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SPLITTING GRAPH ON EVEN SUM CORDIAL LABELING OF GRAPHS

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ABSTRACT

In this paper, we investigate the splitting graph of the family of bipartite graphs, paths and cycles are even sum cordial graphs and proved several classes of graphs such that $P_m(+)\overline{K_n}$,$(\overline{K_n} \cup P_m) + 2K_1$,$\langle W_n^{(1)}, W_n^{(2)} \rangle$, $B_{n,n}$,$S(B_{n,n})$,*Helm graph* H_n *and Flower graph* Fl_n *are even sum cordial graphs.*

Keywords: *Splitting graphs, Helm graphs, Flower graphs, Bistar.*

2010 Mathematics Subject Classification: *05C78.*

1. INTRODUCTION

The origin of graph labeling can be attributed to Rosa[7].In [1],Cahit introduce the concept of cordial labeling of graph. For splitting graph we refer E.Sampath kumar and H.B.Waliker[8]. For different splitting graphs we refer R.Lawrence Razario raj[5, 6]. In [9, 10], Vaidya, et.al., introduced the concepts of Helm, Flower and Bistar graphs are divisor cordial labeling. In this paper, we investigate the splitting graph of the family of bipartite graphs, paths and cycles are even sum cordial graphs and proved several classes of graphs such that $P_m(+)\overline{K_n}$,$(\overline{K_n} \cup P_m) + 2K_1$,$\langle W_n^{(1)}, W_n^{(2)} \rangle$, $B_{n,n}$,$S(B_{n,n})$, Helm graph H_n and Flower graph Fl_n are even sum cordial graphs.

2. PRELIMINARIES

Definition 2.1 [3] Let $G = (V, E)$ be a simple graph and $f: V \to \{1,2,3 \dots . |V|\}$ be a bijection. For each edge uv, assign the label 1 if $f(u) + f(v)$ is even and the label 0 otherwise. f is called an even sum cordial labeling if $|e_f(0) - e_f(1)| \le 1$, where $e_f(1)$ and $e_f(0)$denote the number of edges labeled with 1 and number of edges labeled with 0 respectively. A graph with an even sum cordial labeling is called an even sum cordial graph.

Proposition 2.1: [3] Any path is an even sum cordial graph.

Proposition 2.2: [3] Any cycle C_n is an even sum cordial graph except $n = 6,6 + d, 6 + 2d, \dots$ when $d = 4$.

Definition 2.2: [6] A wheel graph W_n is a graph with $n + 1$ vertices formed by connecting a single vertex to all the vertices of n cycle.

Definition 2.3: [9] The Helm graph H_n is the graph obtained from a wheel W_n by attaching a pendent edge to each rim vertex. It consists three types of vertices:

- an apex of degree n
- n vertices of degree 4
- n pendent vertices.

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Definition 2.4: [9] The Flower graph Fl_n is the graph obtained from Helm H_n by joining each pendent vertex to apex of the Helm H_n . It consists three types of vertices:

- an apex of degree 2n
- n vertices of degree 4
- n vertices of degree 2.

Definition 2.5: [6] Consider two copies of graph G namely G_1 and G_2.Then the graph $G^{'} = \langle G_1, G_2 \rangle$ is a graph obtained by joining the apex vertices of G_1 and G_2 by a new vertex.

Definition 2.6: [10] The Bistar $B_{n,n}$ is a graph obtained by joining the two copies of $K_{1,n}$ by an edge is called bistar graph.

Definition 2.7: [5] The graph $P_m(+)\overline{K_n}$ is a graph with the vertex set $V(G) = \{u_i, v_j, 1 \leq i \leq m, 1 \leq j \leq n\}$ and the edge set $E(G) = \{u_i u_{i+1}, u_1 v_j, u_m v_j, 1 \leq i \leq (m-1), 1 \leq j \leq n\}$

Definition 2.8: [4] The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is another graph G_3 ie.,$G_3 = G_1 \cup G_2$ whose vertex set $V_3 = V_1 \cup V_2$ and the edge set is $E_3 = E_1 \cup E_2$.

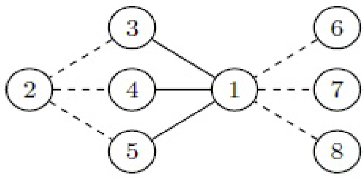
Definition 2.9: [4] The joint $G_1 + G_2$ of G_1 and G_2 consists of $G_1 \cup G_2$ and all lines joining V_1 with V_2.

Definition 2.10: [8] **(Splitting Graph)** For each vertex v of a graph G, take a new vertex $v^{'}$, join $v^{'}$ to all vertices of G adjacent to v. The graph $S(G)$ thus obtained is called splitting graph of G.

3. MAIN RESULTS

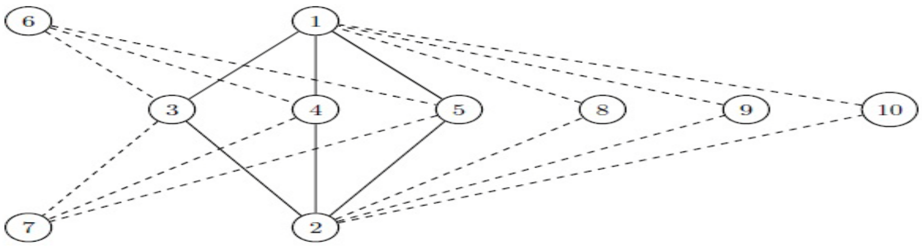
Proposition 3.1: The graph $S(K_{1,m})$ is an even sum cordial graph.

Example 3.1: $S(K_{1,3})$



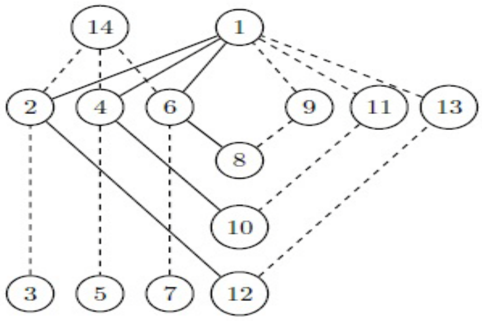
Proposition 3.2: The graph $S(K_{2,m})$ is an even sum cordial graph.

Example 3.2: $S(K_{2,3})$



Proposition 3.3: The graph $S(K_{1,n,n})$ is an even sum cordial graph.

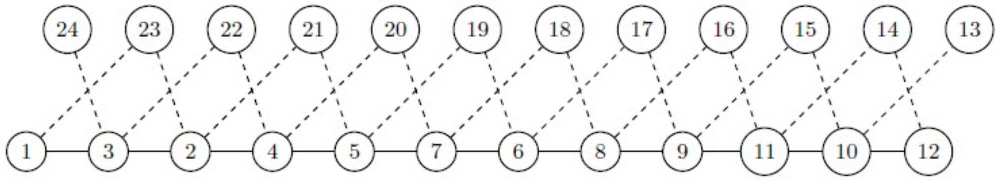
Example 3.3: $S(K_{1,3,3})$



Proposition 3.4: The graph $S(P_n)$ is an even sum cordial graph except n is multiple of 11.

Proof: Let G be a graph $S(P_n)$ Let $v_1, v_2, \dots, v_n$ be the vertices of P_n.By Proposition 2.1. we construct the even sum cordial path P_n. Then add the vertices $v_1', v_2', \dots, v_n'$ to the graph P_n.Assign the labeling for the vertices in the following way. Define $f: V(G) \to \{1,2,3,\dots,2n\}$ as follows $f(v_{n-i}') = n + i + 1, 0 \leq i \leq n-1$ and $|V(G)| = 2n, |E(G)| = 3(n-1)$. Now, we construct the splitting graph $S(P_n)$ by definition 2.10. Thus $|e_f(0) - e_f(1)| \leq 1$. Hence G is an even sum cordial graph.

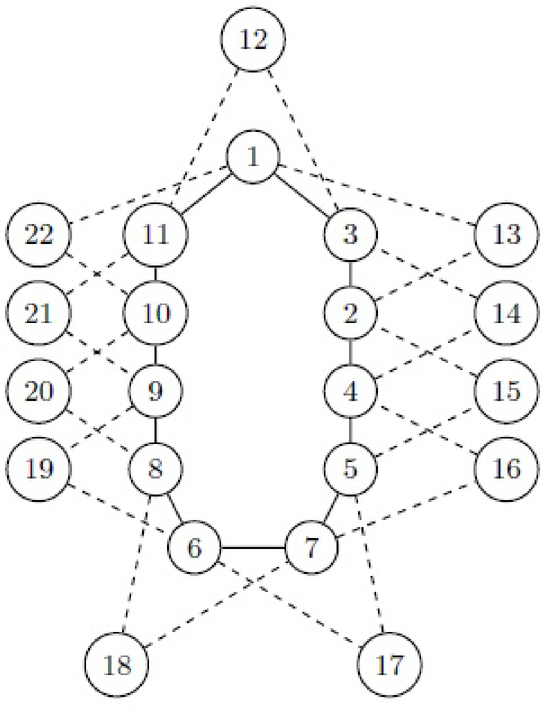
Example 3.4: $\boldsymbol{S(P_{12})}$



Proposition 3.5: The graph $S(C_n)$ is an even sum cordial graph except $n = 6, 6 + d, \ 6 + 2d,$ when $d = 4$.

Proof. Let G be a graph $S(C_n)$ Let $v_1, v_2, \dots, v_n$ be the vertices of C_n. By Proposition 2.2. we construct the even sum cordial cycle C_n. Then add the vertices $v_1', v_2', \dots, v_n'$ to the graph C_n. Assign the labeling for the vertices in the following way. Define $f: V(G) \to \{1,2,3,\dots,2n\}$ as follows $f(v_i') = n + i, 1 \leq i \leq n$ and $|V(G)| = 2n, |E(G)| = 3n$. Now, we draw the splitting graph $S(C_n)$ by definition 2.10. Thus $|e_f(0) - e_f(1)| \leq 1$. Hence G is an even sum cordial graph.

Example 3.5: $\boldsymbol{S(C_{11})}$



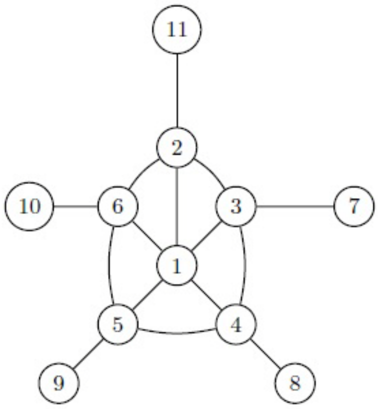
Proposition 3.6: The graph H_n is an even sum cordial graph.

Proof: Let G be a graph H_n. Let $x, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n$ be the vertices of H_n. Here x is the apex vertex of degree n, let $v_1, v_2, \dots, v_n$ are the vertices of degree 4 and $u_1, u_2, \dots, u_n$ are the pendent vertices. Assign the labeling for the vertices in the following way. Define $f: V(G) \to \{1,2,3,\dots,2n+1\}$ as follows $f(x) = 1, f(v_i) = i + 1, 1 \leq i \leq n, f(u_k) = n + k + 1, 1 \leq k \leq n$ and $|V(G)| = 2n + 1, |E(G)| = 3n$. We draw the graph H_n from the wheel graph W_n by attaching a pendant edge to each rim vertex. Then we get

$$\begin{cases} e_f(0) = e_f(1), & \textit{when } n \textit{ is even}; \\ e_f(0) = e_f(1) + 1, & \textit{when } n \textit{ is odd}. \end{cases}$$

Thus $|e_f(0) - e_f(1)| \leq 1$. Hence G is an even sum cordial graph.

Example 3.6: (H_5)

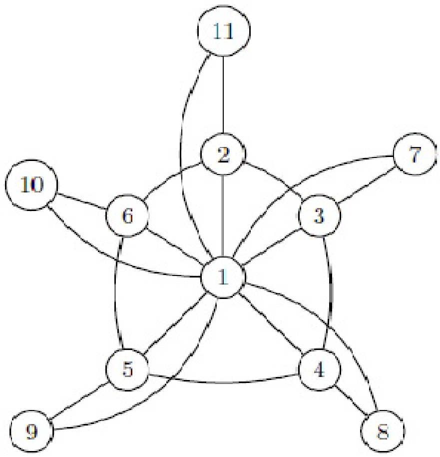


Proposition 3.7: The graph Fl_n is an even sum cordial graph.

Proof: Let G be a graph Fl_n. Let $x, v_1, v_2,$ be the vertices of Fl_n. Here x is the apex vertex of degree $2n$, $v_1, v_2, \ldots, v_n$ are the vertices of degree 4 and $u_1, u_2, \ldots, u_n$ are the vertices of degree 2. Assign the labeling for the vertices in the following way. Define $f: V(G) \to \{1,2,3,\ldots,2n+1\}$ as follows $f(x) = 1$, $f(v_i) = i + 1, 1 \leq i \leq n$, $f(u_k) = n + k + 1, 1 \leq k \leq n$ and $|V(G) = 2n + 1, |E(G)| = 4n$. We draw the graph Fl_n from the graph H_n by jining each pendant vertex of H_n to the apex vertex of H_n by an edge. Then we get $e_f(0) = e_f(1)$.
Thus $|e_f(0) - e_f(1)| \leq 1$.

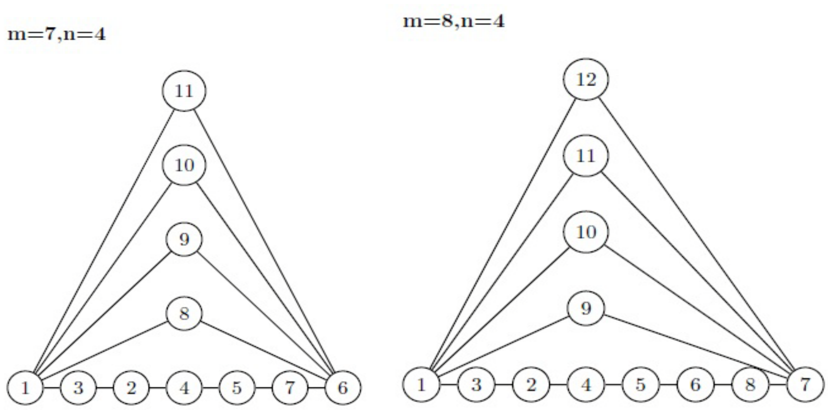
Hence G is an even sum cordial graph.

Example 3.7: (Fl_5)



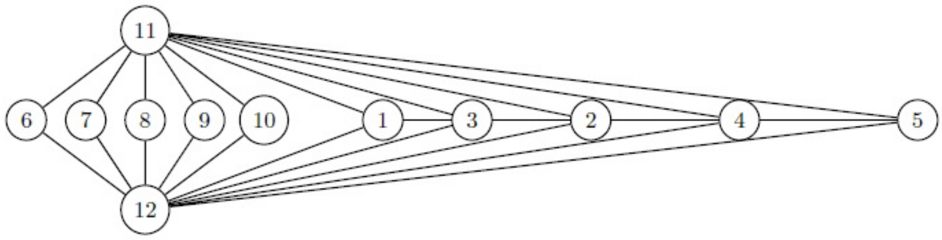
Proposition 3.8: The graph $P_m(+)\overline{K_n}$ is an even sum cordial graph.

Example 3.8:



Proposition 3.9: The graph $(\overline{K_n} \cup P_m) + 2K_1$ is an even sum cordial graph.

Example 3.9: $(\overline{K_5} \cup P_6) + 2K_1$



Proposition 3.10: The graph $\langle W_n^1, W_n^2 \rangle$ is an even sum cordial graph.

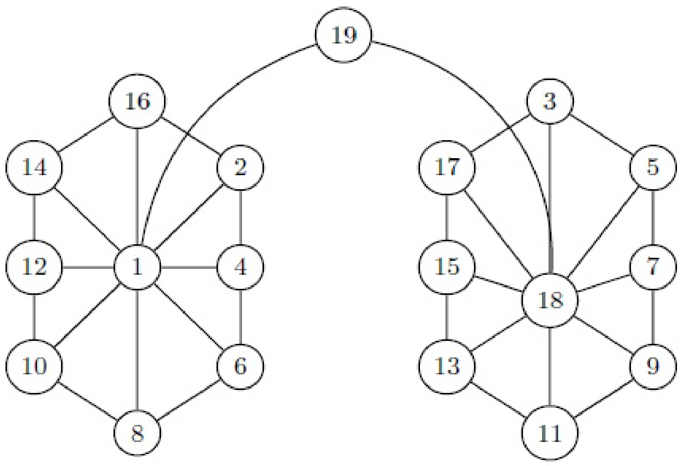
Proof: Let G be a graph $\langle W_n^1, W_n^2 \rangle$.Let $u, v, u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n$ be the vertices of G. Let $u_1, u_2, \ldots, u_n$ be the rim vertices of W_n^1 and $v_1, v_2, \ldots, v_n$ be the rim vertices of W_n^2.

Let u and v be the apex vertices of W_n^1 and W_n^2 respectively and x be a common vertex of W_n^1, W_n^2. Then $|V(G)| = 2n + 3, |E(G)| = 4n + 2$.

Define $f: V(G) \to \{1,2,3, \ldots .2n + 3$ as follows; Construction of W_n^1. Wheel graph W_n^1 is a graph with $n + 1$ vertices formed by connecting single vertex u to all the vertices $u_1, u_2, \ldots, u_n$ of n cycle. Assign the label of W_n^1 by $f(u_i) = 2i, 1 \leq i \leq n$ and $f(u) = 1$.

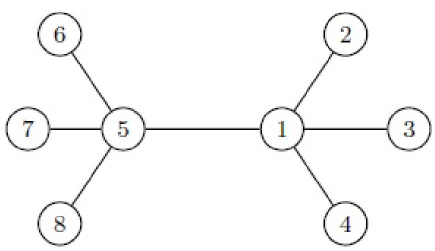
Construction of W_n^2. Wheel graph W_n^2 is a graph with $n + 1$ vertices formed by connecting single vertex v to all the vertices $v_1, v_2, \ldots, v_n$ of n cycle. Assign the label of W_n^2 by $f(v_i) = 2i + 1, 1 \leq i \leq n$ and $f(v) = 2n + 2$ and $f(x) = 2n + 3$. Then connect the vertices u and v to the vertex x by an edge. Then we get $e_f(0) = e_f(1)$.Thus $|e_f(0) - e_f(1)| \leq 1$. Hence G is an even sum cordial graph.

Example 3.10: $\langle W_8^1, W_8^2 \rangle$



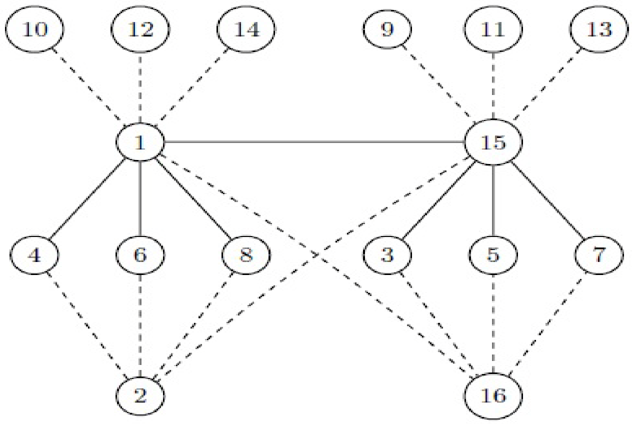
Proposition 3.11: The graph $B_{n,n}$ is an even sum cordial graph.

Example 3.11:



Proposition 3.12: The graph $S(B_{n,n})$ is an even sum cordial graph.

Example 3.12:



4 . CONCLUSIONS

In this paper, we established the splitting graph of the family of bipartite graphs, paths and cycles are even sum cordial graphs and proved several classes of graphs such that $P_m(+)\overline{K_n}$,$(\overline{K_n} \cup P_m) + 2K_1$,$\langle W_n^{(1)}, W_n^{(2)} \rangle$, $B_{n,n}$,$S(B_{n,n})$, Helm graph H_n and Flower graph Fl_n are even sum cordial graphs.

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