrary non empty set, * is a continuous *t-norm,* ◊ continuous *t-conorm* and M, N are fuzzy sets on X^2x [0,∞] satisfying the following conditions:
For each x, y, z ∈ X and t, s > 0
(IFM-1) M(x, y, t) + N(x, y, t) ≤ 1,
(IFM-2) M(x, y, 0) = 0, for all x, y in X,
(IFM-3) M(x, y, t) = 1 for all x, y in X and t > 0 if and only if x = y,
(IFM-4) $M(x, y, t) = M(y, x, t)$, for all x, y in X and t > 0,
(IFM-5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(IFM-6) $M(x, y, .): [0, \infty] \to [0,1]$ is left continuous,
(IFM-7) $\lim_{t\to\infty} M(x, y, t) = 1$,
(IFM-8) $N(x, y, 0) = 1$, for all x, y in X,
(IFM-9) $N(x, y, t) = 0$, for all x, y in X and t > 0 if and only if x = y,
(IFM-10) $N(x, y, t) = N(y, x, t)$, for all x, y in X and t > 0,
(IFM-11) $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s)$,
(IFM-12) $N(x, y, .): [0, \infty] \to [0,1]$ is right continuous,
(IFM-13) $\lim_{t\to\infty} N(x, y, t) = 0$, for all x, y in X and $t > 0$.
Then (M, N) is called an intuitionistic fuzzy metric on X. The function M(x, y, t) and N(x, y, t) denote the degree of nearness and degree of non nearness between x and y with respect to t, respectively.

Remark 2.2: [10] Intuitionistic Fuzzy Metric space, M(x, y, .) is non decreasing and N(x, y, .) is non increasing for all x, y ∈ [0, 1].

Example 2.1: [6] Let (X, d) be a metric space. Define a * b = ab and a ◊ b = min{1, a+b}, for all a, b ∈ [0, 1] and let M and N be fuzzy sets on X^2 x (0, ∞) defined as follows:

$$M(x, y, t) = \frac{t}{t+d(x,y)} \text{ and } N(x, y, t) = \frac{d(x,y)}{t+d(x,y)} \text{ for all } x, y \in X \text{ and all } t > 0.$$

then (M, N) is called an intuitionistic fuzzy metric space on X. We call this intuitionistic fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric.

Remark 2.3: Note that the above examples holds even with the t- norm a*b = min{a, b}and t-conorm a◊b= max{a, b} and hence (M, N) is an intuitionistic fuzzy metric with respect to any continuous t – norm and continuous t – conorm.

Definition 2.4: [6]A sequence $\{x_n\}$ in intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be cauchy sequence if and only if for each ε > 0, t > 0, there exists n_0 ∈ N such that

$$M(x_n, x_m, t) > 1\text{-}\varepsilon \text{ and } N(x_n, x_m, t) < \varepsilon \text{ for all } n, m \geq n_0.$$

The sequence $\{x_n\}$ converse to a point x in X if and only if for each ε > 0, t > 0, there exists n_0 ∈ N such that

$$M(x_n, x, t) > 1\text{-}\varepsilon \quad \text{for all } n \geq n_0.$$

An Intuitionistic Fuzzy metric space (X, M, N, *,◊) is said to be complete if every Cauchy sequence in it converges to a point in it.

Definition 2.5: [10]The intuitionistic fuzzy metric space (X, M, N, *,◊) is said to be (M, N) complete if every (M, N) Cauchy sequence is convergent.

Definition 2.6: [10] A pair of self mapping (A, S) of a intuitionistic fuzzy metric space (X, M, N,◊*),is said to be commuting if $M(ASx, SAx, t) = 1$ *and* $N(ASx, SAx, t) = 0$ *for all* $x \in X$.

Definition 2.7: [1] A pair of self mapping (A, S) of a intuitionistic fuzzy metric space (X, M, N, *,◊) is said to be weakly commuting if

$$M(ASx, SAx, t) \geq M(Ax, Sx, t) \text{ and } N(ASx, SAx, t) \leq N(Ax, Sx, t) \text{ for all } x \in X \text{ and } t > 0.$$

Definition 2.8: [10] A pair of self mapping (A, S) of a intuitionistic fuzzy metric space (X, M, N, *,◊) is said to be compatible if $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ *and* $\lim_{n\to\infty} N(ASx_n, SAx_n, t) = 0$ *for all* $t > 0$. whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = u$ *for some* $u \in X$.

Definition 2.9: [8] A pair (*f*, *g*) of self-mappings of a metric space (*X*, *d*) is said to be weakly compatible mappings if the mappings commute at all of their coincidence points, i.e., *fx* = *gx* for some *x*∈*X* implies *fgx* = *gfx*.

Lemma 2.1: [1] Let (X, M, N, *, ◊) Intuitionistic fuzzy metric space, If there exists k ∈(0, 1) such that for all x, y ∈ X, M(x, y, kt) ≥ M(x, y, t) and, N(x, y, kt) ≤ N(x, y, t) for all t > 0, then x = y.

Lemma 2.2: Let (X, M, N, *, ◊) be an Intuitionistic Fuzzy metric space. f and g be self maps on X and let f and g have a unique point of coincidence, w = fx = gx, then w is the unique common fixed point of f and g.

Theorem 2.1: [7] Let A, B, S, T, I and J be self- mappings of a complete metric space (X, d) satisfying AB(X) ⊂ J(X), ST(X) ⊂ I(X) $\forall\, x, y \in X$

d(ABx, STy) $\leq \alpha_1 \left[\frac{d(Ix,Jy)d(STy,Jx)}{d(Ix,Jy)+d(STy,Jx)}\right] + \alpha_2[d(ABx,Ix) + d(STy,Jy)] + \alpha_3[d(STy,Ix) + d(ABx,Jy)] + \alpha_4 d(Ix,Jy)$

if $d(Ix,Jy) + d(STy,Jy) \neq 0, \forall\, x,y \in X, \alpha_i \geq 0, (i = 1,2,3,4)$ with atleast one α_i is non zero and

$$\alpha_1 + 2\alpha_2 + 2\alpha_3 + \alpha_4 < 1,$$

or $$d(ABx,\ STy) = 0 \ \ if\ d(Ix,Jy) + \ d(STy,Jy) = 0, \forall\, x,y \in X$$

either pair {AB, I} is compatible and pair (ST, J) is weakly compatible, I or AB is continuous.
or
either pair {ST, J} is compatible and pair (AB, I) is weakly compatible, J or ST is continuous.

Then AB, ST, I and J have a unique common fixed point. Further- more if the pairs (A, B), (A, I), (B, I), (S, T), (S, J) and (T, J) are commuting mappings then A, B, S, T, I and J have a unique common fixed point.

The purpose of this paper is to present an extension of theorem 2.1 from metric space to intuitionistic fuzzy metric space by taking the property of intuitionistic fuzzy metric space and weaker conditions compatibility, weak compatibility of maps than that of commutativity condition.

MAIN RESULT

Theorem 3.1: Let A, B, S, T, I and J be self- mappings of a complete Intuitionistic fuzzy metric space (X, M, N, *, ◊) satisfying

AB(X) ⊂ J(X) , ST(X) ⊂ I(X) $\forall\, x, y \in X$ (3.1.1)

M(ABx, STy, t) $\leq \alpha_1 \left[\frac{M(Ix,Jy,t)M(STy,Ix,t)}{M(Ix,Jy,t)+M(STy,Jx,t)}\right] + \alpha_2 \begin{bmatrix} M(ABx,Ix,t) \\ +M(STy,Jy,t)\end{bmatrix} + \alpha_3 \begin{bmatrix} M(STy,Ix,t) \\ +M(ABx,Jy,t)\end{bmatrix} + \alpha_4 M(Ix,Jy,t)$

N(ABx, STy, t) $\geq \alpha_1 \left[\frac{N(Ix,Jy,t)N(STy,Ix,t)}{N(Ix,Jy,t)+N(STy,Jx,t)}\right] + \alpha_2 \begin{bmatrix} N(ABx,Ix,t) \\ +N(STy,Jy,t)\end{bmatrix} + \alpha_3 \begin{bmatrix} N(STy,Ix,t) \\ +N(ABx,Jy,t)\end{bmatrix} + \alpha_4 N(Ix,Jy,t)$ (3.1.2)

if $M(Ix,Jy,t) + M(STy,Jy,t) \neq 1, and\ N(Ix,Jy,t) + N(STy,Jy,t) \neq 0, \quad \forall\, x,y \in X, \alpha_i \geq 0, (i = 1,2,3,4)$ with atleast one α_i is non zero and $\alpha_1 + 2\alpha_2 + 2\alpha_3 + \alpha_4 < 1$,

or $$M(ABx,\ STy,t) = 1\ and\ N(ABx,\ STy,t) = 0,\ if\ M(Ix,Jy,t) + \ M(STy,Jy,t) = 1$$ $$and\ N(Ix,Jy,t) + \ N(STy,Jy,t) = 0, \forall\, x,y \in X$$

either pair {AB, I} is compatible and pair (ST, J) is weakly compatible, I or AB is continuous. (3.1.3)
or
either pair {ST, J} is compatible and pair (AB, I) is weakly compatible, J or ST is continuous. (3.1.4)

Then AB, ST, I and J have a unique common fixed point. Further- more the pair (A, B), (A, I), (B, I), (S, T), (S, J) and (T, J) are commuting mappings then A, B, S, T, I and J have a unique common fixed point.

Proof: Let x_0 be an arbitrary point in X. Since AB(X) ⊂ J(X) then there is a point x_1 in X such that $ABx_0 = Jx_1$. Also since ST(X) ⊂ I(X), there exists a point x_2 with $STx_1 = Ix_2$. Using this argument repeatedly one can construct a sequence $\{z_n\}$ such that

$$z_{2n} = ABx_{2n} = Jx_{2n+1}, z_{2n+1} = STx_{2n+1} = Ix_{2n+2} \ \ for\ n = 0,1,2, \dots..$$

Using (3.1.2), we have

$$\begin{aligned} M(z_{2n+1}, z_{2n+2}, t) &= M\big(STx_{2n+1}, ABx_{2n+2,}t\big) \\ &\leq \alpha_1 \left[\frac{M(Ix_{2n+2}, Jx_{2n+1}, t)M(STx_{2n+1}, Ix_{2n+2}, t)}{M(Ix_{2n+2}, Jx_{2n+1}, t) + M(STx_{2n+1}, Jx_{2n+1}, t)}\right] \\ &+ \alpha_2\big[M\big(ABx_{2n+2}, Ix_{2n+2,}, t\big) + M\big(STx_{2n+1}, Jx_{2n+1,}t\big)\big] \\ &+ \alpha_3\big[M\big(ABx_{2n+2}, Jx_{2n+1,}, t\big) + M\big(STx_{2n+1}, Ix_{2n+2,}, t\big)\big] + \alpha_4 M\big(Ix_{2n+2}, Jx_{2n+1,}, t\big) \end{aligned}$$

and

$$\begin{aligned} N(z_{2n+1}, z_{2n+2}, t) &= N(STx_{2n+1}, ABx_{2n+2,}t) \\ &\geq \alpha_1\left[\frac{N(Ix_{2n+2}, Jx_{2n+1}, t)N(STx_{2n+1}, Ix_{2n+2}, t)}{N(Ix_{2n+2}, Jx_{2n+1}, t) + N(STx_{2n+1}, Jx_{2n+1}, t)}\right] \\ &+ \alpha_2\left[N\left(ABx_{2n+2}, Ix_{2n+2,}, t\right) + N(STx_{2n+1}, Jx_{2n+1,}t)\right] \\ &+ \alpha_3\left[N\left(ABx_{2n+2}, Jx_{2n+1,}, t\right) + N\left(STx_{2n+1}, Ix_{2n+2,}, t\right)\right] + \alpha_4 N\left(Ix_{2n+2}, Jx_{2n+1,}, t\right) \end{aligned}$$

implies that $M(z_{2n+1}, z_{2n+2}, t) \leq kM(z_{2n+1}, z_{2n}, t)$

and $N(z_{2n+1}, z_{2n+2}, t) \geq kN(z_{2n+1}, z_{2n}, t)$, where $k = \frac{\alpha_2+\alpha_3+\alpha_4}{1-\alpha_2-\alpha_3} < 1$.

Similarly $M(z_{2n}, z_{2n+1}, t) \leq kM(z_{2n-1}, z_{2n}, t)$

and $N(z_{2n}, z_{2n+1}, t) \geq kN(z_{2n-1}, z_{2n}, t)$

Hence for all n $M(z_n, z_{n+1}, t) \leq kM(z_{n-1}, z_n, t)$

and $N(z_n, z_{n+1}, t) \geq kN(z_{n-1}, z_n, t)$,

which shows that $\{z_n\}$ is a Cauchy sequence in X. By the completeness of X there exists some z in X such that sequence $\{z_n\}$ and its subsequence $\{z_{2n}\}$ and $\{z_{2n+1}\}$ are also converge to z in X.

Now assuming the continuity of I, $\{I^2x_{2n}\}$ and $\{IABx_{2n}\}$ converges to Iz. Also in view of compatibility of $\{I, AB\}, \{ABIx_{2n}\}$ converges to Iz.

Using (3.1.2), we have

$$\begin{aligned} &M\left(ABIx_{2n}, STx_{2n+1,}t\right) \\ &\leq \alpha_1\left[\frac{M(I^2x_{2n}, Jx_{2n+1}, t)M(STx_{2n+1}, I^2x_{2n}, t)}{M(I^2x_{2n}, Jx_{2n+1}, t) + M(STx_{2n+1}, Jx_{2n+1}, t)}\right] \\ &+ \alpha_2\left[M(ABIx_{2n}, I^2x_{2n}, t) + M\left(STx_{2n+1}, Jx_{2n+1,}t\right)\right] \\ &+ \alpha_3\left[M\left(ABIx_{2n}, Jx_{2n+1,}, t\right) + M(STx_{2n+1}, I^2x_{2n}, t)\right] + \alpha_4 M\left(I^2x_{2n}, Jx_{2n+1,}, t\right) \end{aligned}$$

and

$$\begin{aligned} &N\left(ABIx_{2n}, STx_{2n+1,}t\right) \\ &\geq \alpha_1\left[\frac{N(I^2x_{2n}, Jx_{2n+1}, t)N(STx_{2n+1}, I^2x_{2n}, t)}{N(I^2x_{2n}, Jx_{2n+1}, t) + N(STx_{2n+1}, Jx_{2n+1}, t)}\right] \\ &+ \alpha_2\left[N(ABIx_{2n}, I^2x_{2n}, t) + N\left(STx_{2n+1}, Jx_{2n+1,}t\right)\right] \\ &+ \alpha_3\left[N\left(ABIx_{2n}, Jx_{2n+1,}, t\right) + N(STx_{2n+1}, I^2x_{2n}, t)\right] + \alpha_4 N\left(I^2x_{2n}, Jx_{2n+1,}, t\right) \end{aligned}$$

Taking n→ ∞, M(Iz, z ,t)=1 and N(Iz, z, t)=0 we have Iz = z.

Further using (3.1.2), we have

$$\begin{aligned} &M(ABz, STx_{2n+1}, t) \\ &\leq \alpha_1\left[\frac{M(Iz, Jx_{2n+1}, t)M(STx_{2n+1}, Iz, t)}{M(Iz, Jx_{2n+1}, t) + M(STx_{2n+1}, Jx_{2n+1}, t)}\right] \\ &+ \alpha_2\left[M(ABz, Iz, t) + M\left(STx_{2n+1}, Jx_{2n+1,}t\right)\right] \\ &+ \alpha_3\left[M\left(ABz, Jx_{2n+1,}, t\right) + M(STx_{2n+1}, Iz, t)\right] + \alpha_4 M\left(Iz, Jx_{2n+1,}, t\right) \end{aligned}$$

and

$$\begin{aligned} &N(ABz, STx_{2n+1}, t) \\ &\geq \alpha_1\left[\frac{N(Iz, Jx_{2n+1}, t)N(STx_{2n+1}, Iz, t)}{N(Iz, Jx_{2n+1}, t) + N(STx_{2n+1}, Jx_{2n+1}, t)}\right] \\ &+ \alpha_2\left[N(ABz, Iz, t) + N\left(STx_{2n+1}, Jx_{2n+1,}t\right)\right] \\ &+ \alpha_3\left[N\left(ABz, Jx_{2n+1,}, t\right) + N(STx_{2n+1}, Iz, t)\right] + \alpha_4 N\left(Iz, Jx_{2n+1,}, t\right) \end{aligned}$$

On letting $n \to \infty$ and using Iz = z, we get M(ABz, z ,t) = 1 and N(ABz, z, t) = 0, yield that ABz = z.

Since $AB(x) \subset J(x)$ there exists a point z' such that JZ' = z so that STJZ' = STz.

Further using (3.1.2), we have

$$M(z,STz',t) = M(ABz,STz',t) \le \alpha_1\left[\frac{M(Iz,Jz',t)M(STz',Iz,t)}{M(Iz,Jz',t)+M(STz',Jz',t)}\right] + \alpha_2[M(ABz,Iz,t)+M(STz',Jz',t)] + \alpha_3[M(ABz,Jz',t)+M(STz',Iz,t)] + \alpha_4 M(Iz,Jz',t)$$

and

$$N(z,STz,t) = N(ABz,STz',t) \ge \alpha_1\left[\frac{N(Iz,Jz',t)N(STz',Iz,t)}{N(Iz,Jz',t)+N(STz',Jz',t)}\right] + \alpha_2[N(ABz,Iz,t)+N(STz',Jz',t)] + \alpha_3[N(ABz,Jz',t)+N(STz',Iz,t)] + \alpha_4 N(Iz,Jz',t)$$

implies that M(z, STz', t) = 1 and N(z, STz', t) = 0, yield that STz' = z = Jz' which shows that z' is the coincidence point of ST and J. Now using the weak compatibility of (ST, J), we have STz = ST(Jz') =J(STz') = Jz, which shows that z is also coincidence point of the pair (ST, J).

Now from (3.1.2), we have

$$M(z,STz,t) = M(ABz,STz,t) \le \alpha_1\left[\frac{M(Iz,Jz,t)M(STz,Iz,t)}{M(Iz,Jz,t)+M(STz,Jz,t)}\right] + \alpha_2[M(ABz,Iz,t)+M(STz,Jz,t)] + \alpha_3[M(ABz,Jz,t)+M(STz,Iz,t)] + \alpha_4 M(Iz,Jz,t)$$

and

$$N(z,STz,t) = N(ABz,STz,t) \ge \alpha_1\left[\frac{N(Iz,Jz,t)N(STz,Iz,t)}{N(Iz,Jz,t)+N(STz,Jz,t)}\right] + \alpha_2[N(ABz,Iz,t)+N(STz,Jz,t)] + \alpha_3[N(ABz,Jz,t)+N(STz,Iz,t)] + \alpha_4 N(Iz,Jz,t)$$

Implies that M(z, STz, t)=1 and N(z, STz, t) = 0, yield that z = STz = Jz which shows that z is a common fixed of AB, I, ST and J.

Similarly it can be proved that if we suppose AB is continuous then z is the common fixed point of AB, I, ST and J. Also proof the similar if we take the maps ST or J is continuous, pair (ST, J) is compatible and pair (AB, I) is weakly compatible instead of the maps AB or I continuous, pair (AB, I) is compatible and pair (ST, J) is weakly compatible.

For the uniqueness of z suppose v be another fixed point of AB, I, ST and J then from (3.1.2), we have

$$M(z,v,t) = M(ABz,STv,t) \le \alpha_1\left[\frac{M(Iz,Jv,t)M(STv,Iz,t)}{M(Iz,Jv,t)+M(STv,Jv,t)}\right] + \alpha_2[M(ABz,Iz,t)+M(STv,Jv,t)] + \alpha_3[M(ABz,Jv,t)+M(STv,Iz,t)] + \alpha_4 M(Iz,Jv,t)$$

and

$$N(z,v,t) = N(ABz,STv,t) \ge \alpha_1\left[\frac{N(Iz,Jv,t)N(STv,Iz,t)}{N(Iz,Jv,t)+N(STv,Jv,t)}\right] + \alpha_2[N(ABz,Iz,t)+N(STv,Jv,t)] + \alpha_3[N(ABz,Jv,t)+N(STv,Iz,t)] + \alpha_4 N(Iz,Jv,t)$$

Implies that $M(z,v,t) = 1$ and $N(z,v,t) = 0$, yield that z = v, i.e., z is unique common fixed point of AB, ST, I and J.

Finally, we need to show that z is common fixed point of A, B, S, T, I and J. For this let z be the unique common fixed point of both the pairs (AB, I) and (ST, J). From (3.1.4), we have

$$Az = A(ABz) = A(BAz) = AB(Az), Az = A(Iz) = I(Az),$$

$$Bz = B(ABz) = B\big(A(Bz)\big) = BA(Bz) = AB(Bz), Bz = B(Iz) = I(Bz),$$

which shows that Az and Bz is a common fixed point of both the pairs (AB, I) yield that $Az = z = Bz = Iz = ABz$ in the view of uniqueness of the common fixed point of the pair (AB, I).

Similarly $= Sz = Tz = J = STz$. Hence z is common fixed point of A, B, S, T, I and J.

(II) Suppose $M(Ix,Jx,t) + M(STy,Jy,t) = 1$

implies $M(ABx,STy,t) = 1$

and $\qquad N(Ix, Jx, t) + N(STy, Jy, t) = 0$

implies $\qquad N(ABx, STy, t) = 0$

then we argue as follows:

Suppose there exists an n such that $z_n = z_{n+1}$. Then also $z_{n+1} = z_{n+2}$ because if it not so then from (3.1.3) we have

$$0 < M(z_{n+1}, z_{n+2}, t) \le kM(z_n, z_{n+1}, t)$$

and

$$1 > N(z_{n+1}, z_{n+2}, t) \ge kN(z_n, z_{n+1}, t) \text{ yielding thereby } z_{n+1} = z_{n+2}.$$

Thus $z_n = z_{n+1}$ for k = 1,2,…… It then follows that there exist two w_1 *and* w_2 such that $v_1 = ABw_1 = Iw_1$ *and* $v_2 = STw_2 = Iw_2$. Since $M(Iw_1, Jw_2, t) + M(STw_2, Jw_2, t) = 1$ and $N(Iw_1, Jw_2, t) + N(STw_2, Jw_2, t) = 0$ from (3.1.2) $M(ABw_1, STw_2, t) = 1$ and $N(ABw_1, STw_2, t) = 0$ i.e. $v_1 = ABw_1 = STw_2 = v_2$. Note also that $Iv_1 = I(ABw_1) = AB(Iw_1) = ABv_1$. Similarly $STv_2 = Jv_2$.

Define $y_1 = ABv_1$, $y_2 = STv_2$. Since $M(Iv_2, Jv_2, t) + M(STv_2, Jv_2, t) = 1$ and $N(Iv_2, Jv_2, t) + N(STv_2, Jv_2, t) = 0$ it follows (3.1.2) $M(ABv_1, STv_2, t) = 1$ and $N(ABv_1, STv_2, t) = 0$ i.e. $y_1 = y_2$. Thus $ABv_1 = Iv_1 = STv_2 = Jv_2$. But $v_1 = v_2$. Therefore AB, I, ST and J have common coincidence point.

Define $w = ABv_1$. it then follows that w is also a common coincidence point AB, I, ST and J. If $ABw \ne ABv_1 = STv_1$, then $M(ABw, STv_1, t) < 1$ *and* $N(ABw, STv_1, t) > 0$. But since $M(Iw, Jv_1, t) + M(STv_1, Jv_1, t) = 1$ *and* $N(Iw, Jv_1, t) + N(STv_1, Jv_1, t) = 0$,it follows from (3.1.2) that $M(ABw, STv_1, t) = 1$ *and* $N(ABw, STv_1, t) = 0$ i.e. $ABw = STv_1$ a contradiction. Therefore $ABw = ABv_1 = w$ and w is a common fixed point of AB, I, ST and J .

The rest of the proof is similar to the case (I), hence it is omitted.

This complete the proof.

Corollary 3.1: Let A, S, I and J be self- mappings of a complete Intuitionistic fuzzy metric space (X, M, N, *, ◊) satisfying
A(X) ⊂ J(X), S(X) ⊂ I(X) $\forall\, x, y \in X$

$$M(Ax, Sy, t) \le \alpha_1 \left[\frac{M(Ix,Jy,t)M(Sy,Ix,t)}{M(Ix,Jy,t)+M(Sy,Jx,t)}\right] + \alpha_2 \begin{bmatrix} M(Ax, Ix, t) \\ +M(Sy, Jy, t) \end{bmatrix} + \alpha_3[M(Sy, Ix, t) + M(Ax, Jy, t)] + \alpha_4 M(Ix, Jy, t)$$

$$N(Ax, Sy, t) \ge \alpha_1 \left[\frac{N(Ix,Jy,t)N(Sy,Ix,t)}{N(Ix,Jy,t)+N(Sy,Jx,t)}\right] + \alpha_2 \begin{bmatrix} N(Ax, Ix, t) \\ +N(Sy, Jy, t) \end{bmatrix} + \alpha_3[N(Sy, Ix, t) + N(Ax, Jy, t)] + \alpha_4 N(Ix, Jy, t)$$

if $M(Ix, Jy, t) + M(Sy, Jy, t) \ne 1$, *and* $N(Ix, Jy, t) + N(Sy, Jy, t) \ne 0$, $\forall\, x, y \in X, \alpha_i \ge 0, (i = 1, 2, 3, 4)$ with atleast one α_i is non zero and $\alpha_1 + 2\alpha_2 + 2\alpha_3 + \alpha_4 < 1$,
or

$$M(Ax, Sy, t) = 1 \text{ and } N(Ax, Sy, t) = 0, \text{ if } M(Ix, Jy, t) + M(Sy, Jy, t) = 1 \text{ and } N(Ix, Jy, t) + N(Sy, Jy, t) = 0, \forall\, x, y \in X$$

either pair {A, I} is compatible and pair (S, J) is weakly compatible, I or A is continuous.
or
either pair {S, J} is compatible and pair (A, I) is weakly compatible, J or S is continuous.

Then A, S, I and J have a unique common fixed point.

Proof: Assuming B and T are Identity mappings, result follows from Theorem 3.1.

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