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EXPONENTIAL GROWTH OF MATHEMATICAL SCIENCES: AN ANALYSIS

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ABSTRACT

It is well known that mathematics is the oldest of all sciences and its evolutionary stages are related to the different stages of creativity of civilization. Mathematics has covered a long path right from the primitive process of counting to the abstract axiomatic method of modern times. It is constantly expanding into new areas of investigation and has been instrumental for the development of physical, biological, social and other sciences. Scholars have been endeavouring in the past to locate as to how and at which rate did mathematics grow and develop? It still requires further elaboration. The present paper is an attempt in this direction.

Keywords: *Pure mathematics; Applied mathematics; Modern mathematics; Abstract axiomatic method; Mathematical modeling.*

AMS Subject Classification: *Primary 01A32.*

1. INTRODUCTION

It is universally accepted that mathematics is the oldest of all sciences and its evolutionary stages are related to the different stages of creativity of civilization. It is constantly expanding into new areas of investigation and has been instrumental for the development of physical, biological, social and other sciences. The present paper is an attempt to locate as to how and at which rate did mathematics grow and develop. After discussing history of development of mathematics in brief, the paper has dealt with some factors responsible for tremendous changes in the mathematical sciences especially in the later part of the 20th century. It also includes some data available so far in respect of increase in number of Ph.D. scholars, number of books and research journals, number of research papers, number of mathematical applications in other branches of learning, computer revolution and information technology leading to rapid development. These and such other evidences lead us to conclude that mathematics has been growing at an exponential rate and as such adventurous developments are likely to take place in mathematical sciences in the present century.

2. HISTORY OF DEVELOPMENT OF MATHEMATICS

It is universally accepted that mathematics is the oldest of all sciences. Astronomical observations and calculations based on them are as old as man's perception of his surroundings. With the advent of the human race there developed an opportunity for mathematics to show itself more consciously. Mathematical concepts like (Geometry), number and measure (Astronomy) originated in its early stage. The evolutionary stages of mathematics are related to the stages of creativity of civilization. Mesopotamia, Egypt, China, Greece and India took the lead in the development of mathematics. The two main streams number and form had their beginnings in India and Greece respectively. The Arabs picked them up and finally transmitted these principles to other parts of the world as early as in the 12th century A.D. Besides Greek scholars, Aryabhata, Brahmagupta, Mahavira, Bhaskara and other Indian mathematicians, Wang-Hsiao-Tang (Chinese mathematician of the 7th century A.D.), Alkhowarizmi (9th century) and other Arab mathematicians, Fibonacci (13th century), Leonardo Da Vinci (15th century) and several others contributed a lot to the development of early mathematics. By 1600 A.D. a new growth of science was well under way. Towards the end of the 17th century, calculus was developed independently by Newton and Leibnitz and co-ordinate Geometry was introduced by Rene Descartes. The discovery of calculus is considered as one of men's greatest intellectual achievements. During the 18th and 19th centuries brilliant mathematicians like Euler, Jacob, Gauss, Abel, Lagrange, Dedekind, Cantor, Weierstrass, Riemann and several others led to astonishing conclusions and opened the way for modern mathematics.

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Two of the most notable developments in modern mathematics have been the Set Theory and Symbolic logic. A third development-group theory abets both set theory and symbolic logic. It is recognized as the greatest art of mathematical abstraction. The outstanding characteristic of modern mathematics is its extensive use of abstract axiomatic method. Topology, one of the major mathematical conquests of the 20th century is closely related to algebra and analysis and influences practically every other branch of mathematics. Matrices have become indispensable in various applications of mathematics and can be used in group theory, population dynamics, ecology, genetics, navigation etc. Similarly theories of integration revolutionized by H.L. Lebesgue carried the subject far beyond Cauchy and Riemann into the realm of abstract spaces. Bole, Hilbert, Einstein, Neumann, S. Ramanujam and other's contributions are epoch-making. One of the amazing features of the 20th century mathematics has been its recognition of the power of abstract approach. This abstraction develops the faculty of logical thinking and enhances the intellectual capacity of an individual.

Mathematics is a two-sided coin and both sides of the coin are equally fascinating. Pure mathematics is a great intellectual adventure in which we discover beautiful patterns in number and space and then create patterns in Nature and Society and get these insights into physical, biological and social sciences which will be impossible to obtain without mathematics. Theoretical progress opens new fields of application and in turn applications lead to new problems and fruitful results. There is continuous flow of ideas from one sub-discipline to other and each supplements the other. Consequently studies of both – pure and applied aspects of mathematics have been felt essential and as such courses of some of the applied branches, viz. Bio-Mathematics, Operation Research, Numerical Analysis, Industrial Mathematics etc. are being introduced. Similarly mathematical modeling of large-scale systems has been of great assistance to modern science and technology. Besides mathematical modeling of weather conditions, ground water resources, railway networks, environmental pollution, educational systems, industrial systems etc., modeling of natural calamities, viz. earth quake, hailstorms, flood, Tsunami, etc., is being prepared so that these events may be minimized and sufferings of human being may be reduced. Graph theory and Stochastic processes have also their applications in these areas.

The present century is a century of revolutions in knowledge. It is the age of electricity and electronics, airspace and interplanetary travels, computers and communications, satellites, missiles and atom bombs and lasers and super conductors which have opened up an altogether different concept of development, but these cannot be achieved without mathematics. Mathematics has been centrally involved in these revolutions of social, natural and technological sciences besides revolution within mathematics itself. Mathematics is a continuously undergoing evolution and has been growing very fast. Areas of research have not only increased in number, but also in highly specialized nature. These researches have been instrumental for development in Industry, Oceanography, Astro-physics, Biological sciences, Engineering, etc. and thus have made tremendous impact on the development of a nation. In fact mathematics is widely used in almost all subjects under physical, biological and earth sciences. It is also used in Economics and other social sciences as well as in computer and information sciences. Basically a computer is only a dumb but fast moron. It does repetitive operations with the speed of light. It can manipulate algorithmic operation, it differentiates and integrates, but mathematicians are still required for preparing computer algorithms for solving newer problems. Thus mathematics grew and developed in different ages according to the need and mental development of human beings[1].

Mathematics is constantly expanding into new areas of investigations and works with new concepts. It is the only branch of learning in which major theories of two thousand years ago are still valid, yet never before has been such a flood of fresh ideas. So long as mankind continues to think mathematically, mathematical achievements will continue to grow. In fact developments in other sciences create new types of mathematics and thus mathematics has to grow at least at the same rate at which other sciences grow. Physical, biological, social, defence and management scientists know that revolutional developments in mathematics have greatly influenced the growth of their own subjects and the growth in their subjects have been responsible for the rapid development in mathematics. Mathematics has always thrived on unsolved problems. In fact today, we have enough unsolved problems in mathematics to keep all the mathematicians busy for several years to come. Here it may be noted that problem of squaring the circle, Fermat's last theorem, Zeno's paradoxes, Riemann's hypothesis and such other conjectures inspired scholars for several centuries and paved the path for further developments in mathematical sciences. Fermat's last theorem (1601-65) ($x^n + y^n = z^n$ has no solution for $n > 2$) remained a problem for almost three hundred years and continued to motivate mathematicians till it was solved in 1994 by Prof. Andrew Wiles of Cambridge and Princeton Universities. Similarly there is another important conjecture called Goldback's conjecture (1690-1764) $n = p_1 + p_2$ i.e. any even number greater than 2 is the sum of two prime numbers $p_1 + p_2$ which is yet to be cracked.

3. FACTORS RESPONSIBLE FOR CHANGES IN THE MATHEMATICAL SCIENCES

In the words of Prof. J.N. Kapur, the expression "Mathematics To-day" implies that Mathematics is a function of time and that mathematics today can be different from "mathematics yesterday" and also from "mathematics tomorrow"[2]. It is obviously true since mathematics doubles in a period of about ten years. If we had one unit of mathematical sciences in 1990, we had one thousand by the end of 20th century and we are likely to have two thousand units by the end of first decade of the 21st century. Whatever criterion is adopted, i.e. whether we consider the number of students, number of teachers, number of Ph.D. scholars, number of books written or even the funds spent on mathematics education, in

every case the numbers have been increasing abundantly. In India the explosion of the growth of mathematical sciences has not been less spectacular especially in quantitative terms, but in qualitative terms our contribution is far from satisfactory[3]. However, following are some of the facts which may give an idea of exponential growth of mathematics[4]:

(i) Number of P.D. Theses

In the USA and Canada the number of Ph.Ds. in mathematics during the entire period up to 1955 was 3,696. From 1956 to 1962 an additional 2,056 degrees were awarded. The target has been to produce 1000 Ph.Ds. every year. Almost similar is the case in other countries of the world including India. The number of Ph.Ds. produced in India as per survey conducted by the I.S.C.A. was one during the 1901-10. One in 1911-20, eleven in 1921-30, fourteen in 1931-40, thirty nine in 1941-50, one hundred seventy six in 1951-60, eight hundred seventeen in 1961-70, fifteen hundred twenty six in 1971-80, thirty five hundred in 1981-90 and about five thousand by the end of the 20th century. However, in the first three decades of the 20th century only one university in the country, Calcutta University was producing Ph.D. theses. Today about two hundred fifty universities have been awarding the degree of Ph.D. and thus the number of Ph.Ds. produced every year has been increasing fast.

(ii) Number of branches of mathematics

As a branch grows, it breaks into sub-branches each of which later becomes as big as the original branch and then it further divides. The total number of branches thus increases in a geometrical progression, viz. Algebra→ Modern Algebra → Theory of groups, rings, fields, matrices, etc.

(iii) Number of books

The fast rate at which new books on mathematics from elementary school to research monograph are appearing seems to indicate the doubling of the number of books in ten years.

(iv) Number of research journals

There were about 500 research journals, in mathematics by the end of the 20th century and at least two dozen new research journals are being started every year. Moreover, the pages of the existing journals are also increasing.

(v) Number of research papers

The number of research papers reviewed in Mathematical Reviews (USA) was 6,064 in 1951, 7,874 in 1955, 13,297 in 1965, 28,399 in 1975, 38,901 in 1985 and so on. In India during the period of 50 years from 1912-62 only 823 papers were published, while the number of papers was 1,477 during 1963-72 and so on.

(vi) Pages of new research

About 1,00,000 new pages of research pages of research papers were written every year in the last decade of the 20th century while 15,00,000 new pages are likely to be written in the next ten years.

(vii) Number of research mathematicians

There were about 10,000 mathematicians listed in the world directory of mathematicians published a few years ago. The number is increasing by at least ten percent per year.

(viii) Number of applications

In the 19th century mathematics was applied mainly to physics and engineering. Now these are using at least five times as much as in the 19th century. Besides these two subjects, social, biological and management sciences, etc. have all become highly mathematical. Almost every month a new application is found to be used.

(ix) Influence of computer on mathematics

The computer revolution has led to rapid development in different branches of learning including mathematics. Today mathematics used by computers can handle vastly more complex problems for Nature and society than it could do even thirty years ago. Computers and Information Sciences may change mathematics and mathematics education beyond recognition.

4. CONCLUSIONS

These and such other evidences lead us to conclude that mathematical sciences have been growing at an exponential rate and doubling in a period of about ten years. This is indication of the rapid intellectual growth of the human beings. Without mathematics many developments in our times, such as, nuclear power, supersonic flight, reaching out to Moon and Mars and other perplexing complexities of social and national problems would not be possible. On the whole, adventurous developments are likely to take place in mathematical sciences in the first decade of the 21st century and onwards.

REFERENCE

1. Cf. different books on History of Mathematics.
2. Cf. J.N. Kaur, Fascinating world of mathematical sciences, Vol. I, Delhi, 1955, 53ff.
3. Mathematicians of our country have not yet own any Fields Medal (considered to be equivalent of Nobel Prize). Number of mathematical books and journals published here is very few in comparison to other countries. Similarly a very few seminars and conferences are organized to motivate research scholars. There are such other reasons also to show that we are lagging far behind other countries in the field of mathematics, but efforts are now being made by NBHM, UGC and other Government agencies to place India on the map of World-Mathematics, by helping in the establishment and development of mathematical centers, giving financial assistance to research scholars and also by identifying and nurturing the mathematical talent among students at a young age.
4. It may be explained mathematically in the following manner:
   Let M be the amount of mathematics at any time and M_0 be the amount of mathematics at some initial time (say, 1900). The hypothesis we postulate is that the rate of increase of mathematics at any time is proportional to the amount of mathematics at that time,

   i.e. $\frac{dM}{dt} \propto M$

   $\therefore \frac{dM}{dt} = kM$

   where, *k* is a constant.

   Solving it for

   $M = M_0 e^{k\tau}$

   so that the amount of mathematics increases exponentially.

   If it becomes double in a period of time T,

   $2M_0 = M_0 e^{k\tau}$

   $\therefore e^{k\tau} = 2$

   $\therefore T = \frac{1}{k}\log_e 2$

   Thus the doubling period depends upon *k* and can be decreased by increasing *k*.

   Cf. J.N. Kapur, op. cit., 56.

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