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INTERIOR AND CLOSURE OF IR*-CLOSED SETS IN IDEAL TOPOLGICAL SPACES

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ABSTRACT

In continuation of the already defined IR-closed set by the author, here we study the properties of IR*-interior and IR*-closure and discuss their properties.*

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Key words: I_{g^*r}*-closed,* I_{pr} *closed set,* $int_{IR^*}(A)$, $cl_{IR^*}(A)$.

1. INTRODUCTION

In 1945 R.Vaidhyanathaswamy[12] introduced the notion of Ideal topological space. The behavior of many generalized closed sets in this space forms an interesting area of study and so many topologists studied various topological concepts too. The author [9] has defined the IR*-closed set. In this paper the properties of interior and closure of this set is studied. Also its relation to other generalized sets is compared.

2. PRELIMINARIES

An ideal I on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies (i) $A \in I$ and $B \subset A$ implies $B \in I$ and (ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$. Given a topological space (X, τ) with an ideal I on X and if $\rho(X)$ is the set of all subsets of X, a set operator $(.)^*: \rho(X) \rightarrow \rho(X)$, called a local function of A with respect to τ and I is defined as follows: $A \subseteq X$, $A^*(I, \tau) = \{x \in X \mid U \cap A \notin I \text{ for every } U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau \mid x \in U\}$. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(I, \tau)$, called the *- topology, finer than τ is defined by $cl^*(A) = A \cup A^*(I, \tau)$. Given a topological space (X, τ) with an ideal I on X, cl*(A) and int*(A) will denote the closure and interior of A in (X, τ*).When there is no chance for confusion, A* is substituted for A* (I, τ).

A subfamily of the power set $\rho(X)$ is said to be closed set A in (X, τ) if X –A is open set. Let (X, τ) be a topological space and $A \subset X$. Then interior of A in (X, τ) defined as $\cup\{U: U \subseteq A, U \in \tau\}$ and it is denoted as int(A). Then closure of A in (X, τ) defined as $\cap\{F: A \subseteq F, X - F \in \tau^c\}$ and it is denoted as cl(A). A subset A of (X, τ) is said to be regular open if A = int(cl(A)) and regular closed[7,10] if A = cl(int(A)) and the set of all regular closed sets is denoted as RC(X). A subset A of a space (X, τ) is called regular semi-open set if for every regular open set U, such that $U \subset A \subset cl(U)$.

Definition 2.1: A subset A of an ideal topological space (X, τ, I) is called a

1. Pre_I^*-closed[4] if $cl^*(int(A)) \subset A$
2. IR-closed set [1] if A = cl*(int (A)) and the class of all IR-closed sets is denoted by $r_i^{**}C(X)$. The intersection of all IR-closed sets containing A is denoted by [9] $r_i^{**}cl(A)$.
3. IR-open set[1] if A = int*(cl (A)) and is denoted by $r_i^{**}O(X)$.
4. IR*-closed set [9] if $r_i^{**}cl(A) \subset U$, whenever $A \subset U$ and U is a regular semi-open set.
5. weakly I_{rg}-closed set[7] if $(int\, A)^* \subset H$, whenever $A \subset H$ and H is a regular open set in X.
6. regular weakly closed set [11] with respect to the ideal I(I_{rw}-closed) if $A^* \subseteq U$ whenever $A \subseteq U$ and U is a regular semi-open set.

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7. I_g-closed set [3,6] if $A^* \subset U$, whenever $A \subset U$ and U is an open set in X.
8. generalized closed set with respect to an ideal (Ig-closed) [5] if and only if cl (A)\B $\in$ I whenever $A \subseteq B$ and B is an open set.
9. $I_{\hat{g}}$-closed set [2] if $A^* \subseteq U$, whenever $A \subseteq U$ and U is a semi-open set in X.
10. weakly $I_{\pi g}$-closed set[8] if $(\text{int } A)^* \subseteq H$, whenever $A \subseteq H$ and H is a π-open set in X.

Theorem 2.2: Let (X, τ) be a topological space and $A \subseteq X$, then int (A) = X – cl (X-A).

3. SOME PROPERTIES OF IR*-CLOSED SETS

Remark 3.1: Closed sets are independent with IR*-closed sets

Example 3.2: Let X = {a, b, c, d}, τ = {X , ϕ,{b},{d},{b,d},{a,b,d},{b,c,d}}, I = {{a}, φ }.C(X)={X, φ, {a}, {c},{a,c},{a,b,c},{a,c,d}, IR*-C(X) = {X, φ ,{a,c},{b,d},{c,d}, {a,b,c}, {a,c,d},{a,b,d},{b,c,d}}.

Remark 3.3: Every regular closed set is IR*-closed set.

Remark 3.4: The converse of the above remark need not be true as shown in the following example.

Example 3.5: In Example 3.2, the set of regular closed sets, R.C(X) = {X , ϕ, {a,b,c}, {a,c,d}}

Theorem 3.6: If the regular open and regular closed sets of X coincide, then all subsets of X are IR*-closed sets.

Proof: Let A be a subset of X which is regular open such that $A \subseteq U$ and U is regular open, then $r_i^{**}cl\,(A) \subseteq r_i^{**}cl\,(U) \subseteq U$. Therefore A is IR*-closed sets.

Theorem 3.7: If A is regular open and rg-closed, then A is IR*-closed set in X.

Proof: Let U be any regular semi-open set in X such that $A \subset U$. Since A is regular open and rg-closed we have $r_i^{**}\,cl\,(A) \subset A$. Then $r_i^{**}\,cl\,(A) \subset A \subset U$. Hence A is IR*-closed set in X.

Definition 3.8: The intersection of all pre_I^*-closed -closed sets containing A is called the pre-I-closure of A and is denoted by $p_i^{**}cl\,(A)$.

Definition 3.9: A subset A of an ideal topological space (X, τ, I) is called a I_{pr} -closed set if $p_i^{**}cl\,(A) \subset U$, whenever $A \subset U$ and U is a regular open set and the set of all I_{pr}- closed sets is denoted by I_{pr}- C(X).

Definition 3.10: A subset A of an ideal topological space (X, τ, I) is called a I_{g*r}-closed set if $r_i^{**}cl\,(A) \subset U$, whenever $A \subset U$ and U is an open set.

Remark 3.11: Every IR*-closed set is
1. weakly I_{rg}-closed set.
2. I_{rw}-closed set
3. I_{pr} closed set

Example 3.12 : Let X = { a,b,c,d}, τ = {X , ϕ,{b},{d},{b,d},{a,b,d},{b,c,d}}, I ={ϕ, {a}}.
IR*-C(X) = {X, φ ,{a,c},{b,d},{a,b,c},{a,c,d},{a,b,d},{b,c,d}}.
I_{rg}-C(X) = {X, φ ,{a},{c},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c}{a, b,d},{a,c,d},{b,c,d}}
I_{rw}-C(X) = {X , ϕ,{a},{c},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}
I_{pr}-C(X) = {X, φ ,{a,b},{a,c},{b,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}

All the subsets of IR*-closed set are weakly I_{rg}-closed, $I_{rw\text{-}}$ closed set and I_{pr} –closed.

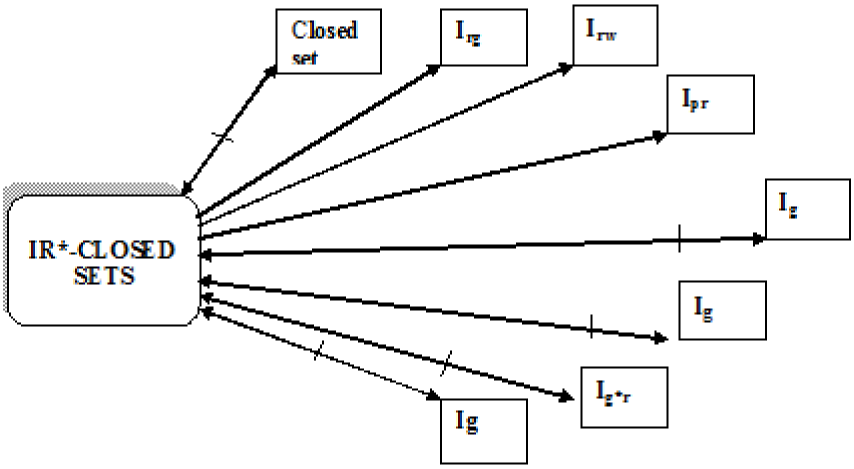
Remark 3.13: The converse of the above remark need not be true as shown in the above example .

Remark 3.14: IR*-closed sets are independent with
1. I_g-closed set.
2. generalized closed set.
3. $I_{\hat{g}}$ closed set.
4. weakly $I_{\pi g}$-closed set
5. I_{g*r}-closed set

Example 3.15: Let X = {a,b,c,d}, τ = {X , ϕ,{a},{c},{a,c},{b,c},{a,b,c},{b,c,d}},
I = {ϕ, {a}}.IR*-C(X)= **{**X , ϕ,{b},{c},{d},{a, b},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, c},{a, c, d},{a, b, d},{b, c,d}}.
I_g-C(X) = {X , ϕ,{a},{b},{d},{a, b},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}}
Ig-C(X) = {X , ϕ,{a},{a, d},{b, d},{c, d},{a, b, d}, {a, c, d},{b, c, d}}
$I_{\hat{g}}$-C(X) = {X , ϕ,{a},{d},{a, d},{b, d},{a, b, d},{b, c, d}}
$I_{\pi g}$-C(X) = P(X)
I_{g*r}-C(X) = {X , ϕ,{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}}

The subset {c} is IR*-closed set but not I_g-closed set; Ig-closed set, $I_{\hat{g}}$ closed set, I_{g*r}-closed set**.** The subset {a} is I_g-closed set; Ig closed set, $I_{\hat{g}}$ closed set and I $_{\pi g}$ but not IR*-closed set.

Figure 3.16: The above remarks are diagrammatically represented below.



4. IR*- OPEN SETS

Definition 4.1: The union of all IR-open set contained in A is denoted by r_i^{**} int(A).

Definition 4.2: A subset A in X is called IR*-open if A^c is IR*-closed in X.

Theorem 4.3: A subset A of X is said to be an IR*-open set if and only if F ⊂ r_i^{**} int (A) whenever F ⊂ A and F is regular semi-open set in X.

Proof:
Necessity: Let F be a regular semi-open such that F ⊆ A. Then X – A ⊆ X – F. Since X - A is IR*- closed set, r_i^{**} cl (X-A) ⊆ X – F ⇒ r_i^{**} int (X) – A ⊆ X - F ⇒ F ⊆ r_i^{**} int (X).

Sufficiency: Let F be any regular semi-open set such that X–A ⊆ X–F and by hypothesis X – F ⊆ X – r_i^{**}int (A). Since r_i^{**}cl (X-A) = X- r_i^{**}int (A) ⊆ F. Therefore X – A is IR*-closed sets and hence A is IR*-open set.

Theorem 4.4: The finite intersection of IR*-open sets is IR*-open sets.

Proof: Let F be a regular semi-open set in X. Let A and B be IR*- open sets in X. Hence F⊂ r_i^{**} cl (A) whenever F ⊂ A and F is regular semi open and F ⊂ r_i^{**} cl (B) whenever F ⊂ B and F is regular semi open. This implies F ⊂ r_i^{**} cl (A) ∩ r_i^{**} cl (B) ⇒ F ⊂ r_i^{**} cl (A∩B) whenever F ⊂ A ∩ B and F is regular semi-open set.

Remark 4.5: The union of two IR*-open sets need not be IR*-open sets.

Example 4.6: Let X = {a, b, c, d}, τ = {X, φ,{a}, {b}, {a, b}, {a, b, c}} and I = { φ,{a}}. IR* open sets = {X, φ ,{a}, {c}, {d}, {c, d}, {b, c, d}} .Let A={a} and B={c}, A ∪ B = {a, c} is not IR*-open.

5. IR*-INTERIOR AND IR*- CLOSURE

Definition 5.1: Let (X, τ , I) be an ideal topological space and A⊂ X. Then the IR*****-interior of A denoted by int $_{IR*}$ (A) defined as int $_{IR*}$ (A) = ∪ {: F ⊆ A and F is ∈ IR*-open sets}.

Definition 5.2: Let (X, τ , I) be an ideal topological space and A$\subset$ X. Then the IR*-closure of A denoted by cl $_{IR*}$(A) defined as cl $_{IR*}$(A) = ∩{F :A ⊆F, and F is ∈IR*-closed sets}.

Theorem 5.3: For the subsets A, B of an ideal supra topological space (X, τ , I) the following statements hold:

1. A ⊆cl $_{IR*}$(A)
2. If A is IR* closed, then A = cl $_{IR*}$(A)
3. x∈cl $_{IR*}$(A) if and only if IR*-open set U containing x, A ∩ U ≠ ϕ.
4. If A⊆ B, then cl $_{IR*}$(A) ⊆cl $_{IR*}$(B)

Proof:

1. If A ⊆ IR*-C(X) ⇒ ∩(A) ⊆ ∩ (IR*-C(X)) ⇒ A ⊆ cl $_{IR*}$(A)
2. For IR*-closed sets, A = cl $_{IR*}$(A)
3. **Necessity:** Suppose that x ∈cl $_{IR*}$(A)Let U be IR*-open set U containing x such that A ∩ U= ϕ. And so, A ⊆ X \ U. But X \ U is IR*-closed and hence cl $_{IR*}$(A) ⊆X \ U. Since x ∉ X / U, we obtain x ∉ cl $_{IR*}$(A) which is contrary to the hypothesis.

   Sufficiency: If, x ∉ cl $_{IR*}$(A) then there exists a IR*-closed set F of X such that A ⊆ F and x ∉ A. Therefore, x ϵ X \ F ϵ IR*-O(X). Hence X \ F is a IR*-open set of X containing x such that (X \ F) ∩ A =ϕ . This is contrary to the hypothesis.
4. Let x ϵ cl $_{IR*}$(A) Then for all U ∈ τ (x) we have U ⊆ A. Since A ⊆ B for all U∈τ(x), U ⊆ B. This x∈ cl $_{IR*}$(A) Thus if A ⊆ B, then cl $_{IR*}$(A) ⊆ cl $_{IR*}$(A)

Theorem 5.4: For the subsets A, B of an ideal topological space (X, τ , I) the following statements hold:

1. int $_{IR*}$ (A) is the largest IR*-open set contained in A.
2. int $_{IR*}$(int $_{IR*}$ (A))= int $_{IR*}$ (A)
3. X \ int $_{IR*}$ (A) = cl $_{IR*}$(A^c)
4. X \ cl $_{IR*}$(A)= int $_{IR*}$ (A^c)
5. If A⊆ B, then int $_{IR*}$ (A) = int $_{IR*}$ (B)
6. int $_{IR*}$ (A) ∪ int $_{IR*}$ (B) ⊆ int $_{IR*}$(A ∪ B)
7. int $_{IR*}$ (A∩ B) ⊆ int $_{IR*}$ (A) ∩ int $_{IR*}$ (B)

Proof:

1. It follows directly from the definition.
2. For IR*-open sets, A = int $_{IR*}$ (A)
3. It follows directly from theorem 2.2
4. From theorem 2.2 we get cl $_{IR*}$(A)= X \ int $_{IR*}$(X-A) ⇒ X \ int $_{IR*}$(A^c). Then int $_{IR*}$(A^c) = X \ cl $_{IR*}$(A) ⇒ X \ cl $_{IR*}$(A)= int $_{IR*}$(A^c)
5. Let x ϵ int $_{IR*}$ (A).Then for all U ∈ τ (x) we have U ⊆ A. Since A ⊆ B for all U ∈ τ (x), U ⊆ B. This x ∈ int $_{IR*}$(B) .Thus if A ⊆ B, then int $_{IR*}$ (A) ⊆ int $_{IR*}$(B).
6. We know that A ⊂ (A ∪ B) and B ⊂ (A ∪ B). Then by (v) int $_{IR*}$ (A) ⊆ int $_{IR*}$(A∪B) and int $_{IR*}$ (B) ⊆ int $_{IR*}$ (A∪B). Hence int $_{IR*}$ (A) ∪ int $_{IR*}$ (B) ⊆ int $_{IR*}$ (A∪B)
7. We know that A ∩ B ⊂ A and A ∩ B ⊂ B. Then by (v) int $_{IR*}$(A ∩ B) ⊆ int $_{IR*}$ (A) and int $_{IR*}$(A ∩ B) ⊆ int $_{IR*}$ (B) Hence int $_{IR*}$(A∩ B) ⊆ int $_{IR*}$ (A) ∩ int $_{IR*}$ (B).

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