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LRS BIANCHI TYPE-III UNIVERSE WITH MAGNETIZED WET DARK FLUID IN GENERAL THEORY OF RELATIVITY

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ABSTRACT
LRS Bianchi type –III cosmological model is studied in General Theory of Relativity with the matter magnetized wet dark energy. We have assumed that F_{13} *is only non vanishing component of* F_{ij} *and obtained an exact solution for the field equation. By assuming a relation between metric potential* $C = A^n$ *and to determine the model,* $\frac{\dot{A}}{A} = \frac{\dot{B}}{B} = \frac{\alpha_1}{t^n}$ *and* $\frac{\dot{C}}{C} = \frac{\alpha_2}{t^n}$ *where* α_1 , α_2 *are constant. Further geometrical and kinematical properties are discussed.*

Key Words: *Locally Rotationally symmetric (LRS) Bianchi type –III, electromagnetic field, wet dark energy, general theory of relativity.*

1. INTRODUCATION

As we know that observational data like la supernovae suggest that the universe is dominated by two dark components containing dark energy and dark matter. Dark energy with negative pressure is used to explain the present cosmic accelerating expansion while dark matter is used to explain galactic curves and large scale structure formation.

Origin of the dark energy and dark matter and their natures remains unknown and we hope that large Hadron Collider can give us these hints.

The equation of state for wet Dark fluid is $p_{WDF} = \gamma(\rho_{WDF} - p_*)$

Where, the parameter γ and p_* taken to be positive restrict ourselves to $0 \le \gamma \le 1$.

And we have energy conservation equation as $\rho_{WDF} + 3H(p_{WDF} + \rho_{WDF}) = 0$.

Using equation of state and $3H = \frac{\dot{V}}{V}$ in the above equation, we get

$$\rho_{WDF} = \frac{\gamma}{1+\gamma}\rho_* + \frac{c}{V(1+\gamma)}$$

Where, c is constant of integration and V is volume expansion.

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Wet dark fluid (WDF) has two components: - one be haves as cosmological constant and other as standard fluid with equation of state $p_{WDF} = \gamma \rho_{WDF}$.

If we take c>0 then this fluid will not violet the strong energy condition $p + \rho \geq 0$;

$$(p_{WDF} + \rho_{WDF}) = (1+\gamma)\rho_{WDF} - \gamma\rho_*$$

$$(p_{WDF} + \rho_{WDF}) = (1+\gamma)\frac{c}{V(1+\gamma)}$$

Lorenz [1] have presented Tilted electromagnetic Bianchi type III cosmological solution. Tikekar and Patel [2] obtained some exact solutions of massive string of Bianchi type –III space time presence and absence of magnetic field. Bali and Jain [3] have studied Bianchi type –III non-static magnetized cosmological model for perfect fluid distribution in general relativity. Amirhaschi H. Zainuddin [4-5] have obtained Bianchi type III string cosmological models for perfect fluid distribution and also studied magnetized massive string with bianchi type III in general relativity. Pradhan [6] have presented Massive string cosmology in Bianchi type III space-time with electromagnetic field. Pradhan and Amirhaschi H. Zainuddin [7] have studied Dark energy model in anisotropic Bianchi type –III space time with variable EoS parameter. Adhav [8] have obtained Bianchi type –III magnetized wet dark fluid cosmological model in general relativity. Upadhaya and Dave [9] have investigated some magnetized Bianchi type –III Massive string Cosmological models in general relativity. S. P. Kandalkar, S. W. Samdurkar, S. P. Gawande [10] have obtained Bianchi type –III string cosmological models in the presence of magnetic field in general relativity, G. S. Rathore, K. Mandawat, D. S. Chauhan [11] have investigated Bianchi type –III string cosmological models with bulk viscosity and electromagnetic field. S. D. Deo, G. S. Punwatkar, U. M. Patil [13] also studied Bianchi type III cosmological model electromagnetic field with cosmic string in general theory of relativity. S. D. Deo, G. S. Punwatkar and U. M. Patil [14-15] have studied Bianchi type I, III cosmological models in general theory of relativity.

2. THE METRIC AND FIELD EQUATIONS

We consider the LRS Bianchi type –III metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2ax}dy^2 + C^2dz^2 \tag{2.1}$$

Where a is non -zero constant and A, B and C are function of t only.

T_i^j is the energy momentum tensor of the source wet dark energy coupled with electromagnetic field is denoted by

$$T_i^j = (p_{WDF} + \rho_{WDF})u_iu^j - p_{WDF}\delta_i^j + E_i^j \tag{2.2}$$

Where, p_{WDF} is the isotopic pressure and ρ_{WDF} is the matter density and u^i is the flow vector of the fluid. The flow of the matter is taken orthogonal to the hyper surface of the homogeneity, so that

$$g_{ij}u_iu^j = 1 \tag{2.3}$$

In a co-moving system of coordinates, from (2.2) we find

$$T_1^1 = T_2^2 = T_3^3 = -p_{WDF} \text{ And } T_4^4 = \rho_{WDF} \tag{2.4}$$

Electromagnetic field is defined as

$$E_i^j = -F_{ir}F^{jr} + \frac{1}{4}F_{ab}F^{ab}g_i^j \tag{2.5}$$

Where, E_i^j is electromagnetic energy tensor and F_i^j is the electromagnetic field tensor.

We assume that F_{13} is the only non-vanishing component of F_{ij} which corresponds to the presence of magnetic field along y-direction.

The Einstein field equation in the general relativity is given by

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi kT_i^j \tag{2.6}$$

Where, R_i^j is known as Ricci tensor and $R = g^{ij}R_{ij}$ is the Ricci scalar and T_i^j is energy momentum tensor for matter

The field equations (2.6) together with the line element (2.1) with equations (2.2) we get

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=8\pi G\left[p_{WDF}+\frac{(F_{13})^2}{2A^2C^2}\right] \tag{2.7}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=8\pi G\left[p_{WDF}-\frac{(F_{13})^2}{2A^2C^2}\right] \tag{2.8}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{a^2}{A^2}=8\pi G\left[p_{WDF}+\frac{(F_{13})^2}{2A^2C^2}\right] \tag{2.9}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{a^2}{A^2}=-8\pi G\left[\rho_{WDF}+\frac{(F_{13})^2}{2A^2C^2}\right] \tag{2.10}$$

$$a\left(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\right)=0 \tag{2.11}$$

The expression for scalar expansion θ and shear scalar σ are

$$\theta=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C} \tag{2.12}$$

$$\sigma^2=\frac{1}{2}\left[\frac{\dot{A}^2}{A^2}+\frac{\dot{B}^2}{B^2}+\frac{\dot{C}^2}{C^2}-\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{B}\dot{C}}{BC}-\frac{\dot{A}\dot{C}}{AC}\right] \tag{2.13}$$

And Hubble parameter H and volume expansion V are defined as

$$H=\frac{\theta}{3}=\frac{\dot{V}}{V} \tag{2.14}$$

$$V=(ABC)^{\frac{1}{3}} \tag{2.15}$$

From equation (2.11) become

$$\frac{\dot{B}}{B}=\frac{\dot{A}}{A} \tag{2.16}$$

Integrating, we get

$$B=kA \tag{2.17}$$

Where k is a constant of integration.

By choosing $k=1$ we have $B=A$

The field equations (2.07) to (2.10) reduce to

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=8\pi G\left[p_{WDF}+\frac{(F_{13})^2}{2A^2C^2}\right] \tag{2.18}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=8\pi G\left[p_{WDF}-\frac{(F_{13})^2}{2A^2C^2}\right] \tag{2.19}$$

$$2\frac{\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}-\frac{a^2}{A^2}=8\pi G\left[p_{WDF}+\frac{(F_{13})^2}{2A^2C^2}\right] \tag{2.20}$$

$$\frac{\dot{A}^2}{A^2}+2\frac{\dot{A}\dot{C}}{AC}-\frac{a^2}{A^2}=-8\pi G\left[\rho_{WDF}+\frac{(F_{13})^2}{2A^2C^2}\right] \tag{2.21}$$

3. SOLUTION OF THE FIELD EQUATIONS

Special Case-I: If $F_{13}=0$

From equation (2.18) and (2.19) we have $F_{13}=0$ (3.1)

Which gives us in this theory an LRS bianchi type III cosmological model does not accommodate electromagnetic field.

We assume that the expansion is proportional to the shear which is physically conditions. This condition leads to

$$C=A^n \tag{3.2}$$

Where n is a constant

From equation (2.19) to (2.20) give us

$$\ddot{A}+(1+n)\frac{\dot{A}^2}{A}=\frac{a^2}{(1-n)A} \tag{3.3}$$

Now put $\dot{A}=g(A)$ in equation (3.3), integrating we get

$$g^2=\left(\frac{dA}{dt}\right)^2=\frac{a^2}{\left(1-n^2\right)}+\frac{k_1}{A^{2(1+n)}} \tag{3.4}$$

Where k_1 is integration constant

The Bianchi type –III model in this case reduces to the form

$$ds^2=-\left(\frac{dt}{dA}\right)^2 dA^2+A^2\left[dx^2+e^{-2ax}dy^2\right]+A^{2n}dz^2 \tag{3.5}$$

If we choose A=T, x=X, y=Y, z=Z then equation (3.5) becomes

$$ds^2=-\left[\frac{dT^2}{\frac{a^2}{\left(1-n^2\right)}+\frac{k_1}{T^{2(1+n)}}}\right]+T^2\left[dX^2+e^{-2aX}dY^2\right]+T^{2n}dZ^2 \tag{3.6}$$

Form the equation (2.21) we get

$$\rho_{WDF}=\frac{-1}{8\pi G}\left[\frac{n(n+2)a^2}{\left(1-n^2\right)T^2}+\frac{(2n+1)k_1}{T^{2(n+2)}}\right] \tag{3.7}$$

Wet dark fluid (WDF) has two components: - one behaves as cosmological constant and other as standard fluid with equation of state

$$p_{WDF}=\gamma\rho_{WDF} \tag{3.8}$$

By using (3.7) in (3.8) we get

$$p_{WDF}=\frac{-\gamma}{8\pi G}\left[\frac{n(n+2)a^2}{\left(1-n^2\right)T^2}+\frac{(2n+1)k_1}{T^{2(n+2)}}\right] \tag{3.9}$$

Special Case-Ii: $\frac{\dot{A}}{A}=\frac{\dot{B}}{B}=\frac{\alpha_1}{t^n},\frac{\dot{C}}{C}=\frac{\alpha_2}{t^n}$

We assume the solution of the system of the equation in the form

$$\frac{\dot{A}}{A}=\frac{\dot{B}}{B}=\frac{\alpha_1}{t^n},\ \frac{\dot{C}}{C}=\frac{\alpha_2}{t^n} \tag{3.10}$$

Where α_1, α_2 are constant.

Integrating equation (3.10), we get

$$A=B=\alpha_3\exp\left[\frac{\alpha_1 t^{-n+1}}{-n+1}\right] \tag{3.11}$$

$$C=\alpha_4\exp\left[\frac{\alpha_2 t^{-n+1}}{-n+1}\right] \tag{3.12}$$

Where α_3, α_4 are constant of integration.

Thus the metric (2.1) reduces to

$$ds^2=-dt^2+\exp\left[\frac{2mt^{-n+1}}{-n+1}\right]\left\{dx^2+e^{-2ax}dy^2+dz^2\right\} \tag{3.13}$$

If without loss of generality we assume that $\alpha_1=\alpha_2=m$ and $\alpha_4=\alpha_5=1$.

Form the equation (2.21) we get

$$\rho_{WDF}=\frac{1}{8\pi G}\left[\frac{a^2}{\exp\left[\frac{2mt^{-n+1}}{-n+1}\right]}-\frac{(2n+1)m}{T^{2n}}\right] \tag{3.14}$$

$$p_{WDF}=\frac{\gamma}{8\pi G}\left[\frac{a^2}{\exp\left[\frac{2mt^{-n+1}}{-n+1}\right]}-\frac{(2n+1)m}{T^{2n}}\right] \tag{3.15}$$

We assume the solution of the system of the equation (3.10) for the particle n=1, we get

$$A=B=\alpha_6 t^{\alpha_1},\ C=\alpha_7 t^{\alpha_2} \tag{3.16}$$

Where, α_6 and α_7 are constant of integration.

If without loss of generality we assume that $\alpha_1=\alpha_2=m$ and $\alpha_6=\alpha_7=1$.

Then the metric (2.1) reduces to

$$ds^2=-dt^2-t^{2m}\left\{dx^2+e^{-2ax}dy^2+dz^2\right\} \tag{3.17}$$

4. CONCLUSION

In the present study, we have investigated LRS Bianchi type –III cosmological model is studied in General Theory of relativity with the matter Magnetized wet dark energy. Einstein's field equations have been solved exactly suitable physical and kinematical parameters. Therefore, the model describes a continuously expanding, shearing and non rotating universe with big-bang.

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